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Salerno, Grazia

## Topology puts solitons in the corner

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1 PHOTONICS

2 **Topology puts solitons in the corner**

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4 Grazia Salerno, [grazia.salerno@aalto.fi](mailto:grazia.salerno@aalto.fi)

5 Department of Applied Physics, Aalto University School of Science

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7 *Nonlinearity and topology are both linked to symmetries, but what happens when the*  
8 *two are combined is not a trivial question. In a nonlinear photonic higher-order*  
9 *topological insulator, solitons localize on the corners together with the topological*  
10 *modes.*

11

12

13 Many physical phenomena are described as linear effects, even though nonlinearities  
14 are important in a range of systems across the natural sciences. From fluids to atomic  
15 condensates and optics, one hallmark of nonlinear systems is the soliton — a robust  
16 and highly localized wave that doesn't change size or shape. Solitons are self-sustained  
17 by a perfect balance between linear and nonlinear effects. But what happens to this  
18 delicate balance when other localization effects are introduced? For example, one can  
19 wonder if solitons can coexist with the localized edge states of a topological insulator.  
20 Writing in *Nature Physics*, Marco Kirsch and colleagues have tackled this issue by  
21 investigating the fate of topological boundary modes in the nonlinear regime of a  
22 photonic system [1].

23

24 Topology is the study of the properties that are immutable under continuous  
25 deformations. In condensed-matter physics, it was first applied to explain the quantum  
26 Hall effect and topological insulators — bulk insulating materials that carry a perfectly  
27 quantized transverse Hall current [2]. We now understand this quantum Hall current as  
28 the transport of electrons through topological one-way states along a single-atom  
29 layer on the edge of the system, and the existence of such edge states depends only  
30 on intrinsic properties of the bulk. These states are characterized by a topological  
31 invariant — a property that is constant under continuous deformations. If this invariant  
32 is zero, the state has no specifically topological properties and is called 'trivial'. If it is  
33 nonzero, the state is topological.

34

35 Topological concepts are no longer limited to condensed-matter systems, but appear  
36 in every aspect of physics — from quantum mechanics to classical-waves like light.  
37 Topological photonics, in particular, uses artificial structures specifically designed to  
38 exhibit topological properties. Like the electronic states in a topological insulator, the  
39 light propagation in such a non-trivial topological structure depends only on the  
40 intrinsic design and not on external perturbations. This ability to control robust optical  
41 transport is one of the main motivations for topological photonics. With the  
42 demonstration of unidirectionally propagating edge states of light and topological  
43 lasers, the field promises to deliver a range of applications.

44

45 If boundary modes of a standard topological insulator are localized at the edge, those  
46 of higher-order topological insulators are located at the edge of the edge — corners  
47 of a 2D system or hinges of a 3D system [3]. Higher-order topological systems require  
48 the existence of crystalline symmetries of the lattice — such as inversion or mirror  
49 symmetries — that affect how the electron density is centred within the unit cell, which  
50 in turn determines the polarization of the crystal. In higher-order topological insulators,  
51 the polarization is quantized, and a fractional charge accumulates on the corners of the  
52 system.

53

54 So far, topological phases have mostly been studied in the linear regime, but work on  
55 the effects of optical nonlinearities in photonic topological insulators has just begun [4,  
56 5]. Kirsch and colleagues contributed to this recent effort by studying the role of  
57 nonlinearities in higher-order topological systems. They used a system of coupled  
58 waveguide arrays, arranged in a breathing kagome lattice (Fig. 1), which is known to be  
59 a higher-order topological insulator model in the linear regime. By changing the  
60 distance between the lattice sites, they realized two possible configurations: a  
61 topological and a trivial one. Laser light entered the waveguides at the corner of the  
62 lattice, propagated through the array by evanescently coupling to neighbouring sites,  
63 and was collected at the end, where the intensity distribution on the detector provided  
64 information about the lattice mode. Kirsch and colleagues introduced nonlinearities by  
65 using a waveguide medium with a Kerr-type nonlinearity, such that the refractive index  
66 depended on the light intensity.

67

68 For sufficiently high input power, solitons formed and the output light was confined  
69 around the input waveguide. In the trivial case, the soliton was localized over the entire  
70 unit cell of the lattice, whereas, for the topological case, the soliton was more strongly  
71 localized on the corner site (Fig. 1). Kirsch and colleagues then calculated the  
72 topological invariant, which for higher-order topological insulators is given by the bulk  
73 polarization, to back up the different nature of these two types of corner modes.  
74 Interestingly, the onset of nonlinearity proved not to be detrimental to the polarization  
75 value, which remains non-zero in the topological case, and strictly zero in the trivial  
76 configuration. Added nonlinearity therefore did not change the topological nature of  
77 the modes. This analysis distinguishes between the emergence of nonlinear topological  
78 corner states and the formation of trivial solitons in such structures.

79

80 The added nonlinearities had another effect: the bulk polarization was no longer  
81 quantized, meaning that the symmetries protecting the topological corner modes were  
82 broken. A very interesting scenario would be if nonlinearities could leave these  
83 symmetries unbroken. Or even more strikingly if they could induce a non-trivial  
84 topological phase starting from a trivial one. Preliminary steps in this direction have  
85 been taken [6], but the study of nonlinear topological phases is still in its infancy. The  
86 regime of strong nonlinearities is particularly interesting because it requires a quantum  
87 optical description and genuine interaction-induced topological effects can appear [7,  
88 8], which could result in the realization of strongly correlated topological phases of  
89 photons.

90

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92

93

94 Figure 1:

95 **Topological soliton formation.** An array of waveguides arranged on a  
96 breathing kagome lattice is an optical model for a linear higher-order  
97 topological insulator. Such a model has topological corner modes  
98 originating from a nonzero bulk invariant. When the system enters a  
99 nonlinear regime, the light becomes more localized and topological  
100 solitons form on the corners of the system. This shows that the topological

101 order is robust against the onset of nonlinearity, leaving the bulk invariant  
102 nonzero.

103

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