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Cohesive-frictional interface model for timber-concrete contacts

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1. Introduction

In civil engineering, the use of hybrid structures is common nowadays. One example are timber-concrete composites (TCC), for which a comprehensive review can be found in Yeoh et al. (2011). While structural behaviour has been widely studied over the years, interface behaviour between timber and concrete has gained a limited attention and not much is known of it even up to date. In modelling the structures, bonding and friction between the components is usually neglected, assuming a frictionless contact. Although this may be reasonable considering that their effects are limited or diminish over the service life of the structure, at least for the authors’ knowledge, there has not been studies systematically focusing on this matter. In modelling the connections, simple Coulomb friction model is usually assumed to account the interface behaviour as in e.g. Dias et al. (2007), Bedon and Fragiacomo (2017) and Mai et al. (2018). However, as it can be seen from Möhler and Herrröder (1979) and Jaaranen and Fink (2020), there may be considerable differences between static and kinetic friction, friction softening may occur over increasing slip and significant initial bonding can exist between the materials if fresh concrete is cast on timber. Neglecting any of these phenomena may lead to unrealistic results in simulations or misinterpretation of test results. To the authors’ knowledge, there has been only one study where a more detailed interface model for timber-concrete contacts has been presented in Suárez-Riestra et al. (2019), where they utilised friction model with so-called “variable coefficient of friction” in modelling tests of TCC connection with perforated plates. However, the friction coefficient is defined unconventionally as a function of ultimate load and current time step, and it seems hard to utilise the approach in a general case. For other materials, cohesive-frictional interface models are often used to model fracture, debonding or delamination and subsequent frictional behaviour. Few examples are fibre-concrete interface behaviour (Nian et al., 2018), masonry walls (Parrinello et al., 2009) and delamination of laminated composites (Parrinello et al., 2016). Despite the active research on the topic, it seems there exists no suitable interface models capture the experimentally observed behaviour of timber-concrete, especially combined variation between static and kinetic friction and friction softening, which was the motivation for developing a interface model especially for timber-concrete contacts.

In this paper, a 2-dimensional cohesive-frictional interface model that can capture the relevant behaviour in timber-concrete interfaces, is presented. The model is based on observations from a set of friction tests conducted on LVL-to-concrete specimens. The paper starts with a short description of the tests and a discussion about the experimentally observed behaviour, followed by the modelling assumptions and complete mathematical formulation of the interface model. Its numerical implementation is discussed and an approach for model parameter setting is
presented. The model has been tested in various cases and verified by comparison on a set of 27 tests on timber-concrete contact pairs under cyclic loading with varying normal pressure and multiple different material pairs, which are presented. Finally, applicability and limitations of the model are discussed along with alternative formulation suitable for modelling timber-to-timber contacts.

2. Cohesive-frictional interface model

2.1. Experimentally observed behaviour

The friction tests were conducted using small-scale double-shear specimens with varying normal stress levels, grain orientation, material (LVL or birch plywood) and surface treatment under cyclic load. For clarity, only the LVL specimens are considered here. This set consisted of 27 specimens in total; nine with LVL surface treatment tested parallel to the grain, nine with LVL surface treatment tested perpendicular to the grain and nine without surface treatment tested parallel to the grain. Test samples consisted of three parts, a wood centre piece and concrete grout side pieces as shown in Fig. 1a. The test pieces were manufactured by casting the concrete directly on wood and the testing was conducted 28–32 days after casting. The test setup is schematically illustrated in Fig. 1b. Normal stress on the specimen was applied by clamping steel plates on both sides of the specimen and adjusting the tension in two tie-rods connecting the plates. During the test, the side pieces were fixed vertically in place by the supports, while prescribed displacement $u_L$ was imposed on the centre piece via loading rod running through the centre piece and fixed to its both ends. The loading consisted two full load cycles, first between $-2.5$ and $2.5$ mm and second between $-5$ and $5$ mm. During the test, force in the loading rod $F$, relative displacement between pieces $u$ and normal force $N$ were recorded, and corresponding stresses were calculated by contact areas measured after the test. Further details on the experimental investigations can be found in Jaaranen and Fink (2020).

Based on the common features observed in all the tests, stress-displacement behaviour of timber-to-concrete contacts under cyclic load can be characterised by initial stress peak in the beginning loading, gradual degradation of the initial peak, smooth transition from sticking to slipping at load reversals, static friction peak after each load reversal, smooth transition from static to kinetic friction over certain sliding distance, interlocking effect when passing the initial position and friction softening over increasing cumulative displacement. These features are illustrated in Fig. 2 with stress-displacement response of three different samples under different normal stresses.

From the observed features, the initial stress peak, and its degradation, is contributed to initial bonding of the interface and de-bonding after the strength of the bond is exceeded. De-bonding is a non-smooth process with large differences between individual specimens. In contrast, transition from static to kinetic friction, softening over cumulated slip and frictional behaviour in general are rather smooth processes with considerably smaller differences between specimens. The data revealed also pressure-dependency of the shear stiffness; the higher the normal stress, the stiffer the shear response is during the load reversals, i.e. the slope of the stress-displacement curve at the reversals increase with increasing normal stress (see Fig. 10 for detail). The pressure-dependency was also observed in the beginning of the test, i.e. before the failure of the initial bond.

