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An interference-adjusted power learning curve for tasks with cognitive and motor elements

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ABSTRACT

Production and operations management (POM) uses learning curve (LC) models to determine the length of training sessions for new workers and predicting future task performance. Empirically validated LC parameters provide managers with quantitative information on the effects of the presumed factors behind the learning process. Previous studies considered LC to compose of cognitive and motor curves. Another widely acknowledged but only recently parameterized phenomenon in the POM field is interference, which assumes some loss of information or experience could occur over a learning session. This paper takes a logical step in this line of research by developing an interference-adjusted power LC model, a composite of cognitive and motor elements. This paper accounts for the decay of cognitive and motor memory traces from repetitions to measure the residual (interference-adjusted) experience and capture these phenomena. Three variants of the model are developed that assume power and exponential decay functions and an approximate version of the exponential one. Assembly data representing various forms of an individual learning profile have been used to test the fits of the developed models. In addition to those models, four potential models from the literature were selected for comparison purposes. The results show that the approximate model fits very well exponential learning profile. The findings highlight the confluence of the three phenomena in learning, component (cognitive/motor) learning, interference, and plateauing.

1. Introduction

Learning curve (LC) models are practical and quantitative industrial engineering tools to measure performance changes as a function of experience. The usefulness of an LC depends on how well it fits empirical data and the meaningfulness of the parameters [1]. In practice, the most often used [2] and meaningful [3] parameters are those that depict the beginning and end performances and the average speed (rate) at which an individual learns over a learning session. LCs that fit empirical data well are valued by practitioners for many reasons, e.g., setting standard times, training needs, estimating cost and production times, among other things. The literature reports that power or exponential LCs best fit learning data, e.g., [3,4].

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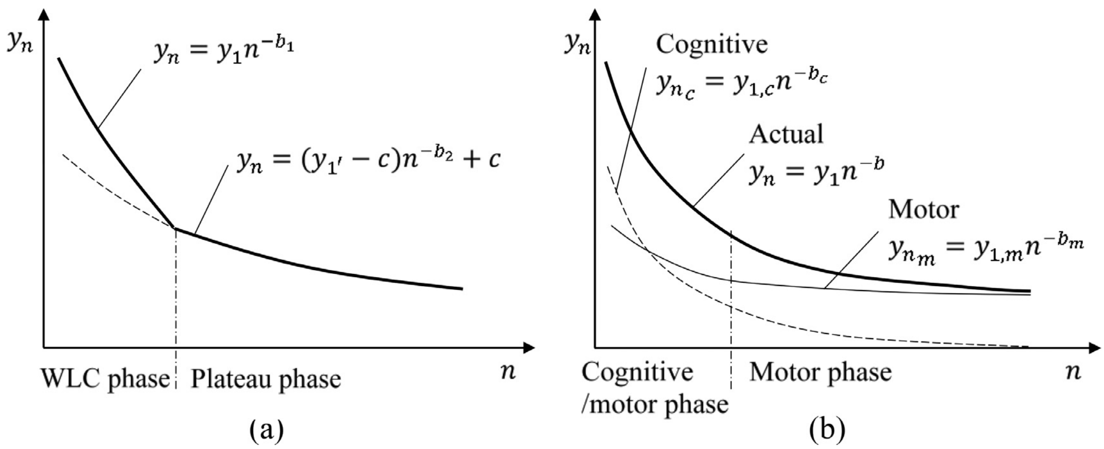

Fig. 1. (a) Hybrid and (b) Dar-El et al. [13] dual-phase LC.

In the production and operations management (POM) literature, Wright [5] was the first to observe learning in an aircraft manufacturing facility and showed that the data follow the power-law, where the unit time (cost) decreases by a constant percentage each time the output doubles. Wright's LC, henceforth WLC, is a standard on which further modifications are built [2]. In applied psychology, early empirical studies showed that quality [6] and time [7] performances follow the power law of practice in perceptual-motor tasks for individuals. However, whether the learning profile is of power or exponential form has been a debatable topic [4], often resolved on the individual learner basis (see Fig. 4). For example, Buck et al. [3] found that the exponential model better describes serial time data (ten repetitions) for about 60% of the subjects in a task that required more cognitive than motor skills.

In the POM field, LC research is most typical in labor-intensive manufacturing, such as manual assembly. Product assembly is a process of joining parts together according to predefined instructions. As such, it involves a large set of skills, both declarative and procedural, workers acquire, or learn, through repetitive cycles. The instructions provide workers with declarative knowledge (what to do and in which order) to reflect the actions to complete the task. A worker then gains procedural, or methodological, knowledge (how to do) from learning-by-doing. Each cycle is broken down into a set of sub-tasks or processes [8], with varying degrees of cognitive and motor activities [9,10], one must execute to accomplish the product. Humans can perform actions sequentially, otherwise concurrent activities interfere with each other [11]. Subsequently, learning different sub-components of a task occurs in isolation [12], whose component times are aggregated into product LC [8]. Since the 1990s, POM researchers have developed dual-phase LCs, aggregations of cognitive and motor task components, for individuals [13,14] and recently for groups of individuals [15], where all of them are of power form. The most influential WLC parameter is the learning rate, which is the average reduction speed of the unit with doubling output [5]. There is evidence that cognitive learning occurs at a faster rate than motor [9] (Fig. 2), which attracted researchers to evaluate the existence and magnitudes of cognitive and motor components based on product learning rate, e.g. [16]. Ideally, only a motor component exists at the later stage of learning, where the curve levels out or plateaus [13] (Fig. 1). The plateau effect may induce managers to exert efforts like process improvements [17] and allocating training hours [18,19]. Progress induced in the short run may appear autonomous or learning-by-doing, which involves automatic improvements resulting from sustained production over long periods [20]. In the POM field, Levy [21] was the first to distinguish autonomous and induced learning. Since then, alongside cumulative production, learning curve models assumed managerial efforts as a proxy to measure knowledge (e.g., [17,18,19,21]). However, the validation of such models requires data on learning investments, which remains an interesting topic to investigate in future work once such data become available.

Disorder and interference impede human performance. Working memory is short-term, where information items are either lost or transferred into long-term memory in about sixty seconds after being presented [12]. This information will decay if not recalled later, and the longer the recall time, it is more likely that associated memory traces decay and gets forgotten [22]. This theory of decay has dominated the forgetting literature in many fields. In the POM, researchers considered, for simplicity, forgetting occurs because of breaks [23,24]. The interference theory of forgetting, researched by psychologists, states that a subject is less likely to retrieve a "*memory image*" as time passes [22,25]. Observations showed that interference is the process of not allowing human brains to have multiple associations for the same stimulus [26]. In practical terms, learning new material interferes with retaining old ones [26], where choice from multiple responses also hampers performance [12]. Learning a task can be thought of as a process where context and attention fluctuate over time. This contextual fluctuation theory (Appendix A) provides a joint measure of memory decay and interference. Empirical studies show that interference occurs in various task types ranging from pure cognitive [22] to psychomotor [27] and pure motor [28]. For about three decades, the POM literature used the term "*continuous forgetting*" or forgetting "*throughout the learning process*" [29], and performance decay due to "*lack of training, reduced retention of skills,…and natural forgetting*" [30] to depict a phenomenon consistent with interference. Recently, Jaber et al. [31] showed that having a model capable of adjusting cognitive interference produces more accurate fits than WLC and its modification with plateau effect.

