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Published in:
Proceedings of the IEEE 32nd Annual International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC 2021

DOI:
10.1109/PIMRC50174.2021.9569544

Published: 22/10/2021

Document Version
Peer reviewed version

Please cite the original version:
Amidzade, M., Al-Tous, H., Caire, G., & Tirkkonen, O. (2021). Orthogonal Multipoint Multicast Caching in OFDM Cellular Networks with ICI and IBI. In *Proceedings of the IEEE 32nd Annual International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC 2021* (pp. 394-399). IEEE. https://doi.org/10.1109/PIMRC50174.2021.9569544

Orthogonal Multipoint Multicast Caching in OFDM Cellular Networks with ICI and IBI

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***Abstract*—We consider optimal cache placement and delivery for Orthogonal Multipoint Multicasting (OMPMC) cellular systems. In OMPMC, all Base Stations (BSs) that cache a file transmit identical signals in a dedicated frequency resource. The simultaneous transmissions create artificial multipath propagation, which creates Inter-Block Interference (IBI) and Inter-Carrier Interference (ICI) in Orthogonal Frequency Division Multiplexing systems where the Cyclic Prefix (CP) is shorter than the maximum propagation delay. The placement of files at BS caches is based on a probabilistic model. A file request is in outage if the average signal-to-interference-and-noise ratio associated with a request is less than a threshold. We formulate the cache policy and bandwidth allocation as a joint optimization problem aiming to minimize the total outage probability, and considering the effect of IBI and ICI. Despite that the outage probability does not have a closed form expression, we are able to devise an algorithm to find the optimal solution based on predictor-corrector approach. Simulations results are used to demonstrate the capability of the proposed algorithm to find the optimum cache policy. Simulation results show that the effect of ICI/IBI has to be considered in designing OMPMC caching policy.**

***Index Terms*—Wireless caching, cache placement and delivery policy, multipoint multicast transmission, resource allocation, inter-carrier interference, inter-block interference.**

I. INTRODUCTION

The unprecedented data growth in cellular networks is a challenge that can be alleviated using wireless caching at the network edge [1]. The ideas is to proactively store the contents at the edge of the network during a placement phase and deliver them to the users at peak time during the delivery phase [2]. Cache placement and delivery need to be jointly considered to design an optimum cache policy.

Stochastic geometry has been exploited to analyze the performance of cache-enabled networks in [3]–[8]. A homogenous Poisson point process (PPP) is used to model the locations of the caching Base Stations (BSs) and/or User Equipments (UEs). In a PPP model, it is natural to model caching at BSs using a probabilistic approach, in which BSs independently and randomly store popular files based on a common caching probability distribution [3], [5]. Different levels of accuracy have been used in modeling the physical layer, and the interferences present in cellular systems. In [3]–[5], the physical layer was abstracted to coverage area, and medium access control was not an issue, while in [6], a random frequency reuse scheme is used to reduce co-channel interference for unicast transmissions to users from serving caches.

In [9] a coordinated multipoint (CoMP) transmission is leveraged to fulfil user demands using multiple cooperative BSs. The network is assumed to possess complete Channel State Information (CSI) between the users and a set of BSs serving the users. Unicast CoMP transmissions from the serving set of BSs are performed towards each user, resulting in unavoidable multiple access interference.

Multipoint *multicast* transmissions on the physical layer have been considered in [7], [8], [10], assuming no CSI at the network. In [7] *coded caching* was applied, and BSs multicast messages towards multiple users. Using advanced receiver processing, users are able to receive transmissions from multiple BSs. There is conventional co-channel interference between the different cells in the network. In [8], we addressed Orthogonal Multipoint Multicasting (OMPMC), where the BSs collaboratively deliver files towards users using a network-level orthogonal transmission. Wideband macro-diversity transmission/reception is assumed, such that each receiver interested in the file receives the transmission over an aggregate multipath channel. transmitting BSs. The network thus operates as a Single Frequency Network (SFN) of the type applied in digital video broadcasting [11]. Assuming Orthogonal Frequency Division Multiplexing (OFDM) with **infinitely** long Cyclic Prefix (CP), there is a priori *no co-channel interference* in the network, and the SFN configuration for each file is separately optimized.

If the CP is not infinitely long, however, the artificial multipath propagation in OMPMC causes interference; signal components from remote transmitters will cause Inter-Block Interference (IBI) and Inter-Carrier Interference (ICI), and affect the Signal-to-Noise-Plus-Interference Ratio (SINR) characterizing file delivery, see e.g. [12]. This leads to a tradeoff between loss in throughput/capacity due to CP overhead on one hand, and SINR improvement on the other.

The objective of this paper is to extend analysis of optimum cache placement and delivery policies to the *non-asymptotic* domain with finite CP length, in a realistic OFDM scenario with IBI and ICI arising from propagation delay taken into account. Realistic modeling of interference leads to an inherently non-linear model of signal quality, and changes the nature of optimization considerably, when compared to an interference ignoring situation. For this, we devise a predictor-corrector method for cache placement and delivery optimization.

We find that for typical OFDM parameterizations, it is necessary to take it into account ICI and IBI when designing the cache placement and delivery policy.

The remainder of this paper is organized as follows: In Section II, the system model is introduced. In Section III, the overall outage probability of OMPMC cellular network is analyzed. In Section IV, a predictor-corrector based algorithm to find the optimum caching policy is devised. The simulation results are presented and discussed in Section V. Finally, Section VI concludes the paper.

II. System Model

We consider a cellular network where the BSs are equipped with caches of limited storage capacity. We assume that the locations of the Equipments (UEs) and BSs come from independent PPPs. Each BS stores files from a predetermined content library according to a common probabilistic model. The UEs request the files from the network based on a known file popularity. The network employs the OMPMC delivery scheme [8] to fulfil the request of UEs. The network allocates resources orthogonally to different files, and each file is transmitted in an SFN manner in these resources by all caching BSs. We adopt an OFDM transmission scheme which is modeled by a symbol length and CP length. In this method, the only interference corrupting the reception of a file comes from ICI/IBI of the SFN transmission of the file itself. Note that both IBI and ICI arise from the multipath nature of the channel, and both time and frequency synchronization is assumed perfect.

The probability that a requested file by a UE is in outage is determined by the SINR of the SFN transmission of that file at the UE. There is an SINR threshold for successful reception; if the SINR of a UE is less than this threshold, the UE is in outage. Applying OMPMC can guarantee a desirable level of SINR measured by the UEs when downloading a file. The overall outage probability of file requests is used as the system performance metric.

A. File Popularity and Placement

The library is assumed to contain N files. The probability that file n is requested is given by the file popularity $f_n(\cdot)$. We model this by the Zipf distribution (see e.g. [13]) as $f_n = n^{-\tau}/\sum_{j=1}^{N} j^{-\tau}$ for $n = 1, \cdots, N$, where τ denotes the skewness of the distribution. Without loss of generality, we assume that all files have the same size, normalized to B. BSs are equipped with a limited storage so that each BS can store at most M files. Cache placement is done based on a probabilistic approach as in [3], [5]. File n is stored with caching weight $q_n \leq 1$. The weight q_n represents the probability that a randomly chosen BS caches file n. Note that caching weights are constrained as $\sum_{n=1}^{N} q_n = M$. The *caching density* per storage unit is denoted by $p_n = \frac{q_n}{M}$, satisfying $p_n \leq \frac{1}{M}$ and $\sum_{n=1}^{N} p_n = 1$. Then, based on the thinning property, file n is cached according to a PPP with intensity $\lambda_{\text{eff}} p_n$, where $\lambda_{\text{eff}} = \lambda M$ is the effective intensity of single-file cache.

B. Orthogonal Multipoint Multicast Transmission

We use OMPMC for cache delivery. All BSs that cache a given file simultaneously multicast it in a network-wide orthogonal resource, and no other transmissions occur in this resource. All UEs needing the file attempt to decode it from this multipoint multicast (MPMC) transmission. We assume that the transmission power of all BSs are identical. The fraction of radio resource reserved for file n is w_n, and $\sum_{n=1}^{N} w_n = 1$. For the multicast delivery, OFDM-based single frequency network transmissions are used.

Based on the MPMC scheme, any user requesting a file will be fulfilled by all BSs which cache that file. Further, a network-level orthogonality over transmission of different files is used to remove the interference among them. We use OFDM-based transmission to deliver files in each dedicated resource. The receiver performance in the presence of multiple transmitters and in an OFDM-based transmission is analysed in [12]. The dominant multipath feature in the system arises from the multipoint transmissions. To simplify the analysis, we model the signal between an individual BS and an UE in terms of a single-path channel. Accordingly, the SINR $\gamma_{n,k}$ for UE k requesting file n is:

$$\gamma_{n,k} = \frac{\sum_{j\in\Phi_n} g_{j,k} \left\|\mathbf{x}_j - \mathbf{r}_k\right\|^{-\beta} b^2(\|\mathbf{x}_j - \mathbf{r}_k\|)}{\frac{1}{\gamma_0} