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An MILP Model for Optimal Placement of Sectionalizing Switches and Tie Lines in Distribution Networks With Complex Topologies

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***Abstract*—Sectionalizing switches (SSs) and tie lines play essential roles in reducing the duration of customer interruptions in electricity distribution networks. The effectiveness of such assets is strongly influenced by their placement in the grid. Operation of SSs and tie lines is also inherently interdependent. Due to the structural complexities regarding the mathematical modeling of such dependencies, optimization of the planning and operation of switches and tie lines has typically required either leveraging heuristic and metaheuristic approaches or oversimplifying the network topology. To tackle such issues, this paper presents a computationally-efficient model for reliability-oriented concurrent switch and tie line placement in distribution networks with complex topologies. The proposed model can be applied to grids with several tie lines and laterals per feeder, and yields the optimal location of tie lines, type of tie switches, namely manual or remote-controlled, and the location and type of SSs. Being cast as a mixed integer linear programming (MILP) problem, the model can be efficiently solved with guaranteed convergence to global optimality using off-the-shelf optimization software. The efficiency and scalability of the proposed model are demonstrated through implementation on five networks and the outcomes are thoroughly discussed.**

***Index Terms*—Electricity distribution, mixed-integer linear programming, optimization, reliability, switch, tie line.**

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NOMENCLATURE

Indices

e	Index for indicating sending or receiving end of feeder sections.
$l, \bar{l}$	Index for feeder sections.
n	Index for load nodes.
r	Index for candidate tie line locations.
s	Index for restoration scenarios.

Sets

E	Index set $\{es, er\}$, where es and er correspond to sending and receiving ends of feeder sections.
E_s	Subset of E indicating which switch of the faulted feeder section can be utilized for fault isolation in restoration scenario s.
L	Set of feeder sections.
L_s	Set of feeder sections whose switches can be utilized to isolate the faulty feeder section in restoration scenario s.
R_s	Set of tie lines which can be utilized in restoration scenario s.
$S_{l,n}$	Set of possible scenarios for restoring load node n during failure of feeder section l.
SR	Set of partitions of a path in Π, where each partition comprises feeder sections being in the same paths in Π.
sr	Set of all feeder sections in a partition in SR.
tr	Set of the *terminal nodes* of the paths in Π that do not include the feeder sections in sr.
Π	Set of all paths from a load node to the terminal nodes.
Ψ	Set of candidate tie line locations.
Ω	Set of all load nodes.

Parameters

g	Annual load growth rate.
IC^M, IC^R	Investment costs for manual and remote-controlled switches, respectively.
IC^T_r	Investment cost for tie line r.
M	Sufficiently large number.
N_n	Number of customers connected to load node n.
OC^M, OC^R	Operation and maintenance costs for manual and remote-controlled switches, respectively.

OC_r^T	Operating cost for tie line r.
P_n	Power demand at load node n.
RT_l	Repair time for feeder section l.
ST^M, ST^R	Switching times for manual and remote-controlled switches, respectively.
T	Demand growth period.
U	Useful lifetime of the switches and tie lines.
α	Annual interest rate.
β	A positive parameter representing the limitation of field crews for manual switch operation.
δ^T	Annualizing factor for revenue lost due to the undelivered energy during power cuts.
λ_l	Failure rate of feeder section l.
ν	Expected revenue loss per unit of undelivered energy.
$\xi_{l,n}$	Binary parameter which is equal to 1 if feeder section l and node n are in the same feeder, being 0 otherwise.

Variables

$EENS$	Expected energy not supplied.
Inv, Op	Investment and operating costs, respectively.
OF	Objective function.
PRS	Cost imposed by the reward-penalty scheme.
$SAIDI$	System average interruption duration index.
x_r^M, x_r^R	Binary investment variables for manual and remote-controlled tie switches, respectively.
$x_{l,e}^M$, $x_{l,e}^R$	Binary investment variables for manual and remote-controlled sectionalizing switches, respectively.
$y_{l,n,s}$	Auxiliary binary variable, which is equal to 1 if isolation of faulty feeder section l from load node n in restoration scenario s cannot be done remotely, being 0 otherwise.
$z_{l,n,s}$	Auxiliary binary variable, which is equal to 1 if restoration of load node n after a failure occurs in feeder section l cannot be done via a remote tie switch operation in restoration scenario s, being 0 otherwise.
$\theta_{l,n,s}$	Annual interruption duration of node n due to the failures in feeder section l for restoration scenario s.
$\tau_{l,n}$	Annual interruption duration for customers connected to node n due to the failures in feeder section l.
υ_s	Auxiliary binary variable for modeling $\tau_{l,n}$ based on $\theta_{l,n,s}$.

I. INTRODUCTION

IN TODAY'S world, electrical energy plays such a vital role in our everyday life that even short service interruptions have become intolerable. As a result, many countries have set regulations to ensure a reliable service for end-users, and enhancing the reliability of power grids has become inevitable [1], [2]. More specifically, as the majority of service interruptions are due to failures at the distribution level [3], increasing the reliability of distribution networks (DNs) has attracted more attention in recent years. In this respect, installing sectionalizing switches (SSs) in DNs has always been considered a fundamental, yet effective means of decreasing the interruption duration of network customers, thereby enhancing the distribution service reliability. Nevertheless, investing and installing such SSs entails significant capital and operational costs. Thus, an optimization should be carried out to determine the balance between enhancing DN reliability and imposed costs, or more precisely, to specify the cost-efficient number of SSs and their placement in the network.

Owing to the combinatorial nature of the SS placement problem, it is classified as a non-deterministic polynomial-time (NP)-hard problem [4], [5]. As a result, distribution companies (DISCOs) have traditionally leveraged their practical experiences to estimate the adequate number and location of SSs [6]. Early research studies which employed mathematical formulations to find the optimal placement of SSs date back to the 1990s and 2000s. In this regard, the researchers proposed heuristic algorithms to solve such a difficult problem, using various approaches, namely genetic [7], simulated-annealing [4], ant colony optimization [8], [9], graph-based solution [10], artificial immune system [11], particle swarm optimization [12], [13], and differential search [14] algorithms, to name but a handful. Even though each of these studies made significant contributions to address the problem of optimal SS placement, they fail to guarantee that their obtained solution is globally optimal since they solved the problem of interest by using either heuristic or metaheuristic techniques.