2.2. Assumption in the model

The interface model is formulated based on an empirical basis, aiming to replicate observations from the friction tests, with some
additional assumptions considering the micromechanical behaviour of rough contacting surfaces. The main emphasis of this section is on the tangential behaviour. In the normal direction, no test data were available. Therefore, simple linear relationships between normal stress and displacement are used to keep the number of needed parameters at minimum.

Generally, two stages can be identified in all the friction tests: initial bonding and frictional, after the bond has failed. The initial bonding, failure of the bond and following purely frictional behaviour can be captured by frictional-cohesive models, e.g. Alfano and Sacco (2006). The chosen cohesive-frictional framework allows relatively free choice of the softening behaviour for the cohesion. The de-bonding of the interface is approximated by smooth softening until the pure friction phase is reached. A linear-cubic traction-separation law was chosen for its good agreement with the test results. It should be noted that interlocking was considered to have only limited significance to overall behaviour and is neglected in the model. However, actual initial stress peak arises likely from combined effect of interlocking and cohesion; and effect of interlocking is implicitly included, lumped together with actual cohesion. A review on interlocking and related modelling aspects can be found in Stupkiewicz and Mróz (2001).

It was noted in Jaaranen and Fink (2020) that both, transition from static to kinetic friction and overall softening of the frictional response, can be closely approximated by exponential functions. Therefore, this type of transition and softening was implemented to the current interface model.

While tangential behaviour under constant stress, including initial bonding and friction, is derived completely on an empirical basis as described, additional micromechanical consideration are made to identify suitable format for sticking behaviour and response under varying normal stress. It is assumed that the contact surfaces are rough and principles for rough contacts apply. A fundamental feature of rough surface is that when they are brought together, they will touch only at small number of protruding asperities. These contacting asperities form the real area of contact that is usually much smaller than the apparent contact area (Persson, 1998). Due to deformability of the material, the real area of contact and number of asperity contacts increases as the normal stress increases. Contacting asperities form junctions that resist tangential motion, and if tangential displacement is applied, shear stresses are generated due to asperity deformations.

In the experimental investigations, pressure-dependency was also observed during initial bonding. A possible explanation for this is incomplete bonding. In Fryborg et al. (2012), process of moisture transfer between wood strands and cement matrix with respect to its effects on the bonding, is discussed. When fresh cement paste comes to contact with dry wood, there is exchange of moisture between the materials. Initially, wood absorbs water from the cement paste and swells, and during the curing, cement will draw back moisture for hydration, shrinking the earlier swollen wood, breaking bonds and partially detaching the materials from each other. The hydration is also interfered by this process weakening the interface. Similar effects are also assumed to occur in timber-concrete contacts considered here, leading to interface only partially bonded and rest filled cavities or weak cement matrix zones.

Implications of these micromechanical features are accounted in the model by following assumptions:
- under higher contact pressure, more asperities are in contact, thus leading to higher shear stiffness
- under constant tangential displacement and increasing contact pressure, more asperities come to contact but shear deformation of the asperities remain constant, leading to virtually constant shear stress
- under constant tangential displacement and decreasing contact pressure, asperity contacts separate gradually, leading to gradual unloading of the shear stress down to zero at zero contact pressure
- in stick phase, under increasing tangential displacement, asperity contacts gradually detach after reaching their shear strengths, leading to apparently smooth transition from sticking to slipping
- initially bonded interface contains cavities that close under pressure, leading to increasing contact stiffness with increasing contact pressure.

It should be noted that at least second and third property are standard for the most interface models. This list of assumptions is treated as rules that need to be satisfied, while related formulation is chosen empirically. In the presented interface model, all these effects, except the last one, are accounted by adjusting the tangential slip. Mathematical details and reasoning for the choice are described in Section 2.3. List of symbols is provided in Table 1.

2.3. Mathematical formulation of the model

2.3.1. General

The interface model presented here is based on the framework presented by Alfano and Sacco (Alfano and Sacco, 2006), where two parallel models, cohesive and frictional, are combined to obtain total response from the interface. In their approach, a representative elementary area (REA) of the interface is assumed to consist of undamaged (cohesive) and damaged (frictional) parts. Initially REA is undamaged and the response is fully cohesive. With increasing relative displacement, larger portion of the REA becomes damaged, leading to fully frictional interface after maximum