Individual learning in labor-intensive manufacturing is complex. Despite this, none of the studies in the literature provide models that integrate both components (cognitive/motor) learning and interference (decay). In this regard, Jaber and Kher [32] and Jaber and Glock [14] modified the dual-phase (cognitive/motor) LC of Dar-El et al. [13] to show that forgetting is still valid. Our study takes a logical step in this line of research by developing three variants of a dual-phase interference-adjusted power-form LC model, combinations of Jaber and Glock [14] and Jaber et al. [31] models. The first two models assume the cognitive and motor memory traces are subject to either power or exponential decay, and the third model is an approximate expression of the latter. The developed and four selected (from the literature) models are fitted to empirical assembly data. The results show that the approximate model performed well for individuals with *exponential* learning profiles, who are likely to struggle at the early learning phase but then improve quickly, which is expected for novices at complex assembly. They also highlight the confluence of three learning phenomena: components (cognitive/motor) learning, interference, and plateauing.

This paper has seven sections. Section 2 reviews the literature on the relevant LC models. Section 3 introduces cognitive and motor learning with interference. Section 4 develops new LC models that, in Section 5, are tested with selected models against assembly learning data. Section 6 discusses the results and managerial insights. Section 7 presents a summary and conclusions and draws some aspects for further research.

2. Literature review

2.1. Learning curve models

The earliest model observed in an industrial setting, the WLC [5], is of a form:

$$y_n = y_1 n^{-b} \tag{1}$$

where y_n and y_1 are the execution times for the nth and first repetitions, respectively, and $b = -\log(\emptyset)/\log(2)$ is the learning exponent, where $\emptyset = \%$ learning rate (LR) divided by 100. Wright determined $0 \leq b \leq 1$, which corresponds to an LR between 100 and 50%. The WLC advocates that y_n reduces by a constant rate ($100\% - LR$) each time the cumulative number of repetitions doubles. However, the WLC has a drawback as it tends to underestimate the early and overestimate the long-term learning performance (e.g., [13]). To overcome this drawback, the Stanford-B model [33] added a constant representing prior knowledge, an equivalent of $B \geq 0$ cycles (or units) because the same or a similar task has been performed; $y_n = y_1(B+n)^{-b}$. For example, Garg and Milliman [34] found the Stanford-B to describe the time to manufacture the Boeing 707 better than WLC. The Plateau model [35] added a constant parameter representing standard time, $c > 0$, and forced the LC to reach a realistic asymptote level above zero, and it is of the form:

$$y_n = (y_1 - c)n^{-b} + c \tag{2}$$

Many researchers have acknowledged the shape of Eq. (2) is good at representing the learning process for different task types, such as perception-motor or reaction-time [6,7], problem-solving [36], and psychomotor [37] tasks. The Plateau model was shown to accurately describe individual and group learning in manual assembly [37]. Seibel [38] found that an S-curve, formed by combining the Stanford-B and Plateau models, fits learning data from a perception-motor task very well. The S-curve is of the form:

$$y_n = (y_1 - c)(B+n)^{-b} + c \tag{3}$$

There has been a lively debate whether learning (and forgetting) is a power or an exponential function. Newell and Rosenbloom [39] found that S-curve, Eq. (3), fitted empirical learning data better than an exponential function that is of the form:

$$y_n = (y_1 - c)e^{-\alpha n} + c \tag{4}$$

Heathcote et al. [4] criticized the bulk of evidence favoring the power-form function, basing their criticism on the fact that the fits were to average data (over subjects, conditions, or practice sessions). They considered only unaveraged data and, contrary to Newell and Rosenbloom [39], advocated using an exponential function over a power function. Heathcote et al. [4] claimed that the extra flexibility (fourth parameter) in Eq. (3) could be expressed exponentially (Eq. (4)). They also proposed an LC, which is the Plateau model added with exponential decay, henceforth the Plateau-E, and it is of the form:

$$y_n = (y_1 - c)e^{-\alpha n} n^{-b} + c \tag{5}$$

where a decay exponent $\alpha \geq 0$. Note that Knecht's [40] LC is like Eq. (5) but without the constant c, making his LC eventually turn up with cumulative repetitions. Heathcote et al. [4] showed that Eq. (5) fits power and exponential learning data quite well. The model also advocates a theory acknowledged in many learning models that performance could be an aggregated measure of all the process components. Kirsner and Speelman [41] proposed dividing such processes into perceptual, premotor planning, decision-making, and motor learning and that they relate to recency and amount of practice. We select the Plateau (Eq. (2)) and Plateau-E (Eq. (5)) models for the computational study in Section 5.

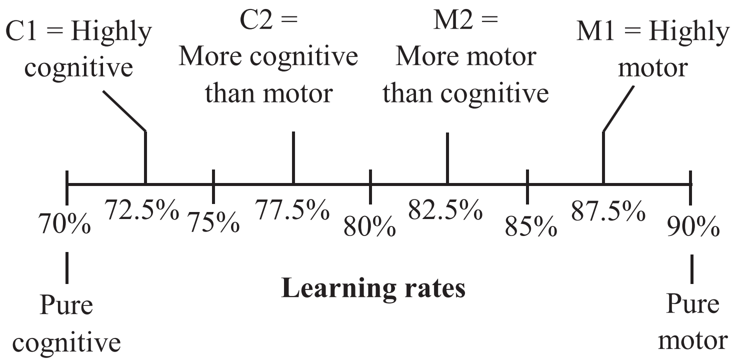

Fig. 2. Classification of learning rates [9].

The above LCs have different relative learning rates (RLRs) [4]. The exponential LC in Eq. (4) has $RLR_{EXP} = \alpha$, a constant. The Plateau in Eq. (2) and the Plateau-E in Eq. (5) have $RLR_{Plateau} = b/n$ and $RLR_{Plateau-E} = \alpha + b/n$, respectively, where the RLR values become slower with n. The ratio $RLR_{Plateau-E}/\ RLR_{Plateau} = \alpha/(b/n) + 1 > 1$, where $\alpha > 0$, increases with n, i.e., exponential decay captures an increasing steepness in learning profiles better than a power decay. The WLC and other traditional models estimate the LR and the corresponding average amount of progress for a specific number of repetitions. In reality, each repetition $i = 1,\ldots,n$ is not equally effective, e.g., [1]. This observation is noteworthy as it makes a learning profile resemble a plain saw-tooth with the largest variations for the first few repetitions. In addition to the random variations, as this suggests, some factors may interfere with learning [31]. Recent extensions of WLC tested RLR decreases over repetitions in aircraft production ([42]; $b(n) = b/(1 + n/d)$, where d is decay parameter) and in order picking ([43]; $b(n) = b + f_p + 1 - \exp(f_p(n^{f_p} - 1))$, where $f_p \in [0, 1)$, and is a fatigue accumulation index).

