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Optimization-Based Distribution System Reliability Evaluation: An Enhanced MILP Model

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***Abstract*—Standard mathematical-programming-based models have attracted considerable attention for optimizing distribution system planning and operation due to their salient advantages. More specifically, mixed-integer linear programming (MILP) has proven very effective in modeling such problems. The availability of optimization software for efficiently solving MILP problems with guaranteed convergence to optimality while providing a measure of the distance to the optimal solution has made MILP models more popular. Although a plethora of efficient MILP formulations have been proposed for planning and operating studies of distribution grids, incorporating reliability into such models is still challenging. Recently, several innovative techniques have been proposed in the literature for optimization-based reliability evaluation of distribution networks. However, either oversimplifications or high computation costs are featured, thereby limiting the applicability of such approaches in practical problems. To overcome this issue, this paper presents an enhanced MILP model for distribution system reliability evaluation. The proposed model boosts the computational effectiveness without sacrificing solution accuracy. Numerical experience with the proposed model demonstrates its superior performance over the state of the art.**

***Index Terms*—Electricity distribution system, mixed-integer linear programming, optimization, reliability assessment.**

NOMENCLATURE

Indices

$l, l', \bar{l}, \hat{l}$	Indices for feeder sections.
m	Index for paths.
n	Index for load nodes.

Sets

E_l	Set of feeder section(s) at the end(s) of a chain of series-connected feeder sections including feeder section l.
Λ	Set of all feeder sections.
$\Lambda_{l,\hat{l}}$	Set of feeder sections in the path from feeder section l to feeder section $\hat{l}$.
Λ^e	Subset of Λ containing feeder sections that are not in $\tilde{\Lambda}$ or Λ^S.
Λ^m	Set of feeder sections in path m.
Λ^S	Subset of Λ containing feeder sections directly connected to substation nodes.
$\tilde{\Lambda}$	Subset of Λ containing feeder sections connected to the first- and second-order nodes, i.e., nodes to which one and two feeder sections are connected.
$\Upsilon_{l,\bar{l}}$	Set of all paths between feeder sections l and $\bar{l}$.
Ω	Set of all load nodes.

Parameters

D_l^r, D_l^s	Repair and switching times.
M	A sufficiently large positive number.
N_n	Number of customers connected to load node n.
P_n	Annual average demand at load node n.
$\alpha_{l,n}$	Binary parameter, which is -1 or $+1$ if feeder section l is connected to node n and the predetermined arbitrary flow direction is toward or away from node n, respectively, being 0 otherwise.
λ_l	Failure rate of feeder section l.
$\omega_D, \omega_E, \omega_F$	Weighting factors for $SAIDI$, $EENS$, and $SAIFI$.

Variables

$EENS$	Expected energy not served.
f_l^o, f_l^p	Continuous variables representing the flow across feeder section l if its flow direction is respectively opposite and identical to the predetermined arbitrary flow direction, being 0 otherwise.
n_l^o, n_l^p	Continuous variables representing the number of customers connected to the nodes downstream of feeder section l if its flow direction is respectively opposite and identical to the predetermined arbitrary flow direction, being 0 otherwise.
N_l^d	Total number of customers connected to the nodes downstream of feeder section l if its switch is closed, being 0 otherwise.

N_l^u Total number of customers connected to the nodes upstream of feeder section l if its switch is closed, being 0 otherwise.

P_l^d Total demand of the nodes downstream of feeder section l if its switch is closed, being 0 otherwise.

P_l^u Total demand of the nodes upstream of feeder section l if its switch is closed, being 0 otherwise.

$SAIDI$ System average interruption duration index.

$SAIFI$ System average interruption frequency index.

y_l^o, y_l^p Binary utilization variables of feeder section l, which are equal to 1 if feeder section l is in service and its flow direction is respectively opposite and identical to the predetermined arbitrary flow direction, being 0 otherwise.

$z_{l,\bar{l}}$ Binary-valued continuous variable, which is equal to 1 if feeder section l is in a feeder whose first branch is $\bar{l}$ and the switch of feeder section l is closed, being 0 otherwise.