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Damage parameter</td>
</tr>
<tr>
<td>$G_c, G_h$</td>
<td>Cohesive fracture energy in normal and shear modes</td>
</tr>
<tr>
<td>$K_N^f$</td>
<td>Frictional interface normal stiffness</td>
</tr>
<tr>
<td>$K_{Nc}, K_{Nf}$</td>
<td>Cohesive interface normal stiffnesses in compression and in tension</td>
</tr>
<tr>
<td>$K_{Tc}, K_{Tf}$</td>
<td>Cohesive interface and frictional interface tangential stiffnesses</td>
</tr>
<tr>
<td>$K_{Tmax}$</td>
<td>Maximum frictional interface tangential stiffness</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Pressure-dependency coefficient for friction</td>
</tr>
<tr>
<td>$p_{iso}, p_{ref}$</td>
<td>Reference pressures for varying friction and tangential stiffness</td>
</tr>
<tr>
<td>$\xi_k, \xi_t$</td>
<td>Relative normal and tangential displacements of the contact surfaces</td>
</tr>
<tr>
<td>$\omega_{0, t, o}$</td>
<td>Normal and tangential displacements at onset of damage</td>
</tr>
<tr>
<td>$\xi, \xi^0$</td>
<td>Cumulative slip, reference value for cumulated slip</td>
</tr>
<tr>
<td>$\xi^e$</td>
<td>Tangential elastic displacement</td>
</tr>
<tr>
<td>$\xi^s$</td>
<td>Total tangential slip</td>
</tr>
<tr>
<td>$\xi^t$</td>
<td>Tangential slip related to smooth transition from sticking to slipping</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Effective separation</td>
</tr>
<tr>
<td>$\omega_{0, t, o}$</td>
<td>Effective separation corresponding to onset of damage</td>
</tr>
<tr>
<td>$\omega_{el}$</td>
<td>Effective separation corresponding to complete separation</td>
</tr>
<tr>
<td>$x_{max}$</td>
<td>Evolution variable for static to kinetic transition</td>
</tr>
<tr>
<td>$x_{s}, x_{k}$</td>
<td>Relative residual kinetic and static friction coefficients</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>$\mu_{k}, \mu_t$</td>
<td>Kinetic friction coefficient, static friction coefficient</td>
</tr>
<tr>
<td>$\mu_{k}, \mu_t$</td>
<td>Initial friction coefficient, initial static friction coefficient</td>
</tr>
<tr>
<td>$\sigma_{s0}, \sigma^i$</td>
<td>Total normal stress, cohesive normal stress, frictional normal stress</td>
</tr>
<tr>
<td>$\sigma_{o}$</td>
<td>Cohesive normal strength</td>
</tr>
<tr>
<td>$\tau, \tau^c, \tau^f$</td>
<td>Total shear stress, cohesive shear stress, frictional shear stress</td>
</tr>
<tr>
<td>$\tau_{0}$</td>
<td>Cohesive shear strength</td>
</tr>
<tr>
<td>$\theta, \phi$</td>
<td>Interface model parameter, vector of interface model parameters</td>
</tr>
<tr>
<td>$\mu, \nu$</td>
<td>Macaulay brackets, $\mu = 0$ if $x &lt; 0, \nu = x$ otherwise</td>
</tr>
</tbody>
</table>
damage has been reached. In their model, the total response is obtained by

\[
\sigma = (1 - D) \left\{ \begin{array}{l}
\sigma^u \\
\tau^u
\end{array} \right\} + D \left\{ \begin{array}{l}
\sigma^s \\
\tau^s
\end{array} \right\}
\]  

\hspace*{1cm} (1)

The damage parameter \(D\) indicates the ratio of damaged area to total area in the REA and damage evolution law in the model. Superscripts \(u\) and \(d\) refer to undamaged and damaged parts, respectively.

In Eq. (1) the model has completely different behaviour depending whether the interface is fully undamaged or fully damaged. Initially, prior to any damage, the behaviour is fully cohesive with constant shear stiffness. The pressure-dependency of the initial interface shear stiffness, contributed to incompletely bonded interface, could be potentially included by assuming some initial damage level. However, to avoid introducing additional parameters, a simplified formulation, which approximates effects of incomplete bonding, is chosen here. In the formulation, frictional interaction is always active and bonding stiffens the interface. To consider this, Eq. (1) is modified to Eq. (2), where the frictional stresses are always active regardless of the current damage state, with modified superscripts referring to cohesive and frictional stresses

\[
\sigma = (1 - D) \left\{ \begin{array}{l}
\sigma^c \\
\tau^c
\end{array} \right\} + \left\{ \begin{array}{l}
\sigma^f \\
\tau^f
\end{array} \right\}
\]  

\hspace*{1cm} (2)

Commonly, positive sign is used for separation and tensile stresses in the interface models. However, opposite sign convention is adopted throughout this paper for consistency with the sign convention in the numerical implementation of the model.

2.3.2. Cohesive stresses

The cohesive stresses in Eq. (2) are obtained from Eq. (3) with stiffness \(K_c^u\) in Eq. (4) that allows different stiffness in tension and compression.

\[
\sigma^c = \left\{ \begin{array}{l}
\sigma^c \\
\sigma^c
\end{array} \right\} = \left[ \begin{array}{cc}
K_c^u & 0 \\
0 & K_c^t
\end{array} \right] \left\{ \begin{array}{l}
\sigma^c \\
\sigma^s
\end{array} \right\}
\]  

\hspace*{1cm} (3)

\[
K_c = \begin{cases}
K_{cc} & \text{if } \sigma^c \geq 0 \\
K_{ct} & \text{if } \sigma^c < 0
\end{cases}
\]  

\hspace*{1cm} (4)

2.3.3. Damage parameter

In Alfano and Sacco (2006), the damage evolution is controlled by bilinear traction-separation laws in pure modes and mixed-mode case is accounted by linear interaction criterion. The formulation they used, poses interdependency between pure modes such that only part of the parameters are independent. For the interface model presented here, it is assumed that the normal behaviour is controlled by the cohesive bond, whereas in tangential behaviour involves interaction between cohesion and asperity interlocking. Thus, stiffness, strength and fracture energies can be all different in normal and tangential directions. In order to allow independent parameters in different modes, mixed-mode formulation in Lorenzis et al. (2013) was adopted, although presented with slightly different notation here and replacing linear softening law by a cubic softening law.

Calculation of the damage is based on a concept of effective separation, where uniaxial damage evolution law is utilised with effective separation variables \((\bar{u}_m, \bar{u}_s, \bar{u}_d)\) is Eqs. (5)–(7). Variable \(\bar{u}_{md}\) is related to onset of damage according to quadratic strength criterion and variable \(\bar{u}_{mf}\) to complete failure based on linear interaction criterion for fracture energies. These particular criteria were chosen for simplicity, since data for mixed-mode interaction for timber-concrete interfaces were not available.

\[
\bar{u}_m = \sqrt{\left(\frac{u_0}{K_{m0}}\right)^2 + u_t^2}
\]  

\hspace*{1cm} (5)

\[
\bar{u}_{mf} = u_m u_{to} \sqrt{\frac{\left(\frac{u_0}{K_{m0}}\right)^2 + u_t^2}{(u_0/n_{to})^2 + u_t^2}}
\]  

\hspace*{1cm} (6)

\[
\bar{u}_{mf} = \frac{2}{u_{m0}} \left(\frac{K_{m0} u_0^2}{n_{to}} \right)
\]  

\hspace*{1cm} (7)

Using the effective separation variables, damage corresponding to current displacement state according to the cubic damage evolution law can be obtained from Eqs. (8)–(9).

\[
D = \begin{cases}
0 & \text{if } u_m < u_{m0} \\
D^* & \text{if } u_{m0} \leq u_m < u_{mf} \\
1 & \text{if } u_m \geq u_{mf}
\end{cases}
\]  

\hspace*{1cm} (8)

with

\[
D^* = 1 - \frac{\left(\frac{K_{m0} u_0^2}{n_{to}} \right)}{u_m^2 + u_t^2}
\]  

\hspace*{1cm} (9)

The damage is considered an irreversible process and attained damage remains even during unloading. In the presented model, this is considered by defining current damage as the maximum attained damage over the whole load history as

\[
D(t) = \max \left(\frac{D(t)}{\bar{D}(t)}\right) \forall t \in [0, T]
\]  

\hspace*{1cm} (10)

2.3.4. Frictional stresses

The frictional stresses in Eq. (2) are obtained from

\[
\sigma^f = \left\{ \begin{array}{l}
\sigma^f \\
\sigma^f
\end{array} \right\} = \left[ \begin{array}{cc}
K_{f0}^t & 0 \\
0 & K_{f0}^t
\end{array} \right] \left\{ \begin{array}{l}
\sigma^f \\
\sigma^f
\end{array} \right\}
\]  

\hspace*{1cm} (11)

The tangential part of Eq. (11) is equivalent to classical plasticity with decomposition of total tangential displacement into elastic and slip parts such that

\[
u_t = u_t^e + u_t^f
\]  

\hspace*{1cm} (12)

In the Section 2.2, the assumption that shear deformation appears only in the asperities that are in contact, was made. Consequently, the frictional shear stress is generated only in these contacting asperities. Now, assuming that the elastic tangential deformation \(u_t^e\) reflects average asperity deformation in a unit area, the shear stress is \(\tau^t = K_{f0}^t \max u_t^e\), when is \(K_{f0}^t\) the asperity stiffness per unit area. The rate of change \(du_t^e/du_t\) would be higher the more asperities are in contact, and consequently, rate of change \(\tau^t/du_t\), i.e. apparent shear stiffness, would be also higher. Following this reasoning, it seems suitable to adjust the shear stiffness of the interface by adjusting \(du_t^e/du_t\) (hence also \(du_t^e/du_t\)) rather than adjusting the stiffness \(K_{f0}^t\) directly, which would also lead to possibility of increase of shear stress by increasing contact pressure alone without any tangential displacement, which is against the assumption that shear stress remains constant under increasing normal stress if tangential displacement is constant.

Gross sliding in the interface is initiated if the frictional shear stress reaches the frictional strength. The initiation is controlled by the slip criterion:
\[ f_s = |\tau'| - \mu_s \tau \]

The criterion was formulated in a general form as mentioned in Wriggers (2006) to incorporate the effects of transitioning from static to kinetic friction, slip softening over cumulated slip and pressure-dependency. These effects are embedded in the variable friction coefficient given by Eq. (14). Variable \( \alpha \) controls the transition from static to kinetic friction. The softening over cumulated slip is controlled by so-called consistency defined by Eq. (17). Furthermore, when the slip criterion is active, cumulated slip is controlled by Eq. (15) where the friction coefficients decay exponentially towards their residual values as the cumulated slip \( u_s \) increases. Pressure-dependency of the friction is accounted by \( k_p \), which monotonically decreases with increasing normal stress. It should be noted that formulation for the slip softening and the transition from static to kinetic friction are analogous to isotropic and kinematic hardening rules in elastoplasticity, respectively.

\[
\mu = \left[ \frac{\mu_s + \mu_k}{2} - \frac{\mu_s - \mu_k}{2} \cdot \frac{\alpha}{\sigma_{\text{max}}^2} \sgn(\tau') \right] k_p
\]  

(14)

with

\[
\mu_s = \mu_{s,0} \left[ \sigma_{\text{fit}} + (1 - \sigma_{\text{fit}}) \exp \left( -\frac{|\tau'|}{C_0} \right) \right] \\
\mu_k = \mu_{k,0} \left[ \sigma_{\text{fit}} + (1 - \sigma_{\text{fit}}) \exp \left( -\frac{|\tau'|}{C_0} \right) \right]
\]  