2.2. Dual-phase learning models

Manufacturing tasks, such as manual assembly, are naturally psychomotor, which involve both cognitive (familiarizing, sequencing, measuring, decision-making, etc.) and motor (body movements, manipulative dexterity) activities. In the POM literature, Hancock and Foulke [44] were the first to distinguish between cognitive and motor learning and used *threshold* and *conditioned learning* as terms to describe their different nature. De Jong [45] presented two reduction phases of the unit time for the beginning of manufacturing. During the first, rapid reduction phase, "*The reduction is all the greater the worse state of manufacture was at the outset and as rational measures for improvement are taken*." Then the LC follows the WLC with the learning exponent $b_1 = 0.234$ (LR = 85%) … 0.621 (LR = 65%), on average $b_1 = 0.322$ (LR = 80%), being dependent on various causes. The second phase starts when work organization and methods have stabilized, and the unit time reduces less and less with practice and progressive increasing skills. Then the LC follows De Jong's [45] model, $y_n = M + (1 - M)y_1 n^{-b}$, with M representing a factor of incompressibility or the percentage of time to perform a unit, which is not subject to improvement. De Jong [45] estimated $M = 0.05 \ldots 0.6$ (e.g., $M = 0.25$ for assembly of doors and cupboards) and $b_2 = 0.322$ (LR = 80%), and acknowledged that in some cases, for example, relatively small series of large assemblies, the second phase is not reached. De Jong's model is like the Plateau model (Eq. (2)) since $My_1 = c$, i.e., standard time, thus the Plateau model represents the second learning phase in a hybrid model in Fig. 1(a).

Dar-El et al. [13] proposed a dualistic modeling approach, where the actual LC is an aggregation of cognitive and motor WLCs, as in Fig. 1(b). The model suggests that a task's cognitive elements dominate its motor elements at the early learning phase and motor elements are subject to a minimum or standard time, whose dominance comes later in the process. Such a model can capture the curvature observed in many learning data better than a single WLC. Dar-El et al. [13] dual-phase learning model, henceforth DPLM, is of the form:

$$y_n = (y_{1c} + y_{1m})n^{-b^*} = y_{1c}n^{-b_c} + y_{1m}n^{-b_m} \tag{6}$$

where y_{1c} and y_{1m} are times at first repetitions under pure cognitive and motor conditions. Similarly, b_c and b_m are learning constants under pure cognitive and motor conditions. The learning constant combining both cognitive and motor learning is given by $b^* = b_c - \frac{\log[(R+n^{b_c-b_m})/(R+1)]}{\log(n)}$, where $R = y_{1c}/y_{1m}$, and $b_c > b^*$. Using learning data from an electric matrix board and electronic components assembly that involved one complex and another simple task, Dar-El et al. [13] approximated the values $b_c = 0.514$ and $b_m = 0.152$, which correspond to LRs of 70% and 90%, respectively. However, they acknowledged that cognitive and motor learning could differ from the presented ones.

In their subsequent study, Dar-El et al. [9] presented four categories (C1, C2, M1, M2, in Fig. 2) of LRs that are distributed evenly between pure cognitive and pure motor rate. They proposed this approach to estimate the average LR of short-cycle tasks (of 1.5 min average standard time), which would fall into one of the four categories. For example, an actual LR of 78.5% is given a C2 classification with an LR of 77.5%.

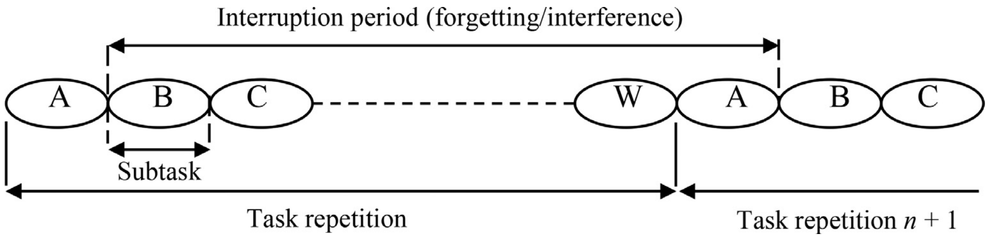

Fig. 3. Interruption period between repetitive subtask causes forgetting/interference (modified from Dar-El et al. [9]).

Classifying a short-cycle time task into one of the four categories is based on the skills required to perform it. Dar-El. et al. [9] used a questionnaire to validate that the categories are consistent with the actual LRs from various experiments. They also used data from previous studies and showed that for tasks that are highly cognitive (C1) and motor (M1), times for the first repetition are 13–15 and 2.5 times the standard time, i.e., where the LC levels out. So, there is much more to learn with highly cognitive tasks (Fig. 1). Dar-El. et al. [9] assumed that repeating a short-cycle task (of 1.5 min standard time) does not cause forgetting and suggested different assumptions for long-cycle tasks, which consists of a series of unique subtasks (equal to short-cycle tasks). Each subtask is interrupted for a period of a cycle length, causing forgetting. Fig. 3 illustrates this behavior.

Yelle [8] proposed that a product manufacturing LC is an aggregate of LCs for each product subprocess. Dar-El et al. [9] have a similar approach for a long-cycle task. More specifically, time to perform repetition n would be the sum of times to perform subtasks within each category *C1, C2, M2,* and *M1*, (without forgetting), i.e., $y_n = \hat{y}_{1_{C1}} n^{-b_{C1}} + \hat{y}_{1_{C2}} n^{-b_{C2}} + \hat{y}_{1_{M2}} n^{-b_{M2}} + \hat{y}_{1_{M1}} n^{-b_{M1}}$. When forgetting is considered, they, for simplicity, suggested slower LRs, with b_i coming b_i^f, ($b_i^f < b_i$), where i = *C1, C2, M2, M1*. They empirically determined the relationship between learning exponents with and without forgetting follows a power-function $b^f = b(q+1)^{-0.152}$, where q is the interruption period (in days).

Jaber and Glock's [14] learning curve model, henceforth JGLCM, is a modification of the DPLM, and is of the form:

$$y_n = xy_1 n^{-b_c} + (1-x) y_1 n^{-b_m} = y_1 \left[x \left(n^{-b_c} - n^{-b_m} \right) + n^{-b_m} \right] \tag{7}$$

The JGLCM differs from the DPLM in two ways. First, it has a parameter x ($0 \le x \le 1$) that splits y_1 into two components, cognitive, $y_{1,c} = xy_1$, and motor, $y_{1,m} = (1-x)y_1$. Second, b_c and b_m are fitting parameters and not inputs, as the DPLM fixes their values. Subsequently, Jaber and Glock [14] fitted four sets of cognitive and motor learning exponents, b_c and b_m, for the DPLM, with the corresponding LRs of (1) 75%, 85%, (2) 72.5%, 87.5%, (3) 70%, 90%, and (4) 67.5%, 92.5%, the best of the four fits recorded. The JGLCM fitted better than the DPLM against learning data from the assembly experiment of Bailey [46] and Bailey and McIntyre [47,48]. This good result has to do with the JGLCM ability to capture the performance of the first few repetitions with much greater accuracy. Therefore, we select the JGLCM, Eq. (7), for the computational study in Section 5. We will use the data from the same Bailey's experiments and compare the fits of the JGLCM with those of alternative models.

The only LC that considers dual-phase learning and forgetting is that of Jaber and Kher [32]. They combined the DPLM and learn-forget curve model, LFCM, of Jaber and Bonney [49] into a dual-phase learning-forgetting model, DPLFM, which assumes that forgetting occurs during breaks. They illustrated the behavior of the DPLFM numerically. Their experiment consisted of performing five learning cycles with 25 repetitions each, with two consecutive cycles separated by ten days and 0.2 days/unit as an initial performance. They also used fixed cognitive and motor LRs of 70% and 90% from [9] and assumed that time to total forgetting for cognitive components (D_c = 30 or 180 days) is shorter than that for motor components (D_m = 300 days), which corroborates previous findings that rapid learning is associated with rapid forgetting.

