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Optimization of absorption placement using geometrical acoustic models and least squares

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Abstract: Given a geometrical model of a space, the problem of optimally placing absorption in a space to match a desired impulse response is in general nonlinear. This has led some to use costly optimization procedures. This letter reformulates absorption assignment as a constrained linear least-squares problem. Regularized solutions result in direct distribution of absorption in the room and can accommodate multiple frequency bands, multiple sources and receivers, and constraints on geometrical placement of absorption. The method is demonstrated using a beam tracing model, resulting in the optimal absorption placement on the walls and ceiling of a classroom.

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1. Introduction

The computation of the impulse response of a space with specific materials at specific surfaces is a well studied topic. When one considers the inverse problem—placing and specifying the materials to obtain a specific response—the problem quickly becomes very complex. Ideally, the optimal placement of materials should be objectively computable, with given target conditions and constraints.

The problem of placing and specifying the materials to obtain a specific response is a nonlinear optimization problem. In this letter, we propose a method for formulating the nonlinear optimization problem as a linear least squares problem. Instead of specifying the target impulse response, we define a target echogram that represents the target energies of all the reflection paths at the receiver location(s). The least squares solution will minimize the deviation of the logarithm of the target energy for each reflection in the echogram by giving optimal absorption coefficients for a set of discretized surfaces in the geometrical model. By using regularization and constraints, we can adapt the solution to real world problems, as discussed later on.

The proposed method will allow us to find a global minimum for the optimization problem. It will also allow us to consider multiple source-listener combinations at the same time. Multiple frequency bands can be considered simultaneously. Different parts of the echogram can be weighted, making errors in the solution at some parts of the echogram more significant than others. Surfaces can also be assigned predefined absorption values.

2. Background

Previously, optimization methods have been used to solve the nonlinear optimization problem, and while they are capable of finding a global maximum, they cannot be guaranteed to do so. Monks *et al.*[1] optimize the material placement and geometry of a concert hall using a combination of simulated annealing and steepest descent. Dühring *et al.*[2] seek to minimize sound pressure level at a target frequency using the method of

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moving asymptotes. Genetic algorithms are additionally used to optimize the geometry of a space.[3,4] Genetic algorithms, simulated annealing, and other heuristic optimization or sampling methods are based on random walks and scale badly in high dimensions—i.e., the number of discretized surfaces in the simulated space.

If we assume the geometry of a space to be fixed, the problem of finding ideal material properties can be formulated as a linear optimization problem. Our optimization problem will be convex, by which it follows that a global minimum can always be found. The optimizable parameters can be varied freely, as any theoretically possible echogram can be defined as the optimization target.

We assume that the early response will be the most important part of the response with respect to the placement of materials, and specular reflections are most prevalent here. We also assume that the surfaces have been divided into small enough segments to represent realistic surfaces to which absorption can be assigned. We use a beam tracing model[5] to calculate specular reflections.

3. Least squares assignment of absorption

In geometrical acoustic models, the total attenuation associated with a single reflection path arriving at a listener can be calculated from the product of reflection coefficients of individual surfaces, along with the distance the sound has traveled. Energies of the arrivals are scaled initially by $(ct)^{-2}$ to account for the geometrical spreading of wave fronts, with c denoting the speed of sound and t the propagation time. Additional attenuation is determined by the reflection coefficients at all reflecting surfaces that a ray or beam encounters.

The sound power reflection coefficient for a surface can be calculated according to $R_j = 1 - \alpha_j$, where α_j is the absorption coefficient describing the absorption of sound energy. Assume an initial energy $E_0(ct_i)^{-2}$ for a single arrival i, first taking only geometrical spreading as a function of time t_i into account. The final energy of the arrival $\hat{E}_i$ can be expressed

$$\hat{E}_i = \prod_j (1 - \alpha_j) \frac{E_0}{(ct_i)^2} = R_{\mathrm{tot},i} \frac{E_0}{(ct_i)^2}, \tag{1}$$

where α_j is the absorption coefficient associated with the surface given by index j and $R_{\mathrm{tot},i}$ is the total energy absorbed by all surfaces interacting with arrival i. Taking the logarithm of both sides, we can write the equation as a linear sum, giving

$$\log(\hat{E}_i) = \sum_j \log(1 - \alpha_j) + c_{\mathrm{att},i} = \sum_j r_j + c_{\mathrm{att},i}, \tag{2}$$

