Concert Hall Geometry Optimization with Parametric Modeling Tools and Wave-Based Acoustic Simulations

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ABSTRACT
Advances in computational capacity made available through graphics processing unit (GPU) processing and developments in parametrically driven design tools are creating new possibilities for acoustic design and analysis. In particular, wave-based numerical simulations are becoming more tractable, and geometry manipulations, which were once cumbersome manual work, can now be automated. A case study of concert hall section profile optimization is presented. Using RHINOCEROS software with the GRASSHOPPER parametric modeling plugin, geometries were automatically generated based on a few parameters, then evaluated using Finite Difference Time Domain (FDTD) numerical simulations using GPU processing in MATLAB. The results from each iteration are used to inform a global optimization algorithm that conducts an intelligent search of the parameter space to find a solution in as few iterations as possible. The optimization is based on a stochastic model of the multidimensional objective function. The objective function is iteratively sampled and a simplified Bayesian approach is used for finding the set of parameters which is most likely to improve the current estimate of the global minimum at each iteration. With this method, curved and linear iterations of the sidewalls and under-balcony surfaces of a concert hall section were investigated. The objective was to deliver the most early energy, in the most uniform distribution, from multiple sources to multiple receiver positions.

1. INTRODUCTION
Computer simulation of room acoustics has been developed since as early as the 1960’s, making prediction and analysis of architectural designs possible. Since those simple initial ray tracing models, many methods have evolved to include more accurate prediction of complex phenomena such as diffraction and scattering using both geometric and wave based techniques. However, simulation is still quite time consuming for large rooms, making iterative optimization intractable. Processing with Graphics Processing Units (GPUs) rather than Central Processing Units provides a significant speed increase, making these problems more feasible.

In conjunction with the advancement of acoustic simulation techniques, parametric architectural design tools now allow fast and flexible generation of spatial geometry based on parametric descriptions. Among many other applications, these tools have been utilized to optimize structural and thermal efficiency of architectural structures.
and have great potential for aiding acoustic optimization problems. Without such tools, generating new geometry for each optimization iteration is much more difficult and limited.

Limitations in acoustic simulation techniques and geometry handling may explain why previous room acoustic optimization attempts have been quite limited, for example, minimizing the low frequency sound pressure in one area of a room by placement of absorptive material\textsuperscript{4}, or selecting the appropriate size and proportions of a room to minimize the effect of low frequency modes\textsuperscript{3}.

The goal of the present work is to utilize FDTD simulations to optimize the section profile of a concert hall to provide early reflections from multiple sources to multiple receivers. The section incorporates curved elements, necessitating wave-based rather than geometric acoustic methods for accurate results. Solutions were scored based on two evenly weighted factors: the uniformity of energy across the audience area (receivers), and the average level of energy at the receivers. This parallels the design problem in which early reflections from multiple orchestra sources need to reach a wide audience area. The results show the applicability of the technique while also revealing appropriate design solutions.

2. METHOD
The optimization was conducted by generating a concert hall section with three parametrically variable parameters, exporting the geometry to a numerical simulation engine which calculated impulse responses for the source-receiver pairs, evaluating the impulse responses, and generating new parameters for the model. Figure 1 illustrates the conceptual structure of the optimization loop.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The optimization software setup.}
\end{figure}
2.1. Hall model
A generic concert hall lateral section was generated, based on a shoebox style hall with a balcony. While geometric optimization problems are often plagued with an unsurmountable number of degrees of freedom, architectural acoustic optimization for concert halls is constrained by so many other factors, that the degrees of freedom are quite limited. For example, sight lines, ceiling clearances, structural span and cantilever limitations, and stair slopes, can constrain a design such that reflecting surfaces must fit within a specific, limited, spatial range. Here, the optimization focused on adjusting the design of sidewalls, the under-balcony ceiling, and the ceiling above the balcony to attain the desired sound distribution across the receivers. These shapes were controlled by three parameters as shown in Figure 2. The technique could also be easily applied to balcony face size and orientation, or reflector sizing and placement, among many other design problems.

The geometry was generated by the Grasshopper plugin for Rhinoceros 3D. This software monitored a text file that contained the three geometry parameters and wrote a text file containing the cartesian coordinates describing the hall section geometry. When the geometry file updated, the simulation commenced in Matlab and the optimization algorithm produced a new set of parameters based on the previous results. Writing the new parameters file then triggered generation of new geometry and the loop continued.

2.2. Simulation engine
Simulations were conducted using a Finite Difference Time Domain numerical model, as implemented by Southern et al. In this case, the 2D wave equation was discretized using a second order central finite difference approximation to the wave equation with update equations implemented for air, boundaries, edges and corners. Pressure values for each time step, at 22,050 Hz sampling frequency, were iteratively calculated over all points in a grid bounded by the section geometry. This was continued until pressure values for the first 100ms of the impulse response had been calculated. The excitation function was a short power-of-cosine window, that yielded accurate results up to

Figure 2. The three geometric parameters under consideration. The same changes were symmetrically applied to both sides of the hall, at the under-balcony and balcony ceiling. Source and receiver positions are also indicated.
approximately 4 kHz at the given sample frequency. A reflection coefficient of 0.95 was applied to all boundaries.

Since the section was symmetrical, ten sources were spaced from the center line, 1 m from the floor, in 0.5 m increments towards the side, and responses were taken from ten receiver positions, offset from the sources by 0.25 m. This resulted in 100 impulse responses for each iteration of the simulation. In the final implementation, the simulation took approximately 2 minutes for each source position. Figure 3 illustrates some of the results.

2.3. Objective function

Two parameters were selected to quantify the acoustic suitability of the geometry. The first was the uniformity of the distribution of the energy between all the source receiver pairs. This corresponds with a design objective of ensuring that all listeners have a similar experience in the hall. This was calculated using an autocorrelation function, similarly to how the diffusion coefficient is calculated\(^1\). The equation is as follows:

\[
U = \frac{\left( \sum_{i=1}^{n_s} \sum_{j=1}^{n_r} \frac{L_{ij}}{10^{10}} \right)^2 - \sum_{i=1}^{n_s} \sum_{j=1}^{n_r} \left( \frac{L_{ij}}{10^{10}} \right)^2}{(n-1) \sum_{i=1}^{n_s} \sum_{j=1}^{n_r} \left( \frac{L_{ij}}{10^{10}} \right)^2}
\]

(1)

where: \(i = \text{Receiver}, j = \text{Source}, L_{ij} = \text{Level at receiver } i \text{ for source } j, n_i = \text{Number of Receivers}, \) and \(n_j = \text{Number of Sources}.

Consequently, if all receivers for all sources receive equal energy the uniformity would be 1, and a score of zero would be attained for all energy being focused to one receiver. The second criteria was the total energy in the impulse response, averaged over every source

![Figure 3. The results of the FDTD simulation at 20 ms, for a low scoring geometry (Left) and a high scoring geometry (Right). The sound field is shown for a single sound source. In the lower right of the right geometry, one can see a second pair of reflections following the sidewall reflections that are not present in the left geometry.](image-url)
receiver combination. Since only the early energy until 100ms is contained in the impulse responses, this is roughly equivalent to designing for high Clarity ($C_{80}$) values or high early Strength (G). These two components were normalized to equally weight their contribution to the score, and added to make the final fitness value.

2.4. Optimization algorithm

The goal of the optimization algorithm is to find the set of parameters that maximizes the objective function, in this case the score based on acoustic criteria. In the case of room acoustics, evaluating the objective function is computationally very expensive as it involves acoustic modeling of the concert hall controlled by the parameters. It is reasonable to spend some time finding the best parameter sets to evaluate, because the evaluation will probably dominate the computational cost of the optimization process. A Bayesian approach is chosen, because it will utilize all the information available to determine the evaluation points which will be the most informative. In this way, an optimal geometry can be determined with the fewest number of simulations.

The Bayesian approach is based on a stochastic model of the objective function. The model is updated incrementally for every new parameter set that is evaluated. It is assumed that the first differences in scores are normally distributed. Conditional means and conditional variances are calculated based on the evaluations of the geometry scores. Then, it is possible to calculate the probability that the evaluation of a given set of parameters will improve the best maximum found so far. For simplicity, a one-step optimal Bayesian sampling is chosen, which means that the next point of evaluation is the one with the highest probability of finding the maximum score. Looking several steps forward is possible, but becomes difficult to calculate.

Because calculating the probability for all possible parameter sets, a further simplification is required. The one suggested by Mockus\textsuperscript{7,8} that is an optimization problem in itself is chosen. Since the current problem is of low dimensionality, i.e. there are only three parameters, an exhaustive search in a regular grid is feasible. For problems with higher dimensions, such as fewer than six, it could be possible to analytically solve the set of local maximum probabilities and find the best one among those. This requires keeping a spatial data structure of the search space, e.g. a Delaunay triangulation or a weighted Voronoi diagram. For even high dimensions, constructing and updating spatial data structures becomes cumbersome and too expensive. In addition, based on Carathéodory’s theorem\textsuperscript{5}, in high dimensions most of the sample points lie on the convex hull of the N-dimensional point cloud, so describing its structure by a complex data structure is not justified from the point of view of computational efficiency. Thus, a simple uniform random sampling method is used to find the parameter set with the highest probability of producing the best score. It is important to note that the random sampling is for the probability, not of the score itself. While evaluations of the geometry score are expensive, evaluations of which parameter set will most probably lead to the best result are relatively cheap. The randomness in the evaluations of this probability does not directly transfer to the evaluations of the scores, because at each step, we choose the best of the random points in the sense of maximizing the probability.
As shown by Mockus\textsuperscript{8}, by using the Bayesian approach, the density of the sampling points will be higher near the global maximum than elsewhere. Thus, with a sufficient number of sampling points, it is expected that the global maximum will be found more accurately than with random sampling. Figure 4 shows the progress of the optimization in the case of a complex 2-dimensional function.

3. RESULTS
The optimization was conducted for two scenarios for comparison. The first was for a single source, and the second was for all 10 sources. The simulations were run until 800 iterations were complete and arbitrarily stopped.

3.1. Single source
Figure 5 illustrates the scores calculated for each parameter combination with a single sound source. The axes represent the range of each of the three parameters, and the diameter of the sphere represents the score at that point in the parameter space. The scores have been normalized with respect to the maximum score, and progressively scaled to emphasize the difference between good and bad solutions. The figure shows that there are many regions and isolated points that produce good solutions, indicating that a wide variety of forms can satisfy these particular design criterion.

Figure 6 displays the geometric forms that correspond to the eight best and eight worst scores for a single sound source in the top and bottom rows respectively. Here too it is seen that many different forms produce nearly equally good results. It can also be seen that some good solutions are very similar to some bad solutions. This reinforces the importance of fine-tuning in architectural acoustic design, as there may be significant differences even within a particular formal typology.
shows some of the simulation results. In the right geometry, two reflections can be seen in the lower right side, arriving after the sidewall reflections, these could provide the early energy boost that resulted in a better score. Optimizing geometry for specific reflection arrival times and directions is an area for further work.

3.2. Multiple sources
Figure 7 illustrates the scores calculated for each parameter combination with ten sound sources. There is a marked difference in the solution space when multiple sources are considered. There are generally fewer good solutions, and they are confined to narrower regions, e.g. Curve/Sidewall ratios between 0 and -0.5. Also, the best solution is significantly better than the next best solution. Figure 8 displays the geometric forms that correspond to the eight best and eight worst scores when considering ten sound sources, in the top and bottom rows respectively. In contrast to the single source case, many of the worst solutions have the same formal typology, namely perpendicular, orthogonal sidewalls, and this type does not show up in the best solutions. This indicating the benefit of shaping these surfaces, if not conclusively defining a single best shape.
4. CONCLUDING REMARKS

This paper describes a system for iteratively generating and testing architectural geometries to attain one that satisfies specific acoustic criteria. The method has demonstrated itself as a useful tool to examine multiple forms’ effectiveness, however the acoustic criteria were not specific enough to specify a single optimal solution. Acoustic design will remain a mixture of art and science, but tools such as this can help to inform the art.

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REFERENCES