2.3. Forgetting/interference-adjusted models

Whether forgetting is of power or exponential form function has also taken its share of the debate [4]. Many empirical studies showed that power-form (not simple exponential) functions better describe forgetting, e.g., [50,51]. In the psychology literature, Wickelgren [50] proposed the theory of single-trace fragility that forgetting is a consequence of two-component processes, time-decay, and interference. His theory suggests that the strength of a memory trace, m, at a retention interval of t units of time is $m = Lt^{-D}e^{-It}$, where L is initial trace strength, $D \ge 0$ is an exponent for a time-decay process, and $I \ge 0$ is an exponent for the loss of memory strength due to interference (directly proportional to the similarity of target stimuli to subsequently encountered stimuli). Wickelgren's memory retention model is similar in mathematical form to Knecht's [40] upturn LC model, to which Plateau-E, Eq. (5), added a constant for a steady-state or standard time. Contrary to Wickelgren [50], Mensink and Raaijmakers [25] proposed a model that could provide a unified account of the interference and forgetting phenomena.

Jaber et al. [31] modified WLC, henceforth MWLC, by adjusting for cognitive interference. The MWLC assumes each task repetition leaves a memory trace which depletes over time [22]. A memory trace is a consequence of limited working (or short-term) memory, which cannot absorb all the information acquired. Note that as soon as the source drops from attention, its activation (trace strength) begins to decay. It further considers that those memory traces could be consolidated (strengthened). Consolidation improves performance as the number of repetitions increases. Interference thus relates to the retrieval of information from long-term memory. Jaber et al. [31] tested both power (P) and exponential (E) decays. The corresponding mathematical expressions of the MWLC-P (Eq. (8)) and the MWLC-E (Eq. (9)) are given by:

$$y_n = y_1 n_e^{-b} = y_1 \left(\sum_{i=1}^{n} (t - t_i)^{-\beta} \right)^{-b} \tag{8}$$

$$y_n = y_1 n_e^{-b} = y_1 \left(\sum_{i=1}^{n} e^{-\alpha(t - t_i)} \right)^{-b} \tag{9}$$

where, in addition to WLC fitting parameters (y_1 and b), β and α are exponents for power and exponential decay, respectively. The resulting interference-adjusted experience, n_e, is less than the nominal experience, measured in cumulative repetitions or units, i.e., $n_e \le n$. Appendix B shows the calculation procedure of MWLC-E in Eq. (9). Jaber et al. [31] found the exponential decay to better fit assembly learning data from Bailey [46] and Bailey and McIntyre [47,48]. They fitted numerous empirical datasets and found that, for a cumulative time of n repetitions, $t_n = \rho_1 n + \rho_0$ is an excellent approximation ($R^2 > 0.95$). Substituting $\rho_1 n + \rho_0$ and $\rho_1(i-1) + \rho_0$ into Eq. (9), they got $\sum_{i=1}^{n} e^{-\alpha\rho_1(n-i+1)}$, which is equivalent to $(1 - e^{-\alpha\rho_1 n})/(e^{\alpha\rho_1} - 1)$. The MWLC-E in Eq. (9) can now be rewritten in an approximate (AMWLC) form as:

$$y_n = y_1 n_e^{-b} = y_1 \left(\frac{1 - e^{-\gamma n}}{e^{\gamma} - 1} \right)^{-b} \tag{10}$$

where $\gamma = \alpha t$ is a fitting parameter. Again, when there is no interference, $\alpha = 0$, applying l'Hopital's rule, $n_e = n$; $0 < n_e < n$ otherwise. The AMWLC is continuous and does not require the calculation of each repetition, as MWLCs, which improves the computational efficiency. AMWLC fitted many experimental and empirical assembly learning data far better than the WLC (Eq. (1)) and the Plateau (Eq. (2)) [31]. When AMWLC does not capture interference, it never performs worse than the WLC and the Plateau. On the other hand, when it does, it tends to outperform the WLC and, in some instances, the Plateau. Thus, the concept and modeling of "*interference*" explained better than "*bounded*" learning the curvature observed in assembly data. A potential candidate for AMWLC is Eq. (5), Plateau-E, which also captures decay and plateau phenomena. In Section 5, we will test both models against experimental learning data.

3. Cognitive and motor learning with interference

Assembly and other labor-intensive tasks are, naturally, psychomotor, requiring both declarative and procedural knowledge. A worker gains declarative knowledge (what to do and in which order) from instructions to reflect the actions to complete the task and procedural or methodological knowledge (how to do) from learning-by-doing. Crossman [7] claimed that the scatter of motor element times (grasp, move, position, reach) has to do with varying the method and not the level of effort. For each repetition, a worker adopts some combination of sensory, perceptual, and motor activities, partly from deliberate choice, habit, and change. In consecutive repetitions, he/she will use either the same or different combinations. According to Fitts and Posner [12], learning a skill occurs when a subject transfers old habits to new situations. They suggested that the learning process goes into three stages: cognitive, associative, and autonomous (readers may refer to Appendix C for more information). This paper adopts the division of *cognitive* and *motor* task elements to distinguish the two types of learning processes or phases.

Pashler [11, p. 220] reviewed laboratory studies and reported that "*many pairs of tasks* [when performed at the same time] *interfere with each other quite drastically, even though they are neither intellectually challenging nor physically incompatible*." In practice, completion of a task, such as product assembly, requires a set of sequential subtasks to be performed (Fig. 3). Each subtask comprises a varying degree of cognitive and motor elements from which estimating the LR values (Fig. 2) for each subtask becomes possible. Dar-El et al. [9,13] suggested that a dormant time (>1.5 min) between each subtask causes forgetting of that subtask (Fig. 3) and its related cognitive and motor processes. The intensity of forgetting depends on, to name a few, retention time, the overlap of context between the subtasks, the instruction material, and those that are worker-specific, such as learning strategy [25]. In the psychology literature, forgetting and interference are handled and modeled either separately [52] or in combination ([25], see Appendix A). The model developed in this paper will use interference to depict forgetting while a subject is learning (performing repetitions).

Anderson [22] showed that interference occurs in a purely cognitive task. For this, he proposed a concept of memory trace decay. The works of [28,52] suggest that forgetting occurs in a purely visuomotor task and is a consequence of two-component processes: time-decay and interference, similar to Wickelgren [50] for cognitive tasks. Sing et al. [52] tested how subjects adapt to a velocity-dependent force field in an arm reach movement experiment. They believed that forgetting would cause ~20% adaptation to decay exponentially over a minute or two, with a time constant somewhere around 15 s. Morehead and Smith [28] experimentally tested how subjects adapt to an alleged implicit sensory adaptation error induced via task-irrelevant clamped visual feedback. Their findings suggest that the state of the motor memory decays by a fixed percentage with every movement, which means that a motor memory will eventually be unlearned if a reach happens in the absence of sensory feedback (interfered). It also implies that there is an asymptote, where it will also decay. Howell and Kreidler [27] earlier showed that feedback that does not conform to instructions creates interference in perception-motor tasks [3]. Based on the literature, both cognitive and motor processes have memory traces, which deplete over time in the absence of learning stimulus, cues/prompts (cognitive), or sensory feedback (motor).