(15)

\[
k_p = \frac{p_0}{\sigma^2 + p_0}
\]  

(16)

When the surfaces are in contact, i.e. \( u_N > 0 \), evolution of the slip and other internal variables is controlled by the slip rule and evolution laws in Eqs. (17)–(23).

\[
du_s = du_{s,1} + dt \frac{df}{d\tau}
\]

\[
d\alpha = \left[ 1 - \frac{\alpha}{\sigma_{\text{max}}} \sgn(du_s^t) \right] dt
\]  

(17)

\[
du_s^t = |du_s^t|
\]  

(18)

\[
d\gamma > 0, f_s \leq 0; d\gamma f_s = 0
\]  

(19)

\[
dt_{u_{s,1}} = \left[ 1 - \left( 1 - \frac{\tau}{\mu^s \sigma^t} \right) \left( \frac{K_{s,1}}{K_{s,\text{max}}} \right) \right] dt_t
\]  

(20)

\[
\tau^* = K_{s,\text{max}} (u_s - u_s^t) \sgn(dt_t)
\]  

(21)

\[
\mu^* = \left[ \frac{\mu_s + \mu_k}{2} - \frac{\mu_s - \mu_k}{2} \cdot \frac{\alpha}{\sigma_{\text{max}}^2} \sgn(dt_t) \right] k_p
\]  

(22)

\[
K_1^* = K_{s,\text{max}} \frac{\sigma^t}{\sigma^t + p_{\text{ref}}}
\]  

(23)

Changes in the tangential slip, variable \( \alpha \) and cumulated slip are defined by Eq. (17). Furthermore, when the slip criterion is active, evolution of the gross slip is controlled by so-called consistency parameter \( \gamma \). Kuhn-Tucker loading–unloading conditions and consistency condition in Eqs. (18)–(19) ensure that the shear stress never exceeds the frictional shear strength and \( d\gamma > 0 \) is zero if the shear stress is below the frictional shear strength. The term \( du_{s,1}^t \) in the first line of Eq. (17) accounts the varying shear stiffness and smooth transition from sticking to slipping by adjusting the total tangential slip \( u_s^t \). It is defined in Eq. (20) with its terms defined in Eqs. (21)–(23) and its magnitude can vary between zero and \( du_t \) depending on the ratio between shear stress and frictional strength as well as normal stress level. It is easier to see the role of Eq. (20) when it is rewritten in terms of elastic displacement \( u_s^c \) as in Eq. (24).

\[
du_s^c = \left( 1 - \frac{\tau}{\mu^s \sigma^t} \right) \left( \frac{K_{s,1}}{K_{s,\text{max}}} \right) dt_t
\]  

(24)

From this it is obvious, that when the ratio between shear stress and frictional strength increases or the normal stress decreases, the apparent shear stiffness \( d\tau^c / du_s^c \) decreases since it is directly proportional to \( du_s^c / dt_t \). The form of Eq. (24), is analogous to the Dahl friction model, e.g. in Pennestri et al. (2015), although pressure-dependency is added here. In the Dahl friction model, the shear force \( F = \sigma_{\text{fit}} \tau \) depends on internal variable \( \tau \) presenting the bristle displacement, equivalent of what is called here the asperity shear deformation. When surfaces move away from the original position, \( \tau \) increases, asymptotically approaching limiting value where shear force is equal to the frictional strength of the contact. This leads to a smooth transition from sticking to slipping.

When the surfaces are separated, i.e. \( u_N \leq 0 \), the slip rule and evolution laws are given in Eq. (25); the slip and the internal variables are simply set to values corresponding to a stress-free state.

\[
u_s^t = u_t, \quad \alpha = 0, \quad du_s^c = 0
\]  

(25)

2.3.5. Modification to variable friction coefficient

In the interface model, the initial frictional shear stress peak is lower than the subsequent ones due to the formulation of the variable friction coefficient in Eq. (14) and transition variable in Eq. (17). Initially, the frictional peak stress corresponds to \( \mu = 1/2 \cdot (\mu_s + \mu_k) \), whereas at the following peaks, assuming adequate sliding distance prior to the load reversal has been reached, the peak stresses correspond to \( \mu = \mu_s \). It can be argued that when the initial bonding is involved, the difference between the initial and subsequent frictional shear stress peaks can be compensated by higher cohesive strength and thus, the difference has negligible effect on the overall behaviour. However, if initial bonding does not exist and initial friction peak is higher than following one, behaviour may be unrealistic. Examples with higher initial friction peak are encountered e.g. In experimental investigations with specimens that were initially detached to break the bonding (Jaaranen and Fink, 2020) or in cyclic timber-to-timber friction tests (see Section 3.5). These observations can be also related to friction softening, but nevertheless, an empirical modification to the model is presented to account the cases where the initial friction peak is higher. When the interface is initially purely frictional, a modified formulation of Eq. (14) and the second line of Eq. (17) may be more suitable:

\[
\mu = \left[ \mu_s - \left( \mu_s - \mu_k \right) \cdot \frac{\gamma_1 + \gamma_2 + 2 \gamma_3 \sgn(\tau')} \right] k_p
\]  