where

$$c_{\mathrm{att},i} = \log(E_0) - 2\log(ct_i). \tag{3}$$

Now the logarithm of the echogram is expressed as a *linear* combination of variables related to the absorption coefficients of surfaces in the room as opposed to the *nonlinear* combination in Eq. (1). Let the entries of $\mathbf{b}$ be defined by $b_i = \log(\hat{E}_i)$, where $\hat{E}_1, \hat{E}_2, \ldots, \hat{E}_k$ are energies of arrivals in the target echogram. We wish to solve for the vector $\mathbf{r}$, defined by $r_j = \log(1 - \alpha_j)$, the logarithm of sound power reflection coefficients for each surface. Given a connectivity matrix G, the system may be written

$$G\mathbf{r} = \mathbf{b} - \mathbf{c} \equiv \mathbf{d}, \tag{4}$$

where the entries of $\mathbf{c}$ are $c_{\mathrm{att},i}$ and $\mathbf{d}$ is the target vector. The dimensions of G are $M \times N$ where M is the number of arrivals in the echogram and N is the number of surfaces in the model.

The matrix entries $G_{i,j}$ are integer values that describe the number of times the path of reflection i hits surface j. For instance, if arrival $i = 3$ hits surface $j = 4$ once

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and surface $j = 12$ twice, the matrix would have zeros in row 3 except for 1 in column 4 and 2 in column 12.

3.1 Regularized least squares

The most straightforward way to solve Eq. (4) is using the ordinary least squares solution, given by the normal equations,

$$\mathbf{r} = (G^T G)^{-1} G^T \mathbf{d}. \tag{5}$$

The system will often be overdetermined as the number of arrivals easily exceeds the number of surfaces, $M > N$. Overfitting is commonly avoided using ridge regression, which also allows for solutions of the underdetermined case. Ridge regression tends toward an even distribution of low absorption values. According to our studies, better results were obtained with the so-called LASSO[6] method, which in this case is defined by

$$\|G\mathbf{r} – \mathbf{d}\|_2 + \beta\|\mathbf{r}\|_1. \tag{6}$$

The free parameter β determines the tradeoff between the fit of the model to the data and the penalty caused by the L_1 norm of the logarithm of the surface sound power reflection coefficients. In practice, the tuning parameter directly affects the maximum absorption area.

3.2 Constrained least squares

For the surfaces to be passive, the coefficients are also subject to hard inequality constraints. First, the sound power reflection coefficients cannot be negative, i.e., a surface cannot remove more energy from the room than is incident upon it. Second, the coefficient must be less than one, as the energy cannot increase due to a reflection.

The first constraint is fulfilled due to the fact that we are solving for the logarithm of the coefficients. As the solution giving the logarithm of the coefficients is real, the sound power reflection coefficients will be positive.

For the second constraint, we need to impose a constraint on the logarithm of the sound power reflection coefficient, $r_j \leq 0$—a non-positive least squares problem. To cast it as a non-negative least squares (NNLS) problem, and thus a commonly encountered quadratic programming problem, we instead solve

$$(-G)(-\mathbf{r}) \equiv \tilde{G}\tilde{\mathbf{r}} = \mathbf{d}, \tag{7}$$

and $\tilde{\mathbf{r}} \geq 0$, or $\mathbf{r} \leq 0$ for the original problem, resulting in $\alpha_j \geq 0$.

The absorption coefficients may be expressed as a solution to the NNLS problem,

$$\alpha_j = 1 - 10^{-\tilde{r}_j}, \tag{8}$$

assuming base-10 logarithms.

3.3 Multiple source-listener paths

To avoid solutions only valid for very specific locations, multiple source-listener paths can be combined into a single problem. As all the source-listener paths share the same surfaces and reflection coefficients, the vector we are solving for, $\mathbf{r}$, remains the same. The target vector $\mathbf{d}$ and the connectivity matrix G is specified for each of the s source-listener combinations:

$$G = \begin{bmatrix} G_1 \\ \cdots \\ G_s \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \cdots \\ \mathbf{d}_s \end{bmatrix}. \tag{9}$$

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The following example does not describe any realistic case, but it has been included to demonstrate the effectiveness of using multiple source-receiver combinations. Figure 1 shows a 2D space with discretized boundaries. The square represents a source, and circles represent receivers. Assigned absorption coefficients are indicated by gray-scale values from 0 (white) to 1 (black). The reflection paths marked with thick lines, representing the earliest reflections, are to be damped; the target echogram is defined as having all reflections arriving before 8 ms damped by 20 dB, while the rest of the reflections are left undamped. Specular reflection paths up to the fifth order are calculated.