4. The proposed learning curve models

In this section, we develop three LC models. They combine the dual-phase power LC model of Jaber and Glock [14], JGLCM (Eq. (7)), with the interference-adjusted power LC models of Jaber et al. [31], which are the MWLC-P (Eq. (8)), MWLC-E (Eq. (9)), and the AMWLC (Eq. (10)). Each model has a parameter x that splits the time at the first repetition, y_1, into two components, cognitive, $y_{1,c} = xy_1$, and motor, $y_{1,m} = (1-x)y_1$. The models have cognitive and motor learning exponents, like JGLCM, but added to decay exponents, representing the rate at which the cognitive and motor memory traces from each repetition decay over time. The first two models, the Dual-Phase Interference-Adjusted Learning Curve with Power or Exponential decay, henceforth DP-IALC-P (Eq. (11)) and DP-IALC-E (Eq.(12)), are of the forms:

$$y_n = xy_1\left(\sum_{i=1}^{n}(t-t_i)^{-\beta_c}\right)^{-b_c} + (1-x)y_1\left(\sum_{i=1}^{n}(t-t_i)^{-\beta_m}\right)^{-b_m} \tag{11}$$

$$y_n = xy_1\left(\sum_{i=1}^{n}e^{-\alpha_c(t-t_i)}\right)^{-b_c} + (1-x)y_1\left(\sum_{i=1}^{n}e^{-\alpha_m(t-t_i)}\right)^{-b_m} \tag{12}$$

The third model, the Approximate Dual-Phase Interference-Adjusted Learning Curve, henceforth A-DP-IALC, assumes exponential decay and is of the form:

$$y_n = xy_1\left(\frac{1-e^{-\gamma_c n}}{e^{\gamma_c}-1}\right)^{-b_c} + (1-x)y_1\left(\frac{1-e^{-\gamma_m n}}{e^{\gamma_m}-1}\right)^{-b_m} \tag{13}$$

In the next section, we will test the fits of the developed models. For that purpose, we use experimental learning data. We use the Mean Squared Errors (*MSE*) to measure the fits between estimated (y_i) and observed ($\hat{y}_i$) learning data [14,31]. The mathematical model is given as:

$$\text{Minimize } MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2 \tag{14a}$$

Subject to:

$$y_1 > 0 \tag{14b}$$

$$0 \le x \le 1 \tag{14c}$$

$$0 \le b_c, b_m \le 1 \tag{14d}$$

$$0 \le \beta_c, \beta_m, \alpha_c, \alpha_m, \gamma_c, \gamma_m \le 1 \tag{14e}$$

5. Testing the models

In this section, we test the fits of the developed dual-phase interference-adjusted LC models, Eqs. (11)–(13), and four models from the literature (JGLCM, Eq. (7), AMWLC, Eq. (10), the Plateau model, Eq. (2), and its extension with exponential decay, Plateau-E, Eq. (5)) against assembly data from Bailey [46] and Bailey and McIntyre [47,48]. All the models are extensions of Wright's [5] learning curve, WLC (Eq. (1)). The AMWLC and the Plateau models are considered reference models, as Jaber et al. [31] found both to fit assembly learning data well. More precisely, AMWLC performed the best for 59% (80/135) and the Plateau for 24% (32/135) of 135 learning datasets taken from Bailey. For 16% (22/135) of the fits, the AMWLC did not capture interference, and the Plateau model did not plateau. Both LC models behaved identically to the WLC.

5.1. The data

We test the LCs against a subset (38) of Bailey's datasets (Fig. 4). The selection of datasets is based on the following three criteria. First, they have at least nine repetitions to allow the LCs to converge, thus finding a satisfactory solution and getting better results. Second, they represent the same share of learning profiles (data), like in Jaber et al. [31], to ensure a realistic

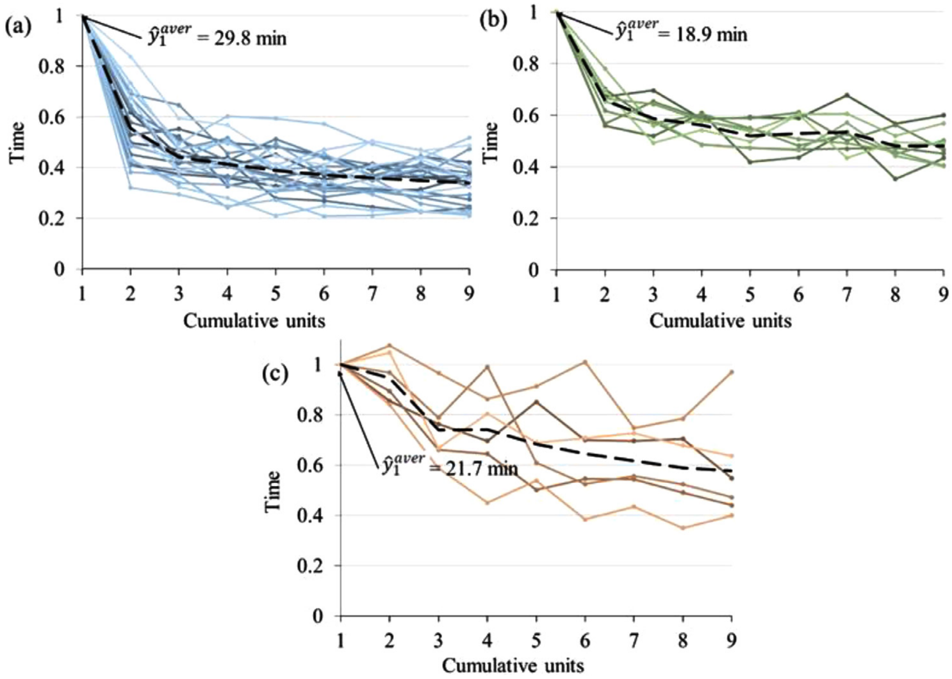

Fig. 4. (a) exponential (N = 23), (b) plateau (N = 9), and (c) power (N = 6) learning profiles.

sample of individuals. The three profiles from [31] are *exponential* (AMWLC performed the best), *plateau* (Plateau performed the best), and *power* (both models behaved like the power-form WLC). Third, for *exponential* data, AMWLC produced at least 9% lower MSE than the Plateau model, and for *plateau* data, vice versa, to ensure learning profiles are sufficiently different. Note that the selection of the *power* learning data was random. Note that in Fig. 4, average times for the first repetitions, $\hat{y}_1^{aver} = \sum_{i=1}^{N} \hat{y}_1^i / N$, are transformed to 1 (reference level). Only the first nine repetitions are presented for a better comparison of the learning profiles. The black dotted lines present averaged data (performance) over individuals, $i = 1,\ldots,N$.

An *exponential* profile (Fig. 4a) shows fast initial learning (on average, a 46% decrease in assembly time from the first to the second repetition). This result is a consequence of poor performance at first repetition (and highly related to interference). The *plateau* profile (Fig. 4b) shows a bit lower initial learning speed (a 34% decrease from the first to the second repetition) as subjects have much less to improve over the first repetition. Henceforth, learning ceases (plateaus) faster (on average, a 28% decrease from the second to the ninth repetition) than for individuals with *exponential* profiles (a 39% decrease). A *power* learning profile (Fig. 4c) improves slowly and steadily with each repetition, and performance shows apparent saw-tooth-like variation. An improvement from the first to the ninth repetition ($\hat{y}_1^{aver} - \hat{y}_9^{aver}$) is the same as with the *plateau* profile; however, *power* learning does not cease.