(26)

\[
d\alpha_1 = \left[ 1 + \frac{1}{2} \sgn(du_s^t) - \frac{\alpha_1}{\alpha_{\text{max}}} \right] du_s^c
\]

\[
d\alpha_2 = \left[ 1 + \frac{1}{2} \sgn(du_s^t) - \frac{\alpha_2}{\alpha_{\text{max}}} \right] du_s^c
\]  

(27)

With the modified formulation, also the first frictional peak stress corresponds to static friction coefficient \( \mu = \mu_s \). The modified formulation is not considerably more complicated, but it increases number of internal variables by one and requires a slightly longer code for implementation.
2.4. Model implementation

Two versions of the interface model have been implemented. In this paper, only the complete implementation of the interface model (Model A) is described. Model B follows the same principles, just excluding the non-related steps. Both models are available by request from the corresponding author:

- **Model A**: Complete interface model implementation (Abaqus UINTER subroutine (Dassault Systèmes, 2017), written in Fortran)
- **Model B**: Shear-only model treating normal stress as input and without tangent stiffness matrix calculation (Matlab script)

When used in the FEM simulation, the subroutine is called from the main program during each increment for each contact points and is provided with relative displacements at the end of the increment, corresponding displacement increment and previously stored internal variables from the end of the last increment. The subroutine needs to update stresses and internal variables as well as calculate interface tangent stiffness matrix, which are then provided back to the main program.

The implementation is summarised here as a pseudo-code in Fig. 3. Any variable with subscript \(n\) refers to values in the end of the previous increment and subscript \(n + 1\) refers to values in the end of the current increment. Update of the cohesive stresses, damage parameter, and frictional stresses if the contact is open, involve only direct calculations. However, if the contact is closed, iterative update is required for the frictional shear stresses. The update is based on predictor–corrector method (Simo and Hughes, 1998). First, the trial stress is calculated in the predictor step (line 8) assuming no gross slip occurs, i.e. \(d\gamma = 0\), and integrating Eqs. (17) over the increment \(D\Delta u_{T,n+1}\). Then the slip criterion is checked (line 9). If the frictional strength is not exceeded, values from the predictor step are taken as final values. Otherwise, the slip and other related internal variables are corrected in the corrector step (lines 13–15) utilising Newton–Raphson method to calculate increments of consistency parameter \(\tilde{\beta}\) and related internal variables until \(f_s\) is lower than the set tolerance \(TOL\). These updated values are then taken as final ones. Calculation of the tangent stiffness matrix is done analytically in the algorithm but the details are omitted here due to their length.

2.5. Parameter setting

To obtain model parameters for applications, the parameters need to be set by fitting the model to experimental data specific to the contact. Here, the parameter setting is done in Matlab (MathWorks Inc., 2020) using Model B that accounts only the tangential part of the interface model. The normal stresses are treated only as an input parameter for the model. The model calculates shear stress for each time step, given the current normal stress and tangential displacement increments as well as internal variables from previous step as inputs. The parameters are set by minimising the least squares error between the model output and the test data. This is done over a set of multiple samples simultaneously so that the pressure-dependency parameters can be approximated as well. The minimisation problem over a set of \(m\) samples is given by

![Fig. 3. Stress update algorithm. Symbol \(\beta\) denotes vector of internal variables.](image-url)
\[
\min \quad \frac{1}{m} \sum_{k=1}^{m} e_k^T
\]
\[
\text{s.t.} \quad \theta_{\text{min}} \leq \theta_i \leq \theta_{\text{max}}
\]

where \( n_k \) is number of data point and \( e^k \) is residual vector for the sample \( k \). The residual vector is defined as

\[
e^k = \tau^k_{\text{exp}} - \tau_{\text{num}}(\theta, u^k_{\text{exp}}, \sigma^k_{\text{exp}})
\]

where \( \tau^k_{\text{exp}}, u^k_{\text{exp}}, \sigma^k_{\text{exp}} \) are measured shear stresses, tangential displacements and normal stresses for the sample \( k \), respectively, and \( \tau_{\text{num}}(\cdot) \) as a vector of simulated shear stresses for given input (\( \cdot \)). The minimisation is performed using the Matlab built-in optimization solver, ‘fmincon’. Robustness of the approach was tested by repeating the procedure multiple times with randomly selected initial values. At least in this case study, each run gave the same set of parameters, verifying that the global optima was obtained.

3. Model validation

The interface model is first tested for different load scenarios. In the end of this section, the model parameters are set by fitting to experimental data (Jaaranen and Fink, 2020) and it’s ability to represent the experimentally observed behaviour is evaluated.

3.1. Shear response to monotonic tangential displacement

In this first test case, the tangential response is modelled with Model B. The loads consist of monotonically increasing tangential displacement from 0 to 0.8 mm and constant normal stress \( \sigma = 2.5 \) MPa. The model parameters are: \( \mu_0 = 1, \mu_0 = 0.6, \sigma_{sT} = 0.5, \sigma_{sT} = 0.8, \sigma_{\text{max}} = 0.5 \text{ mm}, u^T_i = 25 \text{ mm}, K^f_{\text{max}} = 100 \text{ MPa/mm}, K^c_{\text{max}} = 20 \text{ MPa/mm}, \tau_0 = 1 \text{ MPa}, G_{\text{nc}} = 300 \text{ J/m}^2, p_0 = 25 \text{ MPa}, p_{\text{ref}} = 5 \text{ MPa}.