The earliest mathematical-based model which could guarantee to reach the global optimal solution of the reliability-oriented switch placement problem was proposed in [15]. In this study, Abiri-Jahromi *et al.* developed a mixed-integer linear programming (MILP) formulation aiming at minimizing the costs of remote-controlled switch (RCS) deployment and customer outages. While the MILP model presented in [15] was able to find the global optimum, it was too simplified to be used in practical applications. As a result, several research studies have extended the MILP model during recent years. In this respect, authors in [16] and [17] considered the impact of earth faults and distributed generation (DG) in the MILP model, respectively. Izadi and Safdarian in [18] took into account the stochastic nature of contingencies in the MILP switch optimization problem. The authors in [19] extended the MILP formulation so as to determine the RCS placement both in the main feeders and in the laterals. Extending the previous MILP studies, which only considered installing RCSs, Farajollahi *et al.* in [20] and [21] developed MILP models for finding the optimal placement of both the manual switches (MSs) and RCSs. They took into consideration the impact of switch malfunction probability and switch failure in [20] and [21], respectively. Also, the authors in [22] and [23] developed MILP formulations to determine the simultaneous placement of fault indicators, MSs, and RCSs. Lastly, a new MILP model, which could specify the allocation of SSs together with the type of tie switches at reserve connection points, had been proposed in [24], and it was further developed in [25] to optimize also the number and location of tie lines.

While the MILP-based methods in [15]–[25] could be guaranteed to find the global optimal solution in a finite amount of time, they have made a couple of assumptions in their attempt to simplify the models, which have rendered them unable to be implemented on realistic DNs. First and foremost, [15]–[18], [20]–[25] only modeled sections of the main feeders, not their laterals, thereby disregarding the impact of failures in laterals on DN reliability as well as the possibility of SS installment in the laterals. Secondly, authors in [15]–[23] assumed that the placement of tie lines and the type of the tie switches were determined prior to the optimization. While Izadi *et al.* [19] addressed the former simplification by considering laterals together with main feeders as alternative locations for installing switches, they did not deal with the latter. On the contrary, authors in [24] and [25] tackled the second simplification – but not the first one. Accordingly, not only both references ignored the impact of failures and the possibility of SS installation in laterals, but also they assumed the tie switches and tie lines could be installed only at the end of main feeders. Lei *et al.* addressed the first and a part of the second issue of the previous studies by proposing an innovative MILP model in [26] for optimal RCS allocation with the goal of maximizing their reliability benefits. Nonetheless, the proposed model assumes that MSs are installed in all feeder sections prior to the optimization and, based on this assumption, finds the locations of the MSs that should be upgraded to RCSs. In addition to these simplifications, their optimization problem cannot determine on which end of the feeder section (sending or receiving end) the upgrade to RCS should be conducted to obtain the most optimal allocation. This information is particularly valuable in practical applications. Lastly, although the model proposed in [26] can determine whether a tie switch should be upgraded to RCS, it does not consider installing new tie lines. Overall, none of the MILP models in [15]–[26] have addressed the mentioned issues for the optimal sectionalizing and tie switch placement in a distribution network with multiple lateral branches and tie line candidates.

In contrast to the previously mentioned MILP models, some reliability-oriented switch optimization models which leveraged heuristic algorithms, such as [5], [27], and [28], made none of the mentioned simplifications. They utilized a Memetic Algorithm-based approach [27], the Greedy Algorithm [5], and the Genetic Algorithm [28] to concurrently allocate SSs and tie lines in DNs with complex topologies. Although Zhang *et al.* in [28] proposed a more computationally efficient method to jointly optimize tie lines and SSs, compared to [5] and [27], they could not guarantee to find the global optimal solution. Nevertheless, the results of the mentioned studies demonstrated that considering both laterals as candidate installation locations and tie switch allocation in the switch optimization problems is vital, owing to their significant impact on the obtained solutions.

Since the concept of the MILP models developed in [15]–[25] could not be extended to tackle the simultaneous SS and tie line optimization problem, authors in [29] attempted to do so by proposing an MILP formulation, using a different concept, so as to make the global optimal solution accessible. They proposed a two-stage fault management approach, where at its first and second stages, load restoration is accomplished by RCSs solely, and MSs and RCSs jointly, in that order. In this study, they followed a basic concept, considering the failure of each feeder section as a network contingency and defining each network contingency as a particular state. Afterward, a set of operational constraints, such as radiality and Kirchhoff's current law (KCL), should be incorporated into the model for each state to determine the corresponding amount of the demand interrupted. Such a modeling approach introduced two multiplied by the number of load nodes, multiplied by the number of feeder sections, more integer variables, compared to the previous MILP models used in [15]–[22]. While this model may find the global optimal solution for small DNs, such a high number of integer variables would lead to the intractability of the resulting MILP problem for real-size networks. This is due to the fact that in MILP problems, the number of test solutions generated by the branch-and-bound algorithm can grow exponentially with the size of the problem [30]. Galias in [30] addressed this drawback of the MILP switch optimization problem, utilizing tree-structure based algorithms to find the allocation of a given number of SSs in a radially operated DN. Although their method was very fast, the approach not only could not guarantee the convergence to the global optimum in every case but also did not consider the existence of tie lines in the DN, let alone optimizing them. All in all, none of the models in the existing literature could solve the tie line and switch optimization problem for networks with complex topologies, while guaranteeing convergence to the global optimum in a reasonable amount of time. Note that distribution networks with complex topolgies are the ones with several (candidate) tie lines and lateral branches per feeder.

Aiming at tackling such a deficiency, we developed an innovative MILP-based model which determines the allocation of SSs and tie lines in the DNs with complex topologies. As the concepts used in the previous MILP models could not be utilized to solve such a complicated optimization, the formulation proposed in this paper is founded on a novel approach. To be more specific, the idea proposed in this paper generalizes the reliability assessment model such that it can be leveraged to determine the optimal placement of tie lines, type of tie switches installed on them, and type and location of SSs, even if each distribution feeder has multiple tie lines and several laterals. As a result, the proposed model does not make any simplifying assumption regarding the candidate locations for SSs and tie lines. Moreover, as the formulation is developed in an MILP fashion, it can be readily solved by off-the-shelf solvers in a finite amount of time, while guaranteeing convergence to the global optimal solution.

In this method, prior to the switch and tie line optimization, a preprocessing procedure is carried out so as to specify the minimal possible restoration scenarios in the case of every feeder section failure for each load node. The resulting data is then fed into the proposed MILP model to determine the allocation of SSs and tie lines in the DN. Moreover,

TABLE I
COMPARISON OF SWITCH OPTIMIZATION MODELS

Feature	[18]	[19]	[20]–[23]	[24]	[25]	[26]	[28]	[29]	[30]	Proposed model
I	✗	✗	✓	✓	✓	✗	✗	✓	✗	✓
II	✓	✓	✓	✓	✓	✓	✗	✓	✗	✓
III	✗	✓	✗	✗	✗	✓	✓	✓	✓	✓
IV	✗	✗	✗	✗	✓	✗	✓	✓	✗	✓
V	✓	✓	✓	✓	✓	✓	✓	✗	✓	✓
VI	✗	✗	✗	✓	✓	✗	✗	✗	✗	✓