5.2. The fits

Table 1 presents the results from fitting the models to three types of data or profiles. The results show that the Plateau-E model, on average, performs the best (1st) (the total weighted average ranking, WAVG ranking tot.) and is only slightly better than A-DP-IALC (2nd). The other dual-phase models come next: DP-IALC-P (3rd), DP-IALC-E (4th), and JGLCM (5th). AMWLC (6th) and the Plateau (7th), the reference models, performed the worst. The models with exponential decay fit the best *exponential* data. Plateau-E has the best (1st) performance, followed by the two approximate interference-adjusted LCs, with component (motor/cognitive) curves (A-DP-IALC) (2nd) and without (AMWLC) (3rd). The two dual-phase interference-adjusted models, DP-IALC-E and DP-IALC-P, have poor fits (4th), and the models, the JGLCM and the Plateau, without a decay component, have the worst (6th). The dual-phase models, especially those with exponential decay, fit the best *plateau* learning data. The A-DP-IALC has the best (1st) performance, followed by the DP-IALC-E (2nd), the DP-IALC-P, and the JGLCM (3rd). The Plateau-E (5th) and the Plateau (6th) models have the worst performance, but not as much that as the AMWLC (7th). Without exception, the Plateau-E fits the best (1st) and DP-IALC-P the second-best (2nd) *power* learning data. The other models perform equally worse (3rd).

6. Discussion

Having presented the mathematics and fits of the models, we now discuss the performance of the models against the three types of learning profile (data): *exponential* (Section 6.1), *power* (Section 6.2), and *plateau* (6.3), illustrated through

Table 1
Results (MSE) from fitting the models to (a) *exponential* ($N = 23$), (b) *plateau* ($N = 9$), and (c) *power* ($N = 6$) learning data. The *exponential* and *plateau* data are sorted largest to smallest $1-(MSE_{AMWLC}/MSE_{Plateau}) \geq 0.09$ and $1-(MSE_{Plateau}/MSE_{AMWLC}) \geq 0.09$, respectively. For each ID, the color scale green-gold-orange represents the best, midpoint, and worst fit (ID = Individual id, n = number of repetitions, WAVG = weighted average ranking).

(a) Exponential learning data

ID	n	AMWLC	A-DP-IALC	DP-IALC-E	DP-IALC-P	JGLCM	Plateau-E	Plateau
66	11	0.121	0.119	0.214	0.214	0.214	0.119	0.214
64	9	1.502	1.469	2.146	2.146	2.146	0.764	2.146
40	11	2.653	2.595	3.503	3.503	3.503	0.584	3.503
47	9	1.106	1.106	1.416	1.416	1.416	0.972	1.416
149	14	1.483	1.476	1.864	1.864	1.864	1.290	1.864
106	9	1.632	1.627	2.010	2.010	2.010	1.530	2.010
7	9	0.573	0.483	0.700	0.700	0.700	0.549	0.700
10	15	1.609	1.495	1.961	1.961	1.961	1.552	1.961
130	13	1.863	1.793	2.272	2.272	2.272	1.301	2.272
113	16	0.391	0.391	0.445	0.459	0.465	0.322	0.465
110	9	1.588	1.588	1.858	1.858	1.858	1.395	1.858
126	12	0.698	0.698	0.814	0.814	0.814	0.587	0.814
132	12	2.841	2.734	3.276	3.276	3.276	2.251	3.276
128	13	2.359	2.249	2.717	2.717	2.717	1.604	2.717
148	12	5.130	4.982	5.880	5.880	5.880	2.446	5.880
112	10	2.642	2.519	3.009	3.009	3.009	2.131	3.009
157	13	5.381	5.193	6.126	6.126	6.126	0.693	6.126
156	11	1.170	1.145	1.330	1.330	1.330	1.159	1.330
105	10	3.585	3.302	4.055	4.055	4.055	0.543	4.055
46	9	1.201	1.201	1.354	1.354	1.354	0.983	1.354
120	10	2.408	2.408	2.693	2.693	2.693	1.897	2.693
144	22	0.535	0.473	0.596	0.596	0.596	0.514	0.596
104	16	0.785	0.785	0.845	0.837	0.866	0.602	0.866
WAVG ranking		2.70	1.78	4.04	4.04	4.17	1.17	4.17

(b) Plateau learning data

ID	n	AMWLC	A-DP-IALC	DP-IALC-E	DP-IALC-P	JGLCM	Plateau-E	Plateau
172	18	0.664	0.612	0.614	0.614	0.614	0.617	0.617
152	15	1.420	1.300	1.305	1.305	1.305	1.306	1.306
127	16	0.188	0.152	0.156	0.156	0.156	0.159	0.159
155	15	0.885	0.778	0.773	0.778	0.778	0.786	0.786
171	19	0.250	0.222	0.226	0.226	0.226	0.221	0.226
57	10	0.273	0.225	0.225	0.225	0.225	0.244	0.246
134	15	0.750	0.660	0.660	0.660	0.660	0.678	0.678
143	19	0.578	0.508	0.508	0.508	0.508	0.522	0.523
1	17	2.542	2.224	2.302	2.302	2.302	2.302	2.302
WAVG ranking		7.00	1.22	1.67	1.78	1.78	4.22	4.67

(c) Power learning data

ID	n	AMWLC	A-DP-IALC	DP-IALC-E	DP-IALC-P	JGLCM	Plateau-E	Plateau
140	16	0.698	0.698	0.698	0.671	0.698	0.629	0.698
154	11	0.968	0.968	0.968	0.856	0.968	0.806	0.968
121	10	4.713	4.713	4.713	3.803	4.713	2.721	4.713
129	17	0.581	0.581	0.581	0.567	0.581	0.480	0.581
135	9	2.739	2.739	2.739	2.640	2.739	2.105	2.739
4	14	4.757	4.757	4.757	4.723	4.757	4.661	4.757
WAVG ranking		3.00	3.00	3.00	2.00	3.00	1.00	3.00

	AMWLC	A-DP-IALC	DP-IALC-E	DP-IALC-P	JGLCM	Plateau-E	Plateau
WAVG ranking tot.	3.76	1.84	3.32	3.18	3.42	1.87	4.11

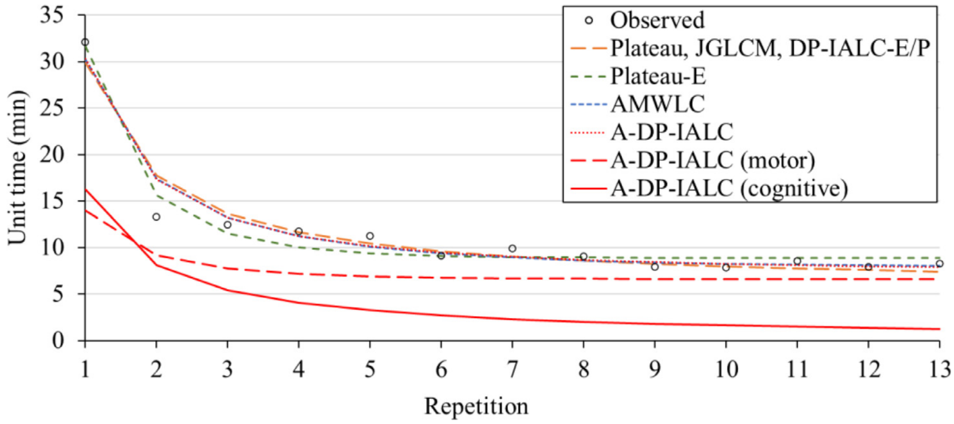

Fig. 5. LCs fitted to observed exponential learning data (ID = 130).