The results is shown in Fig. 4. As it can be seen, the stress increases until the point where the cohesive damage starts accumulating, after which the cohesive stress degrades down to a point where the initial bond has completely degraded, and the response is fully frictional. The model performs as expected.

One important note can be made here: based on selected value \( \mu_0 = 1 \), one would expect to see frictional peak stress \( \sigma_{sT} = (\mu_0 \sigma + \sigma) \approx 2.25 \text{ MPa} \). However, in the present model, the initial frictional stress peak is always lower due to the formulation of variable friction coefficient, as was already discussed in Section 2.3.5.

3.2. Shear response to cyclic tangential displacement

In this second test case, Model B is used. The loads consist of ten tangential displacement cycles between –2.5 and 2.5 mm with constant normal stress \( \sigma = 2.5 \) MPa. The model parameters are same as in the monotonic load test case. The shear response is illustrated in Fig. 5. It shows a large initial peak related to cohesive strength, after which the shear stress tends towards the residual level cycle by cycle. Model response agrees with the assumed cyclic behaviour with slip softening, frictional shear stress tending monotonically towards residual level as the slip accumulates.

![Fig. 4. Initial evolution of total shear stress \( \tau \), cohesive shear stress \( \tau_c \), frictional shear stress \( \tau_f \) components and damage parameter \( D \) over increasing tangential displacement \( u^T \) from the initial position.](image1)

![Fig. 5. Evolution of shear stress under cyclic tangential load of ten cycles and constant normal stress.](image2)

![Fig. 6. Shear stress response under a single load cycle with three different normal stress levels.](image3)

![Fig. 7. Comparison of pure frictional response of the original and modified interface model formulations with identical parameters.](image4)
Effect of the contact pressure on the shear stiffness is demonstrated in Fig. 6. The shear response over a single load cycle has been plotted for three different normal stress levels; 0.5 MPa, 1 MPa, and 2.5 MPa. Amplitude of the loading has been reduced to 1 mm to show the slopes at the beginning and at the load reversals clearly. By observation, it can be seen that the shear stiffness increases with increasing contact pressure, fulfilling the requirement for pressure-dependent shear stiffness. It can be also seen that in the beginning the relative effect to the stiffness smaller due to pressure-independent cohesive component affecting the stiffness.

Additionally, to demonstrate the difference between the original and modified formulations (Section 2.3.5), pure frictional shear response under two full load cycles of the both versions is illustrated in Fig. 7 and corresponding variation of the friction coefficients over cumulative slip are illustrated in Fig. 8. The model parameters are the same as in the previous test cases, except: \( \tau_0 = 0 \), \( G_{IIc} = 0 \). In the modified version, the initial peak stress is now clearly highest compared to the following peaks, whereas in the original version, the first peak is lower than the subsequent one. The difference appears only in the part before the first load reversal, and both versions have identical response after the friction coefficients have stabilised. As an additional note: the modified version is more general in the sense that initial friction coefficient can be adjusted by setting the initial values of \( \alpha \) to any value between 0 and \( \alpha_{\text{max}}/2 \), practically interpolating between the original and modified version of the formulation, which is also illustrated in Fig. 8 by modified formulation response with initial values \( \alpha_{\text{max}}/2 \).

3.3. General response to predefined normal and tangential displacements

In the third test, the interface model A was tested under a predefined load protocol with normal and tangential displacement varying in multiple steps. The simulation was done in Abaqus with a model consisting of an elastic block sliding on a rigid surface with the displacements imposed to the contacting surface of the block directly to eliminate effect of deformation in the block from the results. Purpose of this test case was to verify that the interface model complies with the assumptions made in Section 2.2 by comparing load and response step-by-step (loads and corresponding responses are illustrated in Fig. 9):

1. Loading, unloading and loading in normal direction (no tangential displacement): The normal stress follows normal displacement linearly and is non-zero also in the tensile region since the initial bond is still intact, as is expected.
2. One load cycle in tangential direction (constant normal displacement): There is an initial stress peak, and after the bond fails, the shear response is frictional behaviour with static friction peaks, transition to kinetic friction and overall slip softening over cumulated slip as expected.
3. Unloading and loading in normal direction (no tangential displacement): The normal stress response is similar to the first step, except that zero normal stress is observed in the tensile region due to cohesive bond already completely broken during the second step. Shear stress also decreases to zero as normal stress goes to zero, as expected.
4. One load cycle in tangential direction (constant normal displacement): The shear stress response is similar to the second step, except that no bonding stress peak is observed since the cohesive bond is already broken; the shear response is fully frictional as it is supposed. Also, overall shear stress and the static friction peaks are lower due to increasing slip softening as expected.
5. Additional loading in normal direction (no tangential displacement): The normal stress increases linearly with increasing normal displacement while shear stress remains constant as was intended in the model formulation.
6. One load cycle in tangential direction (constant normal displacement): As the tangential displacement increases, the shear
stress increases up to the level corresponding to current normal stress and after that cyclic frictional response is observed, as expected.

3.4. Model parameter setting and comparison with the experimental investigations

The interface model parameters were set by fitting to the experimental data using the procedure described in Section 2.5. Three test sets from Jaaranen and Fink (2020), treated LVL parallel to grain and treated LVL perpendicular to grain as well as untreated LVL parallel to the grain, were considered. The interface model parameters obtained from fitting are given in Table 2. In two cases, pressure-dependency parameter has value of $p_0 = 100$ MPa, which was upper limit for the parameter. This means that these cases display very limited pressure-dependency on the friction within the applied contact pressure range.

Tangential stress-displacement responses of the interface overlaid with the experimental results are shown in Fig. 10a for treated LVL parallel to the grain and in Fig. 11 for treated LVL perpendicular to the grain and untreated LVL parallel to the grain, respectively. The model shows generally a good agreement with the tests data in all the cases. The transition from static to kinetic friction is captured and the assumed exponential softening seems to agree well with experimental behaviour. The softening over cumulated slip is only moderate in the test data of treated specimens, but it can be seen best by observing decreasing static friction peaks over the load cycles. In the untreated LVL, the softening behaviour can be seen very clearly between subsequent load cycles. The model can capture softening, and no large differences are seen. The largest discrepancies between the model and the experimental data are related to the initial stress peaks. However, this is expected due to large strength variation and non-smooth failure of the initial bond in the experiments in contrast to the smooth de-bonding in the model. In addition, with the untreated LVL, in the transition from the initial bonding to pure friction the model displays slightly different behaviour compared to the experiments. In overall, the difference is small, but for this case different type of traction-separation law for bonding would likely to produce better results. Another clear difference in behaviour can be seen at zero displacements where the model displays a smooth response in contrast to fluctuations seen in the test data. This is due to the effects of interlocking were neglected in the model. Additionally, the initial peak stress seems to be overestimated under lower normal stresses and underestimated under higher normal stresses in the case of treated LVL loaded perpendicular to the grain (Fig. 11). A potential reason for this is ignoring the interlocking, which would introduce, if included explicitly, pressure-dependency to the initial bonding strength.

The interface model can capture the transition from sticking to slipping as well as pressure-dependency of the frictional shear stiffness (see close-up of the first load reversal in Fig. 10b). The interface model response agrees well with test data in this region. This points out that the varying stiffness, smooth transition and their interaction can be captured within the chosen framework.

3.5. Applicability and limitations of the interface model

Comparison between the interface model and the test results showed that the model behaves as intended and can also represent the tangential behaviour under cyclic load from initiation to the end of the test with a good accuracy. However, despite the good agreement with the experimental investigations, further studies are needed in the future to fully validate the model, for example in cases with varying normal stress, different types of cyclic loads, different material combinations and loading including separation of the surfaces. Additionally, predictive capability of the model has not been validated by testing the model against experiments with different loading or normal stresses than that were used for parameter setting. These validation however, are beyond the scope of this study.

The interface model is rate-independent, thus effects like variation of the friction coefficient under different load rates can not be accounted. However, it is unlikely to have major importance for modelling timber-concrete in practice as long as the interface tests...
have been carried out under load rate of similar magnitude. Neglecting the interlocking phenomenon means that in certain load cases the response would not be realistic. For example, if a contact with matching surfaces would be separated in normal direction and returned to its initial position, the part of the initial shear strength corresponding to the interlocking would be still present, which cannot be captured by the model.

Most of the required model parameters are not available in the literature, which may prevent use in practical design. However, when performing tests on the relevant contact pairs is feasible, the interface model can be effectively utilised. In the models involving contact behaviour that has pronounced effect on the overall response, a detailed interface model with experimentally characterised parameters can help to reduce the overall model uncertainty. Examples, where simulations with the presented interface model could utilised, can be found from recent literature. In Müller and Frangi (2018), use of micro-notches as a shear connection between timber and concrete was investigated. Simulations would offer alternative to experiments for investigating behaviour of the joint and effects of interface parameters on it. In Cao et al. (2020), bolt-connected CLT-concrete joints under cyclic loading with increasing amplitudes up to 40 mm displacement and friction was considered as one of the main dissipation mechanisms providing ductility. In this case, considerable effects from friction peaks show variation. In Claus and Seim (2018), highest shear stress is attained in the beginning of the test, while in Steiger et al. (2018), the maximum stress appears after the first load reversal (difference is small). It seems the presented modified formulation would agree better with behaviour of timber-to-timber contacts, but no decisive recommendation can be given based on the limited data and this aspect should be investigated further in the future.

4. Conclusion

In this study, a 2-dimensional interface model for timber-concrete contacts with initial bonding, varying friction coefficient and slip softening has been developed based on the empirical observation from a set of friction tests and additional micromechanical assumptions. Formulation of the model is presented in detail along with approach for setting the parameters in Matlab and a numerical implementation of the model. The interface model has been tested in several cases involving different loadings to validate model implementation and illustrate its behaviour, and it performs as intended. The model was also verified on a set of 27 tests on timber-concrete contact pairs under cyclic loading with varying normal pressure and multiple different material pairs. The interface model is able to capture relevant parts of the experimentally observed tangential behaviour, indicating suitability to present timber-concrete interface behaviour under condition that were studied. Due to large number of parameters, the model may not be well suited for practical design scenarios, but it can be used in research for simulating timber-to-concrete contacts in cases where detailed interface behaviour is important and experimental characterisation of the contact behaviour is feasible. Furthermore, due to similar nature of the cyclic friction response, it is suggested that the interface model can be also utilised for modelling frictional timber-to-timber contact behaviour with a slightly modified formulation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References