In Fig. 1, the result for a single source-receiver combination can be seen, with darker wall segments linearly representing higher absorption values. By placing multiple receivers in the model, the absorption is placed in a way that is optimal on average for multiple positions.

4. Further constraints

4.1 Fixing surfaces

A specific surface S_k can be defined as having a fixed sound power reflection coefficient, $r_k = \log(R_{k,\text{fixed}})$. The kth column of the G-matrix will thus be multiplied by the constant value r_k and can be moved to the right side of Eq. (4). For each surface k with a fixed coefficient, the respective column $\mathbf{a}_k$ is similarly removed from the connectivity matrix G and added to the target vector $\mathbf{d}_{\text{fixed}}$ according to

$$\mathbf{d}_{\text{fixed}} = \mathbf{b} + \mathbf{c} + \sum_k r_k \mathbf{a}_k. \tag{10}$$

4.2 Weighting

Discretized wall segments with different sized areas do not need to be weighted separately, as the importance of the area is taken into account automatically; a larger area will often contribute to more reflections than a smaller one. Instead, we might want to emphasize the importance of specific parts of the echogram, or specific source-listener combinations. If the vector $\mathbf{w}$ contains arbitrary weighting factors w_i for each single arrival i, we can incorporate the weighting factors into the model by multiplying each row i in the connection matrix G, and each element i in the target vector $\mathbf{d}$, by respective weighting factor w_i.

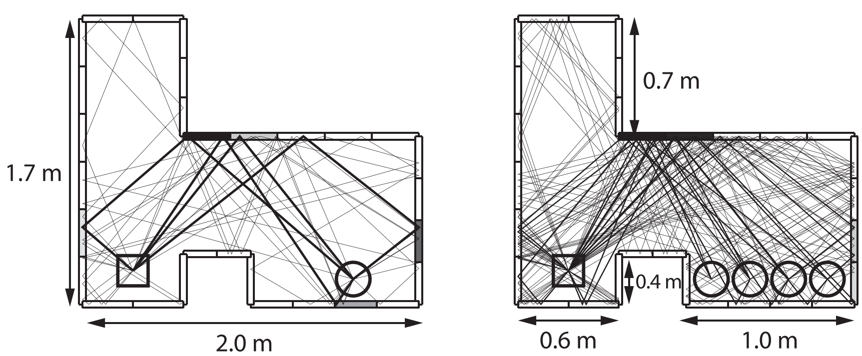

Fig. 1. This is a 2D room with a source (square) and receivers (circles). Lines indicate reflection paths, and rectangles on the boundaries indicate the amount of absorption that is assigned; white indicates no absorption, and black indicates total absorption. The left figure shows the case with one source-receiver combination, while the right figure shows the case with multiple source-receiver combinations.

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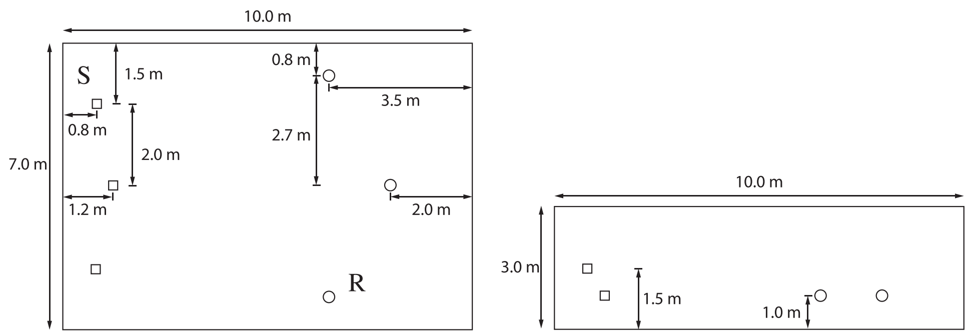

Fig. 2. Top and side view of the symmetric setup of the classroom; squares represent sound sources and circles represent receivers.

5. Example

An example is presented illustrating the result of the method when applied to the case of a large rectangular classroom. The target echogram is roughly defined so as to leave all the reflections contributing to the increased speech intelligibility, e.g., the important early reflections[7] or the "useful energy,"[8] unattenuated, while attenuating the rest of the reflections as much as possible.