I: Both MSs and RCSs as alternatives for installation
II: Guaranteed convergence to global optimum
III: Modeling laterals IV: Tie line optimization
V: Computationally efficient VI: Pragmatic reliability worth evaluation

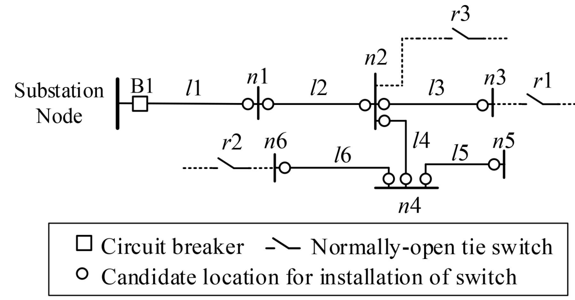

Fig. 1. A sample illustrative distribution feeder.

unlike [5], [15]–[23], [26]–[30] which did not propose any practical method for reliability worth evaluation, we estimated the unreliability cost imposed on the DISCOs, based on a pragmatic method proposed in [24] and [25]. In this regard, the unreliability cost is calculated based upon two widely-used reliability indices, EENS and the system average interruption duration index (SAIDI). The former is of importance to DISCOs since they lose a portion of their revenues with respect to it, while the latter is typically utilized as a measure in reliability incentive schemes. The regulators penalize (or reward) the DISCOs according to such schemes, so as to ensure the provision of reliable service for end-users [1], [2].

Table I summarizes the main differences between this paper and the state of the art on reliability-oriented switch optimization models. In this table, symbols "✓" and "✗" respectively indicate whether or not a particular feature is considered.

In summary, the main contributions of this study are as follows.

- Proposing an efficient MILP model for jointly optimizing the placement of tie lines and SSs (both manual and remote-controlled) in distribution networks with multiple lateral branches and tie line candidates.
- Developing an innovative preprocessing algorithm for efficiently deriving the minimal exclusive restoration scenarios for all single-feeder section failures in distribution networks with complex topologies.
- Integrating a reward-penalty scheme in the MILP model for pragmatic evaluation of reliability benefits.

The rest of this paper is organized as follows. We begin with explaining our modeling principles in Section II. Then, the proposed MILP formulation for the optimal sectionalizing and tie switches and tie line allocation problem is represented in Section III. Section IV describes the proposed preprocessing algorithm and how the whole model is implemented on a distribution network. In Section V, the proposed model is tested on several distribution networks. Finally, concluding remarks are provided in Section VI.

II. MODELING PRINCIPLES

An illustrative example of a typical distribution feeder with laterals and multiple candidate tie line locations is depicted in Fig. 1. As per this figure, in the model proposed in this paper, the connection point of every feeder section to a demand node is considered a candidate location, marked with a small circle, for installation of SSs.

For the sake of comparison, the majority of the state-of-the-art MILP models are only applicable to single-branch or series feeders with a single tie line at their end [15]–[18], [20]–[25]. For instance, keeping $l1$–$l3$ and $r1$ in Fig. 1, and dropping all the other feeder sections and tie lines yield such a feeder structure. As an exception, the approach proposed in [19] considers laterals, yet not only does it not take into account the installation of new tie lines, but also it fails to assess the reliability of distribution feeders with multiple existing tie lines. In other words, a fundamental assumption in [19] is that each feeder has a single tie line whose location is predetermined. Obviously, this is a significantly limiting assumption, since, in practice, modern distribution feeders can have several backup points. Even if a dendritic feeder is allowed to have only one tie line at most, its best location is unknown *a priori* in a pragmatic switch placement model. In other words, multiple candidate locations are typically specified for the installation of tie lines as the investment alternatives. Moreover, the placement of SSs and tie lines should be carried out concurrently to obtain the most efficient placement [25]. However, considering several candidate locations for the installation of tie lines makes the model too sophisticated to be handled by a trivial extension of the existing approaches.

The proposed model in this paper is developed based upon the concept of failure mode and effect analysis (FMEA). Accordingly, the impact of failures in each feeder section on various load nodes should be assessed. Nonetheless, such effects depend on the placement of SSs and tie lines, which are unknown prior to the optimization. Thus, for each feeder section failure, we consider a set of possible restoration scenarios for each of the load points. Each scenario s includes a set of switches required for fault isolation as well as a set of tie lines that can be leveraged in the restoration phase after the fault isolation. For instance, for failures in feeder section $l1$ in Fig. 1, there are two possible mutually exclusive restoration scenarios for load node $n3$. The first scenario is to have at least an SS between $l1$ and $n2$ and also one of the tie lines $r1$–$r3$. The other scenario is to have at least an SS in feeder section $l3$ together with tie line $r1$. Otherwise, in the case of any failures in feeder section $l1$, load point $n3$ would remain unenergized until the faulty feeder section $l1$ is fully repaired.

The proposed model considers two types of switches, namely MS and RCS, as alternatives at each of the candidate locations for both sectionalizing and tie switches (which are employed at installed tie lines). Accordingly, for each switch type (i.e., manual or remote-controlled), a binary decision variable is considered to determine its investment at each candidate location. In order to specify the candidate locations for the SSs, two indices are utilized, namely an index l for the corresponding feeder section and an index e to specify the sending ($e = es$) or receiving ($e = er$) end of that feeder section. Thus, the binary investment variables for SSs are expressed by $x^M_{l,e}$ and $x^R_{l,e}$, where superscripts M and R, standing for MS and RCS, specify the type of the switch, whereas subscripts l and e indicate its location. For instance, $x^R_{l1,es}$ is the binary decision variable for installing an RCS at the sending end of feeder section $l1$, which becomes 1 if an RCS is installed at this location, being 0 otherwise. On the other hand, the location of tie lines are determined by index r, and, therefore, two binary variables, namely x^M_r and x^R_r, are considered for each tie line location, determining the type of its corresponding tie switch.

In the proposed model, in the case of failures in feeder section l, a set of all possible restoration scenarios is considered for load node n, denoted by $S_{l,n}$. For each scenario s in $S_{l,n}$, three sets are defined as E_s, L_s, and R_s. The first contains the SS installed in feeder section l which can be used in the restoration of load node n. It can either be null or contain es or er depending on whether the sending or receiving end of l participates in the restoration scenario s. Set L_s contains all feeder sections whose switches can be utilized to reconnect load node n after a failure in feeder section l. Finally, R_s consists of all tie lines which can be used in restoration scenario s. If this set is null, it means that the load node can be re-energized by its supplying feeder as soon as the faulty feeder section is isolated, without a tie line. As an example, suppose that in the network represented in Fig. 1 feeder section $l2$ fails. In this case, if a switch is placed at the sending end of feeder section $l2$ to isolate the fault, load node $n1$ can be re-energized by its supplying feeder.