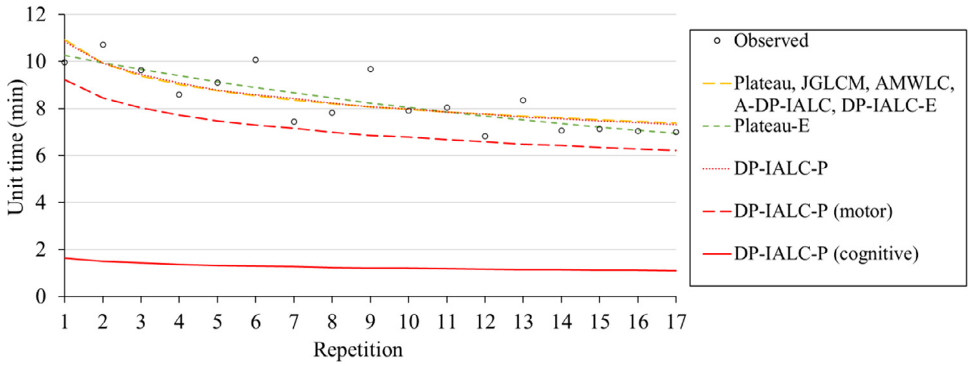

Fig. 6. LCs fitted to observed power learning data (ID = 129).

the fits. The behavior of the models is analyzed with parameters capturing three phenomena: component (cognitive/motor) learning, decay/interference, and plateau. The analysis is followed by managerial insights (Section 6.4).

6.1. Exponential learning

The *exponential* learning profile is estimated the best by the models with adjustment for the decay/interference of each repetition, including the first one. This observation also holds for the Plateau-E and the two approximate models, the A-DP-IALCM and the AMWLC, which is a feature that makes it possible to overcome a common drawback of other models, overestimation of initial (first repetition) performance. The DP-IALC-E and the DP-IALC-P captured interference only for IDs 113 and 104. Fig. 5 illustrates typical trendlines of the fits against *exponential* data (ID = 130). The optimal parameter values of the best-performed model, the Plateau-E, in Eq. (5) that produce the trendline for which the MSE is minimum are $y_1 = 47.357$, $c = 8.849$, $\alpha = 0.524$, and $b = 1.000$, with $MSE = 1.301$. By removing the exponential decay, i.e., $\alpha = 0$, the Plateau-E reverts to the Plateau Eq. (2)) with $MSE = 2.272$. The performance of the dual-phase models, the JGLCM (Eq. (7)) and the DP-IALC-E/P (Eqs. (11) and ((12)) equal to that of the Plateau, where motor learning component is constant (no motor learning) and equal to the standard time, i.e., $y_{n_m} = c = 5.549$. Of the interference-adjusted models, only the approximate ones capture interference, where $\gamma_c = 0$ ($b_c = 1.000$) and $\gamma_m = 0.639$ ($b_m = 1.000$) for A-DP-IALC (Eq. (13)) with $MSE = 1.793$, and $\gamma = 0.300$ ($b = 1.000$) for AMWLC (Eq. (10)) with $MSE = 1.863$. Fig. 5 also illustrates the component curves of the A-DP-IALC, where cognitive dominates motor in the initial learning phase, and motor elements dominate cognitive ones later [13]. About one-third of the *exponential* profile IDs, the A-DP-IALC did not improve the fits over the AMWLC.

6.2. Power learning

In addition to the *exponential*, the Plateau-E also fits the *power* learning profile the best [4]. Its flexibility is based on two measures, the amount learned ($y_1 - c$) and the speed of exponential decay (α). The Plateau-E also decreases the curvature, or straightening the curve, from that of WLC. Fig. 6 illustrates typical fits against *power* data (ID = 129). The optimal parameter values that minimized *MSE* for the Plateau-E are $y_1 = 10.589$, $c = 4.716$, $\alpha = 0.057$, and $b = 0$, with $MSE = 0.480$. Of the developed interference-adjusted models, only DP-IALC-P, which assumes a power-form decay, captures interference

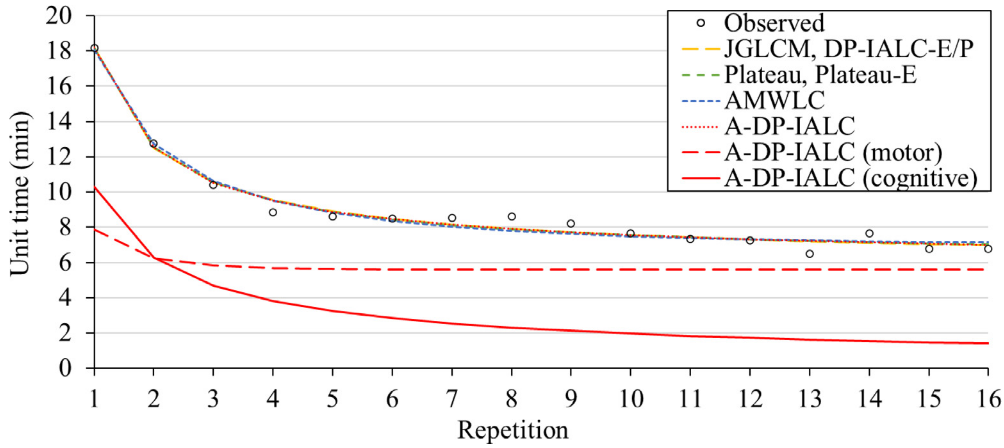

Fig. 7. LCs fitted to observed plateau learning data (ID = 127).

in *power* learning data (y_1 = 10.847, b_c, b_m= 0.243, and a_c, a_m= 0.361, with MSE = 0.567). Other models perform equally worse (y_1 = 10.922, b, b_c, b_m= 0.138, and c, γ, γ_c, γ_m,a_c, a_m= 0, with MSE = 0.581) reverting to simple WLC in Eq. (1). The performance of the dual-phase models is not dependent on the split (x) between cognitive and motor components.

6.3. Plateau learning

The dual-phase models estimate the *plateau* learning profile the best. The approximate interference-adjusted model, A-DP-IALC, fits the best (y_1 = 14.229, x = 0.724, b_c, b_m = 0.719, and γ_c = 0, γ_m = 0.971, with MSE = 0.152, Fig. 7). Other dual-phase models fit equally (y_1 = 18.179, x = 0.593, b_c = 1.000, b_m = 0.057, a_c, a_m = 0, with MSE = 0.156), i.e., they do not capture interference. The Plateau-E and Plateau (y_1 = 18.185, c = 6.040, β, b = 0.905, α = 0, with MSE = 0.159) fit worse. The AMWLC (y_1 = 15.719, b = 0.601, γ = 0.233 with MSE = 0.188) captures interference but fits the *plateau* learning data the worst as it does not consider dual-phase or the plateauing effect. When interference is captured, the AMWLC always performs better than simple WLC (Eq. (1)). The above results suggest that the plateauing effect is related to motor learning, which is slower than cognitive learning, partly due to motor interference (A-DP-IALC).

6.4. Managerial insights

The classification of learners by profile helps in effectively determining the starting point for individual training. We have shown in the previous sections how seven LC models, each a modified version of the WLC, fit three learning profile types in a laboratory assembly. About 60% of individual profiles follow an *exponential* law of practice. They are "*early strugglers*" as they often perform poorly at the first repetitions but improve rapidly at the subsequent ones. This observation suggests significant interference to occur, especially at the first repetition. For this, the models that consider interference-adjusted, residual knowledge, are recommended. The developed approximate dual-phase interference-adjusted LC, the A-DP-IALC, fits such data well. It is consistent with, for example, Fitts and Posner's [12] theory of three stages to acquire a new skill: cognitive, associative, and autonomous (Appendix C). Acknowledging that cognitive and motor interferences can potentially cause significant loss of knowledge over training sessions would help industrial managers to remove learning barriers.

One-sixth of the individual profiles follow a *power* law of practice. They are "*late adopters*" whose performance shows a saw-tooth-like variation, and learning does not noticeably cease. This observation implies that some factors may interfere with learning over the entire training session despite the random variations. A simple power-form WLC is often enough to estimate, and its extensions seldom improve the fit against such data [31]. However, WLC has a minimum curvature that underestimates the performance of early learning stages and overestimates that of later ones. For this, we recommend using Plateau-E that is capable of a very gentle, almost straight, curve (Fig. 6). It has a superior pair of features: exponential decay and the standard time (or amount learned). For managers, determining the standard time for *power* learning data is beneficial and would help them in the decision-making process.

The remaining profiles, about one-fourth, are of *plateau* form. They are "*early adopters*" as they adopt knowledge at the first repetition the fastest and have less than those with an *exponential* profile to learn the next ones. For this reason, their learning also ceases or plateaus the fastest. Acknowledging that learning is an aggregate process of two components, cognitive and motor, would explain such a profile. Cognitive learning is quick, while motor learning is slow. The latter improves less and less with practice (see WLC and Plateau phases in an LC in Fig. 1a). For this, the models with a dualistic approach are also most effective and recommended.

7. Summary and conclusions

This paper presents a modification of the WLC by aggregating cognitive and motor components and considering interference when learning. Interference is modeled by accounting for cognitive and motor memory traces from repetitions that decay over time when there is no learning stimulus. Combining the dual-phase power LC model of Jaber and Glock [14], the JGLCM, and the interference-adjusted power LC models of Jaber et al. [31] resulted in three alternative models. The first two models, the Dual-Phase Interference-Adjusted Learning Curve with Power or Exponential decay, the DP-IALC-P/E, aggregate the cumulative residual experience for each repetition when fitting the data. The third model is an approximate expression, the A-DP-IALC, with exponential decay. Three developed models and another four selected from the literature (two base models, the Plateau model and the Plateau-E) were tested against experimental data from Bailey [46] and Bailey and McIntyre [47,48]. The datasets represent individual learning of an assembly task performed in a laboratory for three profiles: *exponential, power*, and *plateau*. The results showed that the A-DP-IALC fits *exponential* and *plateau* data well (when comparing 85% of the datasets). This finding encourages managers to consider both interference and component (cognitive/motor) processes when planning their assembly tasks. The Plateau-E model fitted well *exponential* and *power* data, in line with previous findings. It uses the components of exponential decay and standard time (or the amount learned) to capture a wide range of learning profiles. The overall results highlight the confluence of component (cognitive/motor) processes, interference, and plateauing phenomenon.

The results presented in this paper seed for further studies. One of them is modeling the cognitive and motor processes together with interference and plateau effects. This approach should consider memory traces, depletion, recency, and amount of practice. More detailed data will help better understand the effects of cognitive and motor interference. This includes cognitive and motor component times, perceived cognitive and motor load, and information on worker-related factors that may interfere with learning. How information is presented does affect cognitive load, interference, and performance. Therefore, it would also be interesting to study the effects of the type of instruction and the amount of information available for workers over training sessions. The recent technologies would be helpful as they provide customized and adaptive instructions that consider worker-specific needs. Lastly, extensions of this study could consider sequential learning phases in terms of a hybrid LC. For example, assuming the first phase follows the WLC and the second the Plateau model, as in Fig. 1(a), one would determine the optimum number of cumulative units where the two models or functions intersect.

Declarations of Competing Interest

None.

Acknowledgments

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Appendix A. Mensink and Raaijmakers' [25] context fluctuation process

The associative strength of a stimulus item S_i (which is controlled by cues/prompts) to the stored memory trace or "*image*" I_i is $S(I_i,S_i) = bt_i$, where t_i equals the presentation time (for example, looking at assembly drawing) in seconds and b denotes the amount of associative information transferred per second. Parameter b depends on many factors such as prior associative strength (experience), imageability (how the cues/prompts are presented), and the encoding (learning) strategy. This associative strength is not dependent on the length of the retention interval. The innovative feature of the model by Mensink and Raaijmakers [25] is the contextual fluctuation process that can handle time-dependent changes in retrieval strengths (memory performance). Consider a context that includes $K + k$ elements in total, of which k elements are only active at any time (current elements), and other K elements are inactive. In the context of a fluctuation process and during a time interval, active elements may become inactive while inactive ones may become active at some rates. At study time (encoding), only active elements can be stored in the episodic image. The probability of retrieving correct memory image is a function of associative strength of the cues/prompts to that image, relative to strengths of all associations (including the interfering, unrelated ones). Once retrieved, recovery depends on the absolute strength of the trace (equal to the overlap in features between cue/prompt and trace). The contextual overlap is a decreasing function of the retention interval.

Appendix B. Calculation procedure of MWLC-E (Eq. (9))

Consider an assembly worker who performs four repetitions $i = 1, 2, 3$, and 4 in times y_i, where y_1=14.28, y_2=11.15, y_3=9.37, y_4=8.27. Let us denote the starting and completing times t_i and t_{i+1}. Then t_1= 0, t_2 = 14.28, t_3 = 25.43, t_4 = 34.8, and 28 t_5=43.07. At first repetition, K units of information are recalled, which will decay over $t_5 - t_0$, and only $e^{-\alpha(t_5-t_0)}$ units of information are remembered by the time the fourth repetition is completed; i.e., t_5. The second repetition starts at time t_2 (time to complete the first repetition), where again K units of information are recalled and by time t_5, only $e^{-\alpha(t_5-t_1)}$ units of information are remembered. Then, we have $M = K(e^{-\alpha(t_5-t_1)} + e^{-\alpha(t_5-t_2)} + e^{-\alpha(t_5-t_3)} + e^{-\alpha(t_5-t_4)} + e^{-\alpha(t_5-t_5)})$. Now, when there is no interference, the decay exponent $\alpha = 0$, we have $M = K(1 + 1 + 1 + 1 + 1) = 5K$, meaning that $n_e = n = 5$, i.e. full transfer of information. Now, to illustrate, assume $\alpha = 0.1$, then $M/K = 0.013+ 0.056 + 0.171 + 0.437 + 1.000 = 1.678 < 5$. So, an assembly worker would have accumulated an actual (non-interfered) experience of 1.678 repetitions, and not 5.

Appendix C. Fitts and Posner's [12] three stages of skill acquisition

Fitts and Posner's [12] theory proposes that acquiring new skills requires passing three stages. In the cognitive stage of a task (first few repetitions), a learner tries to understand how to do it and what is required. This requires, for example, reviewing the rules, the actions to perform, and the strategies to be used. Through instructions, observations, and feedback, the learner gains an elementary understanding of a task. This information enables making preliminary attempts at the task. Different subcomponents of skill are typically tackled in isolation. A learner at the intermediate or associative stage can detect and eliminate all conceptual and procedural errors of a task. The learner has determined the most effective way of doing the task and starts refining the skill. The length of this stage is dependent on task complexity. At the final or autonomous phase, a low degree of attention is required for performance. A learner is less likely to be subject to cognitive control and interference from other activities and environmental distractions. This stage is the longest of a learning session, where task speed and efficiency will increase but at a continually decreasing rate.

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