Reflections are calculated up to the 6th degree. Nine source-receiver combinations are used. All the sources (shown by squares in Fig. 2) emit an impulse at $t_0 = 0$ s, with a sound pressure level of 80 dB at 1 m from each respective sound source. In this example, all the reflections are shown summed together into a single echogram. The summed echogram, without any absorption, is shown as the first plot in Fig. 3. The second plot of Fig. 3 shows the target echogram, where all the reflections arriving after $t_1 = 60$ ms are to be damped for each source-receiver combination. The floor is defined as fixed with an absorption coefficient of 0.

The least squares problem was solved using the scikit-learn package[9] in Python, which includes a solver for LASSO-type problems. The value $\beta = 0.01$ was used for the regularization penalty $\beta\ \|\mathbf{r}\|_1$.

The resulting echogram is shown in Fig. 3, while the result for the absorption values is shown in Fig. 4. Black represents an absorption coefficient of 1, while white represents an absorption coefficient of 0. The absorption coefficients for the discrete surfaces are very nearly perfectly symmetric with respect to the symmetry of the room.

A speech sample was auralized in the room, for the source-receiver combination marked by the respective first letters in Fig. 2. Air absorption was not taken into account. The sample in Mm. 1 demonstrates the result of shuffling the discretized absorption surfaces in Fig. 4 along the ceiling and walls, as a comparison.

Mm. 1. Shuffled absorption. This is a file of type "wav" (2.5 MB).

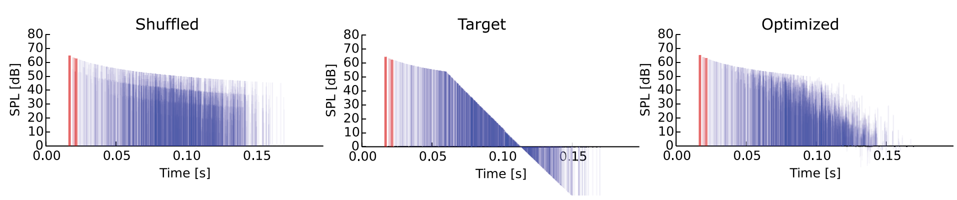

Fig. 3. (Color online) The summed echogram for all the source-receiver combinations. Echograms are shown for the shuffled case together with the target echogram and the result of the calculation. The thick lines in the beginning indicate direct sound.

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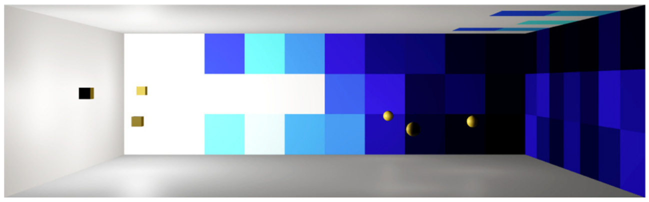

Fig. 4. (Color online) The optimal surface absorption coefficient values, as given by the solution to the LASSO problem. White represents an absorption coefficient of 0.0, while black represents an absorption coefficient of 1.0. The solution is almost perfectly symmetrical.

The optimized result, with the absorption placed according to Fig. 4, can be heard in Mm. 2.

Mm. 2. Absorption placed using least squares. This is a file of type "wav" (2.5 MB).

6. Future work

In our current system, each surface can have separate sound power reflection coefficients for different frequency bands, allowing for multiple target echograms on a range of frequency bands

$$G \cdot [\mathbf{r}_{\mathbf{f},0} \quad \mathbf{r}_{\mathbf{f},1} \quad \cdots \quad \mathbf{r}_{\mathbf{f}_N}] = [\mathbf{d}_{\mathbf{f},0} \quad \mathbf{d}_{\mathbf{f},1} \quad \cdots \quad \mathbf{d}_{\mathbf{f}_N}]. \tag{11}$$

Without additional constraints, Eq. (11) defines a range of separate problems with independent least squares solutions for each frequency band. This is problematic, since free variations between adjacent frequency bands result in unrealistic material properties. One possible solution to this problem is to tie the separate frequency bands together, by penalizing variations between adjacent frequency bands on a single surface.[10]

7. Conclusions

This letter presents a method for optimally assigning absorption values for a discrete set of surfaces, given a target echogram. In contrast with previous room optimization literature, we formulate the problem as linear least squares. While the procedure is able fit an arbitrary target response in the least-squares sense, an example related to speech intelligibility is given. The result implies that the method is plausible for real world scenarios, especially in cases where the optimal placement of materials with broad-band absorption properties is to be chosen.

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