Let us investigate the restoration scenarios of load nodes $n1$–$n6$ for the DN depicted in Fig. 1 in the case of a failure in feeder section $l5$, as presented in Table II. As per this table, for node $n3$, two restoration scenarios exist: 1) having a switch in feeder section $l3$ (based on L_s) to isolate the fault and a tie line at $r1$ (according to R_s) to transfer the demand of node $n3$ to an adjacent feeder, and 2) having at least a switch at the sending end of $l5$ (as reflected in E_s) or in feeder section $l4$ (based on L_s) to isolate the faulty feeder section $l5$. As R_s is an empty set in the latter scenario, no tie line is required since node $n3$ can be directly supplied through its corresponding feeder. Also, in the case of load node $n5$, there is no restoration scenario according to the table; hence, when a failure occurs in $l5$, the service of customers at this node remains interrupted until the faulty feeder section is repaired.

It is worth emphasizing that the restoration scenarios and corresponding sets do not provide any information about the restoration times; rather, they indicate the possibility of restoring every load node in the case of a feeder section failure as well as SSs and tie lines required to do so. On the other hand, the binary investment variables for SSs, $x^M_{l,e}$ and $x^R_{l,e}$, and tie

TABLE II
RESTORATION SCENARIOS FOR FAILURES IN FEEDER SECTION $l5$

Node	$n1$	$n2$	$n3$		$n4$	$n5$	$n6$	
Scenario	$s1$	$s1$	$s1$	$s2$	$s1$	-	$s1$	$s2$
E_s	$\{es\}$	$\{es\}$	$\emptyset$	$\{es\}$	$\{es\}$	$\emptyset$	$\emptyset$	$\{es\}$
L_s	$\{l2, l4\}$	$\{l4\}$	$\{l3\}$	$\{l4\}$	$\emptyset$	$\emptyset$	$\{l6\}$	$\emptyset$
R_s	$\emptyset$	$\emptyset$	$\{r1\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\{r2\}$	$\emptyset$

switches, x^M_r and x^R_r, are employed to model the restoration times. Developed based upon the concepts explained above, a mathematical model is presented in the next section to determine the optimal location and type of both SS and tie lines concurrently.

III. PROBLEM FORMULATION

This section represents the proposed mathematical formulation. The objective is to find a trade-off between the costs of unreliability and allocation of SSs and tie lines. Thus, the optimal solution of this optimization contains the location of tie lines, type of tie switches, and type and location of SSs.

A. Objective Function

Equation (1) minimizes the objective function, OF, which is the annualized system cost including the annualized value of switch and tie line investment cost, Inv, the operational cost, Op, the cost imposed by the reward-penalty scheme, PRS, and the annualized value of revenue lost due to undelivered energy. To annualize the revenue lost due to undelivered energy, (2) specifies its annuity factor, δ^T, assuming that the demand of each load node grows at a rate of g annually for T consecutive years. Note that PRS is calculated based on SAIDI which reflects the total annual interruption duration for an average customer and is not affected by the assumed load growth. Thus, no annuity factor is required for PRS in (1). In the cases where the assumption of a constant load growth rate is not applicable, the infinite perpetuity approach described in [31] can be used for deriving the objective function. Equations (3) and (4) determine the total investment cost of SSs and tie lines, and their operational cost, respectively. Expressions (5) and (6) respectively imply that at each candidate location for SS and tie switch only one switch can be installed. Finally, equation (7) indicates the binary nature of the investment variables.

$$\min OF = \frac{\alpha}{1-(1+\alpha)^{-U}} Inv + Op + PRS + \delta^T \nu EENS \quad (1)$$

$$\delta^T = \alpha \left(\frac{(1+g)^T - (1+\alpha)^T}{(g-\alpha)(1+\alpha)^T} + \frac{(1+g)^{T-1}}{\alpha(1+\alpha)^T} \right) \quad (2)$$

$$\begin{aligned} Inv = & \sum_{l \in L} \sum_{e \in E} \left(x^R_{l,e} IC^R + x^M_{l,e} IC^M \right) \\ & + \sum_{r \in \Psi} \left(x^R_r IC^R + x^M_r IC^M + \left(x^R_r + x^M_r \right) IC^T_r \right) \end{aligned} \quad (3)$$

$$\begin{aligned} Op = & \sum_{l \in L} \sum_{e \in E} \left(x^R_{l,e} OC^R + x^M_{l,e} OC^M \right) \\ & + \sum_{r \in \Psi} \left(x^R_r OC^R + x^M_r OC^M + \left(x^R_r + x^M_r \right) OC^T_r \right) \end{aligned} \quad (4)$$

$$x^R_{l,e} + x^M_{l,e} \leq 1; \forall l \in L, \forall e \in E \tag{5}$$

$$x^R_r + x^M_r \leq 1; \forall r \in \Psi \tag{6}$$

$$x^R_{l,e},\ x^M_{l,e}, x^R_r, x^M_r \in \{0,1\}; \forall l \in L, \forall e \in E, \forall r \in \Psi. \tag{7}$$

B. Proposed Reliability Assessment Model

In this part, we develop a novel reliability assessment model, which not only considers the installation of MSs and RCSs in the main feeders as well as their laterals but also takes into account the deployment of new tie lines equipped with either manual or remote-controlled tie switches. As noted earlier, to obtain a pragmatic evaluation of the reliability worth for DISCOs, we determine the EENS and the SAIDI of distribution network. Accordingly, (8) and (9) calculate EENS and SAIDI, respectively.

$$EENS = \sum_{l \in L} \sum_{n \in \Omega} \tau_{l,n} P_n \tag{8}$$

$$SAIDI = \frac{\sum_{l \in L} \sum_{n \in \Omega} \tau_{l,n} N_n}{\sum_{n \in \Omega} N_n}. \tag{9}$$

Equation (10) models the annual interruption duration, $\tau_{l,n}$, for the customers at load node n in the case of failures at its supplying feeder if their power cannot be restored until the faulted section, l, is fully repaired (i.e., no restoration scenario exists for load node n during failure of feeder section l). If at least one scenario exists for the restoration of load node n in the case of a fault in feeder section l, the annual interruption duration of such a node is calculated as $\tau_{l,n} = min\{\theta_{l,n,s} | \forall s \in S_{l,n}\}$, where $\theta_{l,n,s}$ denotes the annual interruption duration of node n due to the failures in feeder section l for restoration scenario s. This is because there can exist more than one scenario for power restoration of load node n in the case of a failure in feeder section l, and, rationally, it will be restored through the fastest scenario. To linearly formulate the minimum function, (11)–(14) are utilized. Equation (11) specifies that $\tau_{l,n}$ is lower than or equal to $\theta_{l,n,s}$ for all scenarios, while (12)–(14) jointly ensure that $\tau_{l,n}$ is higher than $\theta_{l,n,s}$ for exactly one of the scenarios.

$$\tau_{l,n} = RT_l \lambda_l; \left\{\forall l \in L, \forall n \in \Omega \mid S_{l,n} = \emptyset, \xi_{l,n} = 1\right\} \tag{10}$$

$$\tau_{l,n} \leq \theta_{l,n,s}; \forall l \in L, \forall n \in \Omega, \forall s \in S_{l,n} \tag{11}$$

$$\tau_{l,n} \geq \theta_{l,n,s} - M(1 - \upsilon_s); \forall l \in L, \forall n \in \Omega, \forall s \in S_{l,n} \tag{12}$$

$$\sum_{s \in S_{l,n}} \upsilon_s = 1; \{\forall l \in L, \forall n \in \Omega \mid S_{l,n} \neq \emptyset\} \tag{13}$$

$$\upsilon_s \in \{0,1\}; \forall s \in S_{l,n}, \forall l \in L, \forall n \in \Omega. \tag{14}$$

Equations (15)–(19) determine the values of $\theta_{l,n,s}$ for every restoration scenario. It goes without saying that the optimization solver sets the values of $\theta_{l,n,s}$ to their lower bounds, since the proposed optimization problem minimizes an objective function which is monotonically increasing with respect to the unreliability costs, and such costs will also rise when $\tau_{l,n}$ increase. As a result, the optimization tries to minimize the values of $\theta_{l,n,s}$ and, therefore, set them to their lower bounds, which are specified in (15)–(19).

Equations (15)–(17) jointly determine the minimum time required to isolate the fault in each restoration scenario. In this respect, (15) ensures that the minimum time for isolating a faulted feeder section and, therefore, the restoration time of customers located in the corresponding feeder is higher than the switching time of an RCS, ST^R, in any restoration scenario. If at least an RCS is available to isolate the faulted section from the load node in a restoration scenario, the right-hand sides of both (16) and (17) are non-positive, so the restoration time is determined by (15). If no RCS, but at least one MS, exists to isolate faulted feeder section l, (16) sets the tightest lower bound of $\theta_{l,n,s}$ in restoration scenario s to $ST^M\lambda_l$, which is greater than $ST^R\lambda_l$, while the right-hand side of (17) would be non-positive as there is at least one MS to isolate the fault. Finally, if neither an RCS nor an MS can isolate the fault, (17) sets the lower bound of $\theta_{l,n,s}$ to $RT_l\lambda_l$ in that restoration scenario. This means that if, in a restoration scenario, no SS exists to isolate the fault from load node n, the restoration time for that load node will be equal to the repair time of the faulted section.

$$\theta_{l,n,s} \geq ST^R \lambda_l; \forall l \in L, \forall n \in \Omega, \forall s \in S_{l,n} \tag{15}$$

$$\theta_{l,n,s} \geq ST^M \lambda_l \left(1 - \sum_{e \in E_s} x^R_{l,e} - \sum_{\bar{l} \in L_s} \sum_{e \in E} x^R_{\bar{l},e}\right); \quad \forall l \in L, \forall n \in \Omega, \forall s \in S_{l,n} \tag{16}$$

$$\theta_{l,n,s} \geq RT_l \lambda_l \left(1 - \sum_{e \in E_s} \left(x^R_{l,e} + x^M_{l,e}\right) - \sum_{\bar{l} \in L_s} \sum_{e \in E} \left(x^R_{\bar{l},e} + x^M_{\bar{l},e}\right)\right); \quad \forall l \in L, \forall n \in \Omega, \forall s \in S_{l,n}. \tag{17}$$

As (15)–(17) determine the minimum time for fault isolation, they jointly determine the restoration time in the scenarios where the power is restored through the node's supplying feeder after the isolation of faulted section. However, in some scenarios, the service of customers is restored through a tie line and by its adjacent feeder, if the faulted section is successfully isolated. Such tie lines should be installed in candidate locations and be equipped with either a manual or remote-controlled tie switch, which is normally-open. For scenarios where a tie line is required for service restoration, expressions (18) and (19) are considered alongside (15)–(17) to determine the service restoration time. In this regard, (18) determines the lower bound of $\theta_{l,n,s}$ whenever at least one of the candidate tie lines which can supply interrupted load node n is installed and is equipped with an MS, but none is equipped with an RCS. This is because, in this case, the right-hand side of (19) is non-positive, but that of (18) is $ST^M\lambda_l$. If none of the candidate tie lines which can supply interrupted load node n after fault isolation is installed, the right-hand side of (19) will be non-zero and it will determine the lower bound of $\theta_{l,n,s}$. In other words, the restoration time in this scenario is equal to the repair time of the faulted section. It is worth noting that, in the cases that both sectionalzing and tie switches are manual, the limitation of field crews might result in a longer restoration time than what is calculated according to the proposed model. To consider this aspect in the model, equations (20)–(24), presented in the Appendix, should be added

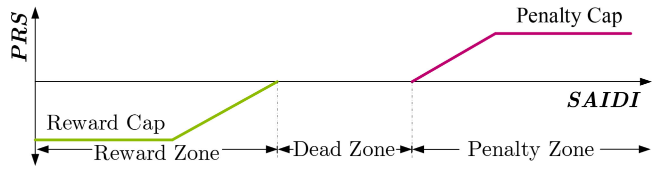

Fig. 2. Reward-penalty graph.

to the model.

$$\theta_{l,n,s} \geq ST^{M}\lambda_l\left(1-\sum_{r\in R_s} x_r^R\right); \quad \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\} \tag{18}$$

$$\theta_{l,n,s} \geq RT_l\lambda_l\left(1-\sum_{r\in R_s}\left(x_r^R+x_r^M\right)\right); \quad \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\}. \tag{19}$$

C. Reward-Penalty Scheme

As noted earlier, in this paper, we integrate a reliability incentive scheme into the model to carry out a reliability worth evaluation, similar to practical situations. Reward-penalty schemes are implemented in many countries as regulatory tools to ensure that DISCOs provide a reliable service for their customers [1], [2]. Fig. 2 shows the general structure of the reward-penalty scheme considered in this paper, which is based on the models proposed in [24] and [25]. In this figure, the positive value of *PRS* means that the DISCO is penalized, whereas the negative value indicates that the DISCO is rewarded. As a typical practice in such schemes, both reward and penalty amounts are restricted to fixed levels in order to limit the financial risks associated with such schemes, namely the reward cap and the penalty cap. The cost imposed by such a reward-penalty scheme (i.e., *PRS*) is modeled with a set of linear equations in [24] and [25], to which interested readers can refer. It is worth mentioning that the mixed-integer linear expressions presented in [24] and [25] exactly model the non-convex reward-penalty function depicted in Fig. 2 and do not entail any approximations.

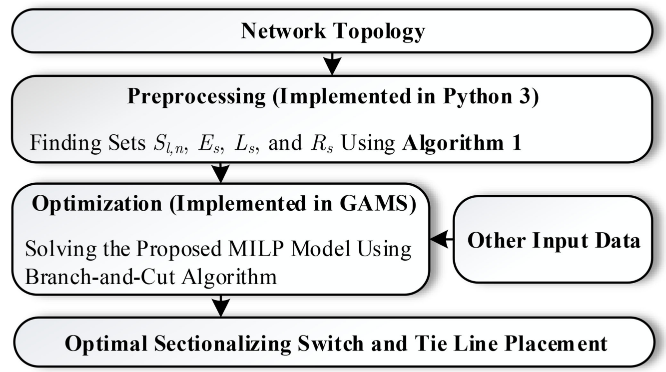

Fig. 3. Flowchart of the implementation procedure.

Algorithm 1: Finding Sets $S_{l,n}$, E_s, L_s, and R_s

Result: $S_{l,n}$, E_s, L_s, and R_s
for *each node n in* Ω **do**
  Find all paths (Π) to the terminal nodes;
  for *each path in* Π **do**
    for *each feeder section l in the selected path* **do**
      if *feeder section l has not been investigated* **then**
        Find the route from feeder section l to node n;
        Partition the route into subsets (SR) such that each subset includes feeder sections which are in the same paths of Π;
        for *each subset sr in SR* **do**
          Add a restoration scenario s to $S_{l,n}$;
          Find all paths from Π which do not include the feeder sections in set sr, and assign terminal nodes of such paths to set tr;
          if *tr includes a substation node* **then**
            Set R_s to $\emptyset$;
          else
            Assign R_s as the set of tie lines connected to the nodes in set tr (if any);
          end
          if *feeder section l is in subset sr* **then**
            Determine E_s based on the direction of the route from feeder section l to node n;
            Drop feeder section l from subset sr, and assign the result to L_s;
          else
            Set E_s to $\emptyset$;
          end
        end
      end
    end
  end
end

IV. IMPLEMENTATION OF THE PROPOSED MODEL

Fig. 3 depicts the implementation procedure of the proposed model. Initially, a preprocessing is carried out, where the network topology is used to obtain sets $S_{l,n}$, E_s, L_s, and R_s through the pseudocode represented in Algorithm 1. This algorithm firstly determines the set of paths, Π, from each network node n to the *terminal nodes*. The terminal nodes consist of the substation nodes, the nodes connected to candidate tie lines, and any node which is connected to only one feeder section (i.e., located at the end of feeders or laterals). Afterward, the route from each feeder section l in each path to node n is partitioned into several subsets, which form set SR. By following the procedure in the algorithm, for each subset sr in SR, a restoration scenario s is created and its corresponding sets $S_{l,n}$, E_s, L_s, and R_s are derived.

To explain Algorithm 1 in a clear way, we review the procedure based upon which the program developed according to Algorithm 1 determines $S_{l,n}$, E_s, L_s, and R_s for the sample distribution feeder shown in Fig. 1. According to the algorithm, the program should carry out this procedure for all load nodes n. As an example, suppose the program begins with node $n1$ in the algorithm, determines all paths from this node to the terminal nodes, and assigns the paths to set Π. This set would be equal to $\{\{l1\}, \{l2\}, \{l2, l3\}, \{l2, l4, l5\}, \{l2, l4, l6\}\}$. In the next step, the program should select one feeder section l from one of the paths in the derived set, Π. If the selected feeder section l was not investigated in previous iterations, the program conducts the following procedure to obtain the set of restoration scenarios for node n in the case of a failure in feeder section l. Assume that the program selects feeder section $l3$ from path $\{l2, l3\}$, which has not been investigated before. Afterward, according to the algorithm, the program determines the route from $l3$ to $n1$, which is $\{l2, l3\}$, and partitions it into subsets (we label each subset sr and the set of all subsets SR) such that each subset must include feeder section(s) that are all in the same paths of Π. In this example, SR is $\{\{l2\}, \{l3\}\}$ (has two subsets), because $l2$ is in different paths of Π compared to $l3$ ($l2$ is in paths $\{l2\}$, $\{l2, l4, l5\}$, and $\{l2, l4, l6\}$, while $l3$ is not). In the next step, for each sr, the program adds a restoration scenario to $S_{l,n}$ and determines E_s, L_s, and R_s according to the following steps. It is worth noting that because two subsets exist in SR in this example, $S_{l,n}$ consists of two restoration scenarios, one for each subset in SR. The program first selects $sr = \{l2\}$ from $SR = \{\{l2\}, \{l3\}\}$ to determine the corresponding sets for the first restoration scenario. After adding the first restoration scenario to $S_{l3,n1}$, all terminal nodes of the paths that do not include $l2$ are assigned to tr, which would only include one element, the substation node. In the next step, because tr includes a substation node (the condition of the if-clause is true), R_s is set to $\emptyset$, and then because $l3$ is not in $sr = \{l2\}$, E_s is set to $\emptyset$ and $\{l2\}$ is assigned to L_s, according to the algorithm. Lastly, the program selects $sr = \{l3\}$ from $SR = \{\{l2\}, \{l3\}\}$ to determine the corresponding sets for the second restoration scenario. In this case, the terminal nodes of the paths in Π that do not include $l3$, which are assigned to tr, are the substation node, $n2$, $n5$, and $n6$. Similar to the previous restoration scenario, because tr includes a substation node, R_s is set to $\emptyset$. Also, since feeder section $l3$ is in $sr = \{l3\}$ (the condition of if-clause is true), E_s is determined according to the direction of the route from the feeder section $l3$ to node $n1$; then, this feeder section is dropped from sr, and the derived set is assigned to L_s. As a result, for this restoration scenario, E_s and L_s are respectively equal to $\{es\}$ (which denotes the sending end) and $\emptyset$. This way, the program has derived E_s, L_s, and R_s for both restoration scenarios in $S_{l3,n1}$.

After obtaining sets $S_{l,n}$, E_s, L_s, and R_s, these data as well as other economic and technical data are fed into the mathematical formulation, stated in the previous section. Solving the proposed MILP model through commercially available optimization software, the global optimal solution would be determined, demonstrating the optimal SS and tie line placement plan for the distribution network.

V. Numerical Study

In this section, in order to investigate the applicability and scalability of the proposed model, it is applied to five test distribution networks comprising 37, 85, 137, 145, and 230 nodes, and the obtained results are discussed. The mathematical model has been implemented in GAMS 25.1, while IBM CPLEX 12.8 is utilized as the solver with an optimality gap of 0% as the stopping criterion. All the simulations have been carried out on a Fujitsu CELSIUS W530 Power PC with an Intel Xeon E3-1230 processor at 3.20 GHz and 32 GB of RAM. Also, for the sake of reproducibility of the results, test networks data are available in [32].

TABLE III
Outcomes for Case I

Test network	37-node	85-node	137-node	145-node	230-node
Objective function (k$)	15.557	9.423	17.872	2.670	1.546
Annualized inv. cost (k$)	9.709	4.393	21.602	6.134	28.121
Operating cost (k$)	1.362	0.602	3.098	0.750	3.164
PRS (k$)	-1.427	0	-9.983	-4.514	-31.432
$\delta^T \nu EENS$ (k$)	5.914	4.428	3.155	0.300	1.693
SAIDI (hours/cust./year)	0.915	1.200	0.767	0.287	0.094
EENS (MWh/year)	40.785	30.540	21.761	2.071	11.676
Preprocessing time (s)	0.02	0.04	0.46	1.37	0.54
Simulation time (min)	0.02	0.12	2.08	16.67	0.24

TABLE IV
Outcomes for Cases II and III

Test network	Case II			Case III		
	OF (k$)	SAIDI	EENS	OF (k$)	SAIDI	EENS
37-node	24.039	1.798	81.608	35.031	3.571	158.828
85-node	9.535	1.276	31.617	15.307	1.751	48.530
137-node	34.143	1.626	47.142	59.048	3.196	93.435
145-node	5.297	0.464	3.348	22.127	0.934	6.737
230-node	7.714	0.190	23.644	45.002	0.296	36.786

SAIDI: Hours/customer/year EENS: MWh/year

In the simulations, the useful lifetime of SSs and tie lines, U, is set to 15 years, while an annual interest rate of 8% is taken into account. The investment costs of an MS, an RCS, and a tie line are set to \$500, \$4,700, and \$15,000, respectively. Also, the annual operating cost of switches is considered 2% of their investment cost, while that of tie lines is assumed to be 1% of their initial investment cost. The switching times for RCSs and MSs installed in the distribution network are assumed to be 0.1 and 1 hour, respectively. In addition, it is considered that the network demand will rise for 10 years with a constant annual growth rate of 3%. Lastly, the expected revenue loss per unit of undelivered electrical energy to the customers, ν, is assumed to be \$120/MWh.

To assess the impact of simultaneous optimization of SSs and tie lines, three following cases are studied for each test distribution network; **Case I)** concurrent installation of SSs and tie lines, **Case II)** installation of SSs without any investment in tie lines, and **Case III)** no investment in neither SSs nor tie lines.

The results of implementing the model on the five distribution networks in Case I and in Cases II and III are represented in Tables III and IV, respectively. As could have been predicted, for all the test networks, the total system cost (i.e., the objective function of the model) and both of the reliability indices are highest in Case III, where neither an SS nor a tie line can be installed. In Case II, installing SSs has led to better service reliability (i.e., lower reliability indices) for all the five networks, compared to those in Case III. Nevertheless, the best outcomes in terms of system cost and reliability indices are obtained when simultaneous placement of SSs and tie lines are considered in Case I. To be more specific, by comparing the outcomes of Cases I and II, we realize that when investment in tie lines is considered besides the installation of SSs, the EENS index is improved by 50.0%, 3.4%, 53.8%, 38.1%, and 50.6% in the networks with 37, 85, 137, 145, and 230 nodes, respectively. In a similar manner, the SAIDI is improved by

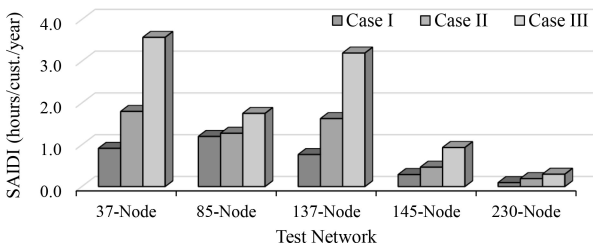

Fig. 4. SAIDI of the test networks for various cases.

TABLE V
NUMBER OF INSTALLED SWITCHES AND TIE LINES IN CASES I AND II

Test network	37-node		85-node		137-node		145-node		230-node	
Case	I	II	I	II	I	II	I	II	I	II
No. of RCSs	8	9	3	4	22	15	0	0	1	0
No. of MSs	31	21	17	18	43	40	45	48	142	62
No. of tie lines	2	0	1	0	4	0	2	0	11	0

49.1%, 5.9%, 52.8%, 38.1%, and 50.5% in the networks with 37, 85, 137, 145, and 230 nodes, respectively.

Fig. 4 vividly demonstrates the improvement made to the SAIDI in Case II compared to Case III, and in Case I compared to Case II. While installation of tie lines in addition to SSs has significantly improved both reliability indices for the networks with 37, 137, 145, and 230 nodes, for the 85-node network, the reliability improvement is marginal. This is due to the fact that the 85-node network comprises 11 feeders, which is a high number relative to its total number of nodes. As a result, investing in tie lines in this network is not as beneficial as it is in the other networks, which have fewer, but longer, feeders. It is worth mentioning that the only tie line installed in the 85-node network connects the two longest feeders to each other. These results validate that the allocation of tie lines together with SSs would efficiently increase the reliability level in every network; however, such improvement might be marginal in some cases and significant in others.

To further assess the impact of considering the allocation of tie lines besides the SS optimization, the number of installed SSs in the five networks, for Cases I and II, are compared in Table V. As can be deduced from this Table, the number of installed switches may either increase or decrease based on the network topology when the allocation of tie lines is considered in addition to the installation of SSs. For example, in the 137-node network, the numbers of both manual and remote-controlled SSs increase when the allocation of tie lines is considered, whereas, in the 85-node network, fewer MSs and RCSs are installed as a result of the installation of one tie line. Nevertheless, even with fewer SSs, investing in one tie line in the 85-node network has caused both the EENS and SAIDI to decrease in Case I, compared to those in Case II, which shows the importance of efficient installation of tie lines. Such results evidently show that simultaneous installation of SSs and tie lines is essential, and a distribution utility may achieve the global optimal solution for tie line and SS allocation problem only by concurrent optimization of both assets.

Figs. 5 and 6 depict the allocation of tie lines and SSs in Case I and the optimal SS placement in Case II for the 37-node

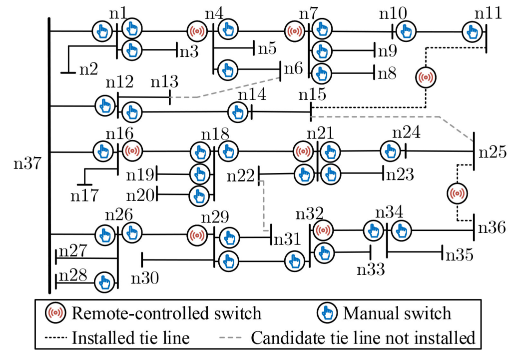

Fig. 5. The optimal sectionalizing switch and tie line allocation plan for the 37-node test network in Case I.

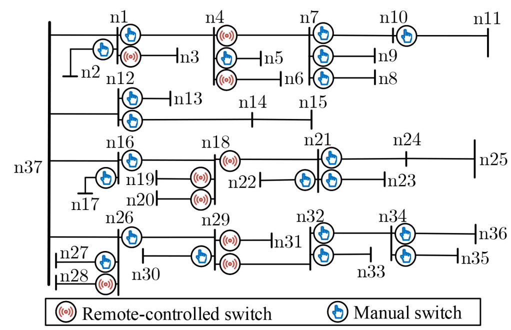

Fig. 6. The optimal sectionalizing switch placement plan for the 37-node test network in Case II.

network. Considering the allocation of tie lines together with SSs not only changes the number of installed SSs but also reorganizes the location of SSs so that the maximum benefit can be obtained from the installed tie lines. The changes made to the placement of SSs mean that not optimizing tie lines and SSs jointly would bring high redundant costs for the distribution utility. For example, when sole placement of SSs is conducted in Case II, all of the SSs are installed at the sending end of feeder sections, and all laterals are equipped with SSs; however, neither of those happens when the installation of tie lines is considered in Case I. Also, the optimal location of installed RCSs completely changes when the allocation of tie lines is taken into account. Moreover, in Case I, an MS is installed in each of the feeder sections connected to the substation node, while in Case II, installing SSs in such locations would bring no benefit at all since there is no tie line to restore the interrupted load after those switches isolate the fault. These significant differences in the location of SSs in these two cases endorse the fact that only by concurrent placement of SSs and tie lines, the most efficient investment plan for those assets can be obtained.

The simulation times as well as the preprocessing times for Case I of the five networks are represented in Table III. Note that the preprocessing time is the time required for obtaining the scenarios and the corresponding sets in Python using Algorithm 1, while simulation time is the time required for solving the optimization model, presented in Section III, in GAMS. It is evident that the preprocessing times in all cases are negligible, compared to the simulation times. Nevertheless, the input data provided by the preprocessing procedure can significantly facilitate the mathematical modeling and improve the computational performance of the optimization model since the restoration scenarios for every single-feeder section failure are prespecified.

The simulation times for Case II are also 0.01, 0.01, 0.02, 0.02, and 1.46 minutes for the test networks with 37, 85, 137, 145, and 230 nodes, respectively. By comparing the computation times in Case I to those of Case II, we realize that while the simulation times are low in Case II (i.e., using the proposed model for sole installation of SSs) of the first four networks, considering the allocation of tie lines together with SSs (i.e., Case I) increases the simulation time to a great extent for these networks. However, this is not true regarding the 230-node network. In Case II of this network, the simulation time is much longer than Case I of this network. This happens in spite of more decision variables in Case I than in Case II owing to the combinatorial nature of mixed-integer linear programs. Regardless of which case has a higher simulation time, the computational burden of the proposed model would not be an issue for its implementation since the simulation times, even in the worst scenario (i.e., in Case I of the 145-node network), are less than twenty minutes, which indicates that the developed MILP model is computationally efficient. Moreover, considering the allocation of tie lines is vital for achieving the globally optimal solution that an increase in the computation time would not be of considerable importance. Therefore, the global optimal solution for the tie line and SS allocation problem is accessible in a finite amount of time using the proposed MILP model.

VI. Conclusion

A novel approach has been presented for optimizing the concurrent placement of SSs and tie lines in the distribution networks with complex topologies. The proposed model aimed to reach a compromise between the total costs of the assets and the reliability-related costs. To pragmatically account for the reliability-oriented costs, a reliability incentive regulation in terms of a reward-penalty scheme based on SAIDI as well as the revenue lost due to undelivered energy to the end-users, estimated based on EENS, were considered. In order to quantify these costs, an innovative technique was proposed to model SAIDI and EENS. Developed based on the FMEA concept, the presented reliability evaluation method relied on finding the minimal exclusive failure states, for which a systematic algorithm was proposed. Leveraging the proposed reliability assessment technique, the optimal switch and tie line placement problem was formulated as an MILP model. Implementation of the proposed model on various test cases demonstrated its applicability, efficiency, and scalability. Future research will focus on modeling higher-order failures, e.g., multiple feeder section outages, uncertainty of parameters, and impact of distributed generation on optimizing placement of sectionalizing and tie switches.

Appendix

In distribution networks, when two MSs are required to isolate a fault and restore the power in a restoration scenario, the limitation of field crews may result in a longer restoration time than what is calculated based on the model presented in Section III. This only happens when the sectionalzing switch, which is isolating the fault, and the tie switch, needed to be utilized in the restoration scenario, are manual. To consider this practical issue in the mathematical model, (20)–(24) should be added to the model developed in Section III. The additional equations involve two sets of binary variables, $y_{l,n,s}$ and $z_{l,n,s}$, respectively defined in (21) and (23), to model the sequential operation of two MSs. One the one hand, equation (20) sets binary variable $y_{l,n,s}$ to one, whenever no RCS is installed to isolate load node n from the faulted feeder section l in a case where the power restoration is through a tie line. On the other hand, according to (22), $z_{l,n,s}$ should be one if none of the switches installed on the tie lines to restore the power to node n are remote-controlled. If both binary variables are set to one for a restoration scenario s, it means that two manual switch operations are required to reconnect load node n after a failure occurs in feeder section l. In a case where there are not enough field crews to operate the manual sectionalizing and tie switches in parallel and, rather, one crew should operate them sequentially, then (24) ensures that the restoration time of customers at load node n is at least βST^M for a fault in feeder section l; where β is a positive parameter, equal or greater than one, which represents the limitation of field crews that causes the longer restoration time. This parameter should be set based on the additional required time (e.g., for traveling to or between the sites) for operating the MSs sequentially in the distribution network. However, if there are enough crews, it must be set to 1.

$$y_{l,n,s} \geq 1 - \sum_{e \in E_s} x^R_{l,e} - \sum_{\bar{l} \in L_s} \sum_{e \in E} x^R_{\bar{l},e}; \quad \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\} \tag{20}$$

$$y_{l,n,s} \in \{0,1\}; \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\} \tag{21}$$

$$z_{l,n,s} \geq 1 - \sum_{r \in R_s} x^R_r; \quad \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\} \tag{22}$$

$$z_{l,n,s} \in \{0,1\}; \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\} \tag{23}$$

$$\theta_{l,n,s} \geq \beta ST^M \lambda_l \big(y_{l,n,s} + z_{l,n,s} - 1\big); \quad \forall l \in L, \forall n \in \Omega, \{\forall s \in S_{l,n} \mid R_s \neq \emptyset\}. \tag{24}$$

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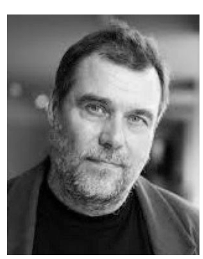

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