I. Introduction

Cost, operational limits, and reliability are considered the three principal dimensions based upon which distribution systems are planned and operated [1], [2]. Although a plethora of efficient techniques have been developed to attain the optimal solution for planning and operating distribution systems considering the first two aspects, i.e., cost and technical factors [3]–[7], the integration of reliability into the state-of-the-art mathematical models for topology-variable-based studies, e.g., expansion planning and network reconfiguration, is still a major challenge. This is mainly because 1) reliability indices depend on the network topology, which is an outcome of such studies, and, thus, unknown *a priori*, and 2) classical reliability assessment approaches rely on algorithmic procedures that are easy to simulate but hard to formulate mathematically in a closed form. Restricted by these limitations, the majority of reliability-constrained distribution system studies in the existing literature have been carried out based on heuristic [8] and metaheuristic [9] techniques. The principal idea behind these approaches is the evaluation of several candidate solutions with definite topologies using traditional reliability assessment methods. Such candidate solutions can be either members of a solution pool obtained by iteratively solving mixed-integer linear programming (MILP) models [8] or individuals generated at each iteration of nature-inspired metaheuristic algorithms [9]. However, neither of these techniques guarantees convergence to optimality.

Considering the stressed significance of continuity of electricity supply in today's world [1] and the notable share of distribution systems in the power interruptions of end users [2], accounting for reliability in distribution system studies is essential. In this respect, several attempts have been made to explicitly model standard distribution system reliability indices as a set of mixed-integer linear expressions [10]–[17]. Such expressions can be readily incorporated into the state-of-the-art MILP models for distribution system operation and planning to account for the reliability aspect. In [10], a set of MILP expressions is developed to model three widely used distribution reliability indices, namely system average interruption duration index (SAIDI), system average interruption frequency index (SAIFI), and expected energy not served (EENS). These expressions are then incorporated into a mixed-integer second-order conic programming model for optimal distribution network reconfiguration. A novel MILP formulation is presented in [11] for analytical reliability assessment of distribution networks based upon which an MILP model for reliability-constrained distribution system expansion planning is developed in [12]. Aiming to enhance the computational performance, an algebraic approach is described in [13] for distribution network reliability evaluation. In [14] and [15], fault-enumeration-based reliability assessment models are proposed to consider post-fault network reconfiguration. Alternatively, MILP models are respectively devised in [16] and [17] for reliability-constrained expansion planning of passive and active distribution networks.

Despite their significant contributions to the field of optimization-based distribution reliability assessment, the models previously reported in the literature either are computationally expensive [11]–[15] or rely on far drastic simplifications [10], [16], [17]. As a consequence, the former may lead to intractability for practical large-scale networks, whereas the latter may yield inaccurate reliability indices, thereby potentially deteriorating the solution quality in terms of optimality. To address such issues, in [18], we recently presented a novel MILP-based model that outperforms the existing formulations [10]–[13], [16], [17] without sacrificing accuracy.

The main contribution of this paper is the innovative extension of our work developed in [18] comprising several modeling improvements that yield a smaller, albeit equivalent, instance of MILP when compared to the state of the art. This advancement significantly boosts the computational performance – especially for real-size networks.

In order to validate its performance, the proposed reliability assessment model is incorporated into a reliability-constrained optimal distribution network reconfiguration problem. The outcomes are benchmarked using equivalent models based on the reliability expressions presented in [12] and [18], which are relevant accurate formulations that can be applied to reliability-constrained topology-variable-based optimization problems. Note that the model presented in [13] cannot be straightforwardly leveraged in such problems, whereas the formulations devised in [10], [16], [17] entail oversimplifications that lead to underestimating reliability metrics. Our extensive numerical experience corroborates the computational superiority of the proposed model, which becomes more evident as the system size grows.

The remainder of this paper is organized as follows. Section II is devoted to the proposed optimization-based reliability assessment model, where mixed-integer linear formulations are devised for widely used distribution system reliability measures, namely EENS, SAIDI, and SAIFI. Section III presents the application of the proposed model to optimize reliability-constrained distribution system reconfiguration. Numerical results are provided in Section IV. Finally, Section V concludes the paper.

II. Proposed Reliability Assessment Model

This section explains the proposed formulations for EENS, SAIDI, and SAIFI. These models are developed in the context of topology-variable-based distribution system reliability evaluation, and thus adopt the common assumptions and considerations presented in [11]–[13] and [18] about the sequence of events regarding failure impact mitigation and arrangement of protection and sectionalizing devices in the network.

A. Expected Energy Not Served

According to [18], EENS is modeled by (1):

$$EENS = \sum_{l\in\Lambda} \left(\lambda_l D_l^r P_l^d + \lambda_l D_l^s P_l^u\right), \tag{1}$$

where $\lambda_l D_l^r P_l^d$ and $\lambda_l D_l^s P_l^u$ respectively express the expected annual energy not supplied due to the repair and switching actions associated with failures on feeder section l.

To calculate these two terms, the total power connected to the nodes downstream and upstream of feeder sections, i.e., P_l^d and P_l^u, respectively, must be modeled. To that end, we consider the operation of a fictitious lossless network with the same structure as the distribution network under study, as formulated in (2)–(7):

$$\sum_{l\in\Lambda} \alpha_{l,n}\left(f_l^p - f_l^o\right) + P_n = 0; \forall n \in \Omega \tag{2}$$

$$0 \le f_l^p \le M y_l^p; \forall l \in \Lambda \tag{3}$$

$$0 \le f_l^o \le M y_l^o; \forall l \in \Lambda \tag{4}$$

$$y_l^p + y_l^o \le 1; \forall l \in \Lambda \tag{5}$$

$$y_l^p, y_l^o \in \{0,1\}; \forall l \in \Lambda \tag{6}$$

$$\sum_{l\in\Lambda|\alpha_{l,n}=-1} y_l^p + \sum_{l\in\Lambda|\alpha_{l,n}=1} y_l^o = 1; \forall n \in \Omega. \tag{7}$$

Expressions (2)–(5) represent Kirchhoff's current law (KCL) for this fictitious network, where the power balance at load nodes is imposed in (2), while (3) and (4) prevent the flow of fictitious power across unused feeder sections. In (5), fictitious power is precluded from simultaneously flowing in both directions across feeder sections. Expressions (6) represent the binary nature of utilization variables y_l^p and y_l^o. Finally, radial operation of the network is ensured using (7), where the number of supplying feeder sections for every node is enforced to equal 1 [12]. It is worth noting that the use of binary variables y_l^p and y_l^o to characterize the direction of fictitious flows across feeder sections is a salient modeling aspect compared to the formulation of [18].

Considering the radial operation of the network imposed by (7), the fictitious power flow across each feeder section is equal to the total demand connected to its downstream nodes, as expressed in (8):

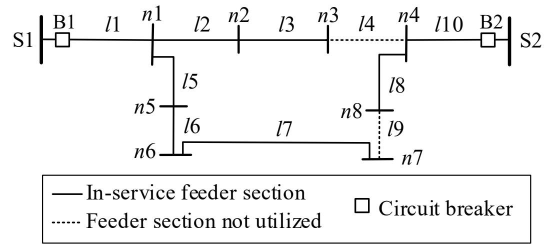

Fig. 1. Illustrative distribution network.

$$P_l^d = f_l^p + f_l^o; \forall l \in \Lambda. \tag{8}$$

The total power connected to the nodes upstream of a given feeder section l can also be modeled as the difference between the fictitious flow of the first feeder section of the corresponding feeder and that of feeder section l. This is because the fictitious flow of the first branch of each feeder is equal to the total demand of that feeder. However, the feeder in which feeder section l is located is not known *a priori*. As an example, let us consider the illustrative distribution network depicted in Fig. 1, where feeder section $l7$ is in the left-hand-side feeder whose first feeder section is $l1$. If we reconfigure the network by switching off feeder section $l5$ and bringing feeder section $l9$ in service, feeder section $l7$ becomes part of the right-hand-side feeder whose first feeder section is $l10$.

In order to capture the impact of network configuration on the demand upstream of feeder section l, i.e., P_l^u, we consider binary-valued continuous variables $z_{l,\bar{l}}$. Note that $z_{l,\bar{l}}$ is equal to 1 if feeder section $\bar{l}$ is the first branch of the feeder to which feeder section l belongs, being 0 otherwise. These auxiliary variables allow modeling P_l^u for the feeder sections that are not directly connected to a substation node as per (9):

$$P_l^u = \sum_{\bar{l}\in\Lambda^S} z_{l,\bar{l}}\left(\left(f_{\bar{l}}^p + f_{\bar{l}}^o\right) - \left(f_l^p + f_l^o\right)\right); \forall l \in \Lambda \setminus \Lambda^S, \tag{9}$$

which can be expressed in a linear form using (10) and (11):

$$P_l^u \ge 0; \forall l \in \Lambda \setminus \Lambda^S \tag{10}$$

$$P_l^u \ge \left(f_{\bar{l}}^p + f_{\bar{l}}^o\right) - \left(f_l^p + f_l^o\right) - M(1 - z_{l,\bar{l}}); \quad \forall l \in \Lambda \setminus \Lambda^S, \forall \bar{l} \in \Lambda^S. \tag{11}$$

As for the feeder sections directly connected to substation nodes, P_l^u is set to 0 in (12) as no demand is connected upstream:

$$P_l^u = 0; \forall l \in \Lambda^S. \tag{12}$$

The model for $z_{l,\bar{l}}$ is presented next. The non-negativity of $z_{l,\bar{l}}$ is characterized in (13). According to (14), if feeder section l is in service, the summation of $z_{l,\bar{l}}$ over $\bar{l}$ must be equal to 1 since, in this case, feeder section l must have one corresponding first feeder section $\bar{l}$.

$$z_{l,\bar{l}} \ge 0; \forall l \in \Lambda, \forall \bar{l} \in \Lambda^S \tag{13}$$

$$\sum_{\bar{l} \in \Lambda^S} z_{l,\bar{l}} = y_l^p + y_l^o; \forall l \in \Lambda. \quad (14)$$

The characterization of $z_{l,\bar{l}}$ is completed based on the partition of the set of feeder sections Λ into two subsets: 1) subset $\tilde{\Lambda}$ of feeder sections connected to the first- and second-order load nodes, i.e., load nodes to which one and two feeder sections are respectively connected, e.g., feeder sections $l3$, $l6$, $l7$, and $l9$ in Fig. 1, and 2) subset Λ^e of other feeder sections not directly connected to substation nodes, e.g., $l2$, $l4$, $l5$, and $l8$ in the toy network of Fig. 1. This partition-based model is the main distinctive enhancement over the state-of-the-art formulation presented in [18] upon which the proposed model is built.

The feeder sections belonging to the first subset $\tilde{\Lambda}$ are, in fact, part of a chain of feeder sections connected in series. As an example, feeder section $l7$ is part of the chain of feeder sections $l5$–$l9$. Or, as another example, if we drop feeder section $l4$ from the illustrative network, the degree of load nodes $n2$ and $n4$–$n8$ would be 2 and that of $n3$ would be 1. Hence, feeder sections $l3$ and $l6$–$l9$ would belong to $\tilde{\Lambda}$. For such feeder sections, the tightest lower bound for $z_{l,\bar{l}}$ can be determined based on the corresponding values for feeder sections at the extreme(s) of the chain as expressed in (15):

$$z_{l,\bar{l}} \geq z_{\hat{l},\bar{l}} + \sum_{l' \in \Lambda_{l,\hat{l}}} \left(y_{l'}^p + y_{l'}^o - 1\right); \forall l \in \tilde{\Lambda}, \forall \bar{l} \in \Lambda^S, \forall \hat{l} \in E_l, \quad (15)$$

where each feeder section $\hat{l}$ is at one extreme of the chain including feeder section l. Expressions (15) imply that if all series-connected feeder sections l' that connect feeder section l to feeder section $\hat{l}$ are in service, i.e., $y_{l'}^p + y_{l'}^o = 1$, then $z_{\hat{l},\bar{l}}$ sets the tightest lower bound for $z_{l,\bar{l}}$. This is because, in such cases, both feeder sections l and $\hat{l}$ are in the same feeder, and, hence, have the same corresponding first feeder section $\bar{l}$. As an example, let us consider feeder section $l7$ in Fig. 1, for which two possible first feeder sections $l1$ and $l10$ exist, as $\Lambda^S = \{l1, l10\}$. Feeder section $l7$ is part of chain $l5$–$l9$, hence, $E_{l7} = \{l5, l8\}$, $\Lambda_{l7,l5} = \{l5 - l7\}$, and $\Lambda_{l7,l8} = \{l7 - l9\}$. In addition, as per Fig. 1, feeder section $l9$ is not in service, i.e., $y_{l9}^p + y_{l9}^o = 0$, whereas feeder sections $l5$–$l8$ are in service, i.e., the summation of their respective utilization variables, $y_{l'}^p + y_{l'}^o$, is equal to 1. Accordingly, expressions (15) yield:

$$z_{l7,l1} \geq z_{l5,l1} + \sum_{l' \in \{l5-l7\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l7,l1} \geq z_{l5,l1}$$
$$z_{l7,l10} \geq z_{l5,l10} + \sum_{l' \in \{l5-l7\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l7,l10} \geq z_{l5,l10}$$
$$z_{l7,l1} \geq z_{l8,l1} + \sum_{l' \in \{l7-l9\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l7,l1} \geq z_{l8,l1} - 1$$
$$z_{l7,l10} \geq z_{l8,l10} + \sum_{l' \in \{l7-l9\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l7,l10} \geq z_{l8,l10} - 1.$$

Considering that variables $z_{l,\bar{l}}$ can take non-negative values less than or equal to 1 based on (13) and (14), $z_{l5,l1}$ and $z_{l5,l10}$ respectively determine the tightest lower bounds for $z_{l7,l1}$ and $z_{l7,l10}$. Note that this is desired in this example, since both feeder sections $l5$ and $l7$ are in the same feeder for the radial operating topology depicted in Fig. 1.

For the feeder sections belonging to the second subset Λ^e, e.g., feeder section $l5$ in Fig. 1, expressions (16) set the tightest lower bound for $z_{l,\bar{l}}$ to 1 in case all the feeder sections in the path between feeder section l and first feeder section $\bar{l}$ are in service:

$$z_{l,\bar{l}} \geq 1 + \sum_{l' \in \Lambda^m} \left(y_{l'}^p + y_{l'}^o - 1\right); \forall l \in \Lambda^e, \forall \bar{l} \in \Lambda^S, \forall m \in \Upsilon_{l,\bar{l}}. \quad (16)$$

For the sake of exemplifying (16), let us consider feeder section $l5$ in Fig. 1. Bearing in mind that the network must be operated radially, there are three possible paths from this feeder section to one of the first feeder sections $l1$ and $l10$, namely $\Lambda^{m1} = \{l1, l5\}$, $\Lambda^{m2} = \{l2 - l5, l10\}$, and $\Lambda^{m3} = \{l5 - l10\}$. Accordingly, considering the radial operating topology illustrated in Fig. 1 to determine the value of the binary utilization variables, expressions (16) become:

$$z_{l5,l1} \geq 1 + \sum_{l' \in \{l1,l5\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l5,l1} \geq 1$$
$$z_{l5,l10} \geq 1 + \sum_{l' \in \{l2-l5,l10\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l5,l10} \geq 0$$
$$z_{l5,l10} \geq 1 + \sum_{l' \in \{l5-l10\}} (y_{l'}^p + y_{l'}^o - 1) \Rightarrow z_{l5,l10} \geq 0,$$

while, also, expressions (13) and (14) give rise to:

$$z_{l5,l1}, \ z_{l5,l10} \geq 0$$
$$z_{l5,l1} + z_{l5,l10} = 1,$$

which can only be satisfied if $z_{l5,l1}$ becomes 1, whilst $z_{l5,l10}$ is 0, as desired. Using these values in the relations devised for feeder section $l7$ yields $z_{l7,l1} \geq 1$ and $z_{l7,l10} \geq 0$. Thus, considering (14) for $l7$ yields $z_{l7,l1} = 1$ and $z_{l7,l10} = 0$ as expected.

B. System Average Interruption Duration Index

Expression (17) models SAIDI in terms of the failure rates of feeder sections, λ_l, the repair and switching durations, D_l^r and D_l^s, respectively, and the total numbers of customers connected to the nodes downstream and upstream of feeder sections, N_l^d and N_l^u, respectively:

$$SAIDI = \sum_{l \in \Lambda} \left(\lambda_l D_l^r N_l^d + \lambda_l D_l^s N_l^u\right). \quad (17)$$

In order to model N_l^d and N_l^u, we apply KCL equations to a fictitious network where the demand at every load node is set equal to the number of customers connected to such a node, as expressed in (18)–(20):

$$\sum_{l \in \Lambda} \alpha_{l,n} \left(n_l^p - n_l^o\right) + N_n = 0; \forall n \in \Omega \quad (18)$$

$$0 \leq n_l^p \leq M y_l^p; \forall l \in \Lambda \quad (19)$$

$$0 \leq n_l^o \leq M y_l^o; \forall l \in \Lambda. \quad (20)$$

N_l^d and N_l^u can be modeled by (21)–(24), which are structurally identical to (8) and (10)–(12), respectively, with n_l^p and n_l^o playing the role of f_l^p and f_l^o:

$$N_l^d = n_l^p + n_l^o; \forall l \in \Lambda \tag{21}$$

$$N_l^u \geq 0; \forall l \in \Lambda \setminus \Lambda^S \tag{22}$$

$$N_l^u \geq \left(n_{\bar{l}}^p + n_{\bar{l}}^o\right) - \left(n_l^p + n_l^o\right) - M\left(1 - z_{l,\bar{l}}\right); \quad \forall l \in \Lambda \setminus \Lambda^S, \forall \bar{l} \in \Lambda^S \tag{23}$$

$$N_l^u = 0; \forall l \in \Lambda^S. \tag{24}$$

C. System Average Interruption Frequency Index

Considering its intrinsic similarities with SAIDI, SAIFI can be readily modeled by (25):

$$SAIFI = \sum_{l \in \Lambda} \lambda_l \left(N_l^d + N_l^u\right). \tag{25}$$

III. Application of the Proposed Reliability Assessment Model

In this section, the application of the proposed topology-variable-based reliability assessment model is exemplified within the context of optimal reliability-constrained distribution network reconfiguration, and its implementation procedure is explained.

A. Optimal Reliability-Constrained Network Reconfiguration

The optimization problem is cast as:

$$\min \ \omega_E EENS + \omega_D SAIDI + \omega_F SAIFI \tag{26}$$

Subject to:

$$\text{Expressions (1)–(8) and (10)–(25).} \tag{27}$$

As expressed in (26), the optimization goal is the minimization of the weighted summation of the reliability indices, namely EENS, SAIDI, and SAIFI. Admittedly, the optimal distribution network reconfiguration problem can be driven by other technical and economic terms. Nonetheless, such aspects are disregarded in (26) in order to focus on the performance of the proposed reliability model. This problem is subject to the set of constraints (27) comprising the proposed models for EENS, SAIDI, and SAIFI, as well as the radiality constraint.

B. Implementation Process

The flowchart of the implementation procedure is represented in Fig. 2, where a preprocessing stage is considered to build the sets E_l, $\Lambda_{l,\hat{l}}$, Λ^e, Λ^m, Λ^S, $\tilde{\Lambda}$, and $\Upsilon_{l,\bar{l}}$ based on the network structure characterized by the sets Λ and Ω as well as parameters $\alpha_{l,n}$. The outcomes of this stage together with the other input parameters, including the reliability data, nodal demands, and the number of customers connected to load nodes, are then used to render the optimization model (26)–(27).

As the optimization problem (26)–(27) is an instance of MILP, finite convergence to optimality is guaranteed by efficient off-the-shelf software based on the branch-and-cut algorithm, while also providing a measure of the distance to the optimum during the solution process [19], [20].

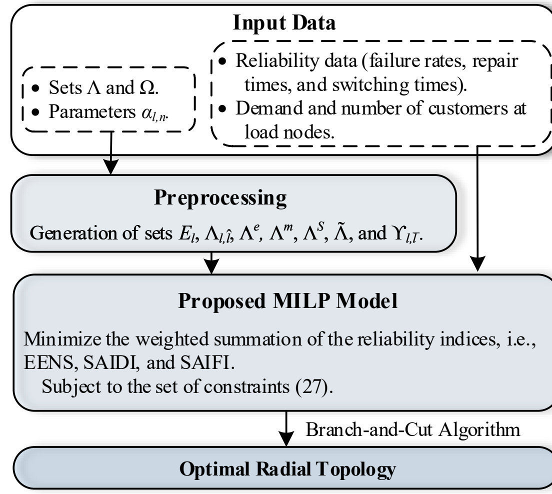

Fig. 2. Implementation procedure.

IV. Numerical Results

This section presents the numerical experience with four distribution test systems with 24, 54, 86, and 136 nodes. Data for these test systems are posted online [21].

The performance of the proposed reliability evaluation model has been assessed with the state of the art of topology-variable-based characterization of distribution system reliability. To that end, we have also considered two different, albeit equivalent, mixed-integer linear instances of the distribution reconfiguration problem wherein the reliability model is respectively built upon the enumerative formulation devised in [12] and the formulation without feeder set partitioning presented in [18], which, similar to the proposed reliability model, is non-enumerative. Such benchmarks are hereinafter referred to as EM and N-EM, respectively. All the optimization problems have been implemented in GAMS 24.9 and solved by CPLEX 12.6, with an optimality gap of 0, running on a Fujitsu CELSIUS W530 Power PC with an Intel Xeon E3-1230 processor at 3.30 GHz and 32 GB of RAM.

For the four distribution grids, Table I summarizes the optimal results, namely the reliability indices for the optimal radial configurations and the feeder sections that must be switched off to reach such optimal topologies. As expected, identical solutions are achieved by the three equivalent models.

Notwithstanding the identical solutions obtained, the choice for the reliability assessment model has a significant impact on the computational performance. As reported in Table II for each case, the formulation relying on the proposed reliability model and N-EM both require the same number of decision variables, which is up to two orders of magnitude less than that of EM. Note that, for each system, the number of binary decision variables is identical for the three models, being respectively 66, 126, 196, and 290 for the 24-, 54-, 86-, and 136-node networks. Moreover, the proposed approach entails fewer constraints than EM and N-EM do, as per Table II. This difference becomes more evident for larger networks comprising longer feeders with more series-connected feeder sections,

TABLE I
OPTIMAL SOLUTIONS

	24 Nodes	54 Nodes	86 Nodes	136 Nodes
EENS	5.784	11.249	52.271	13.245
SAIDI	0.153	0.784	1.247	0.713
SAIFI	0.260	0.610	1.026	0.769
Switched-Off Feeder Sections	1, 3, 5, 8, 11–13, 18–21, 25, 29	6, 11–13, 15–17, 21, 28, 30, 54, 58, 62	1, 7, 11, 12, 17, 23, 33, 41, 45, 68, 78, 82–84, 87, 96	9, 35, 53, 58, 70, 83, 92, 113, 137, 138

EENS: MWh/year SAIDI: hours/customer/year
SAIFI: failures/customer/year

TABLE II
PROBLEM DIMENSION AND COMPUTATIONAL PERFORMANCE

Test Grid	Approach	Number of Decision Variables	Number of Constraints	Simulation Time (s)
24 Nodes	Proposed	562	1,547	0.30
	EM	16,400	20,612	0.97
	N-EM	562	1,564	0.27
54 Nodes	Proposed	1,144	2,677	1.20
	EM	67,096	75,934	11.11
	N-EM	1,144	2,798	1.04
86 Nodes	Proposed	2,132	5,819	5.56
	EM	227,698	248,172	23.11
	N-EM	2,132	6,350	7.10
136 Nodes	Proposed	2,276	5,692	257.29
	EM	264,836	291,480	1,593.71
	N-EM	2,276	10,650	862.71

thereby resulting in a considerable boost in the computational performance for such cases. As can be observed, for the 24- and 54-node systems, the proposed model behaves similarly to N-EM, both substantially outperforming EM. Interestingly, for the 86-node system, the use of the proposed formulation considerably reduces the computing time by 75.94% and 21.69% compared to EM and N-EM, respectively. Such a computational benefit is increased for the 136-node test system, for which the respective reduction factors are equal to 83.86% and 70.18%.

V. CONCLUSION

This paper has presented an enhanced topology-variable-based model for distribution system reliability assessment, where widely used distribution network reliability indices, namely EENS, SAIDI, and SAIFI, are formulated using mixed-integer linear expressions. Accordingly, the proposed model can be effectively incorporated into different reliability-constrained problems associated with planning and operating distribution systems. In order to demonstrate its applicability, the proposed reliability model has been integrated into a reliability-constrained distribution network reconfiguration problem. Our simulations reveal that, in the case of a large network with over 100 nodes, the proposed technique can reduce the computational cost by 70.18% and 83.86% compared to two state-of-the-art models.

REFERENCES

[1] H. L. Willis, *Power Distribution Planning Reference Book*, 2nd ed. New York City, NY, USA: Marcel Dekker, 2004.

[2] R. E. Brown, *Electric Power Distribution Reliability*, 2nd ed. Boca Raton, FL, USA: CRC Press, 2008.

[3] S. Haffner, L. F. A. Pereira, L. A. Pereira, and L. S. Barreto, "Multistage model for distribution expansion planning with distributed generation—Part I: Problem formulation," *IEEE Trans. Power Deliv.*, vol. 23, no. 2, pp. 915–923, Apr. 2008.

[4] J. F. Franco, M. J. Rider, M. Lavorato, and R. Romero, "A mixed-integer LP model for the reconfiguration of radial electric distribution systems considering distributed generation," *Electr. Power Syst. Res.*, vol 97, pp. 51–60, Apr. 2013.

[5] A. Tabares, J. F. Franco, M. Lavorato, and M. J. Rider, "Multistage long-term expansion planning of electrical distribution systems considering multiple alternatives," *IEEE Trans. Power Syst.*, vol. 31, no. 3, pp. 1900–1914, May 2016.

[6] D. F. Teshome and K. L. Lian, "Comprehensive mixed-integer linear programming model for distribution system reconfiguration considering DGs," *IET Gener. Transm. Distrib.*, vol. 12, no. 20, pp. 4515–4523, Nov. 2018.

[7] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Distribution system expansion planning considering non-utility-owned DG and an independent distribution system operator," *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 2588–2597, Jul. 2019.

[8] R. C. Lotero and J. Contreras, "Distribution system planning with reliability," *IEEE Trans. Power Deliv.*, vol. 26, no. 4, pp. 2552–2562, Oct. 2011.

[9] I. Ziari, G. Ledwich, A. Ghosh, and G. Platt, "Optimal distribution network reinforcement considering load growth, line loss, and reliability," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 587–597, May 2013.

[10] J. C. López, M. Lavorato, and M. J. Rider, "Optimal reconfiguration of electrical distribution systems considering reliability indices improvement," *Int. J. Electr. Power Energy Syst.*, vol. 78, pp. 837–845, Jun. 2016.

[11] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Reliability assessment for distribution optimization models: A non-simulation-based linear programming approach," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3048–3059, Jul. 2018.

[12] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Distribution network expansion planning with an explicit formulation for reliability assessment," *IEEE Trans. Power Syst.*, vol. 33, no. 3, pp. 2583–2596, May 2018.

[13] A. Tabares, G. Muñoz-Delgado, J. F. Franco, J. M. Arroyo, and J. Contreras, "An enhanced algebraic approach for the analytical reliability assessment of distribution systems," *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 2870–2879, Jul. 2019.

[14] Z. Li, W. Wu, B. Zhang, and X. Tai, "Analytical reliability assessment method for complex distribution networks considering post-fault network reconfiguration," *IEEE Trans. Power Syst.*, vol. 35, no. 2, pp. 1457–1467, Mar. 2020.

[15] Z. Li, W. Wu, X. Tai, and B. Zhang, "Optimization model-based reliability assessment for distribution networks considering detailed placement of circuit breakers and switches," *IEEE Trans. Power Syst.*, vol. 35, no. 5, pp. 3991–4004, Sep. 2020.

[16] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, H. Farzin, M. Moeini-Aghtaie, and M. Lehtonen, "A MILP model for incorporating reliability indices in distribution system expansion planning," *IEEE Trans. Power Syst.*, vol. 34, no. 3, pp. 2453–2456, May 2019.

[17] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, M. Moeini-Aghtaie, and M. Lehtonen, "MILP model of electricity distribution system expansion planning considering incentive reliability regulations," *IEEE Trans. Power Syst.*, vol. 34, no. 6, pp. 4300–4316, Nov. 2019.

[18] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, G. Muñoz-Delgado, J. Contreras, M. Lehtonen, and J. M. Arroyo, "Linear formulations for topology-variable-based distribution system reliability assessment considering switching interruptions," *IEEE Trans. Smart Grid*, vol. 11, no. 5, pp. 4032–4043, Sep. 2020.

[19] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*. New York, NY, USA: Wiley-Interscience, 1999.

[20] IBM ILOG CPLEX. (2021). *IBM CPLEX Optimizer*. [Online]. Available: https://www.ibm.com/analytics/cplex-optimizer

[21] M. Jooshaki, M. Lehtonen, M. Fotuhi-Firuzabad, G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Distribution test systems for reliability assessment," IEEE Dataport. [Online]. Available: https://dx.doi.org/10.21227/qhq1-gp87. Accessed: Mar. 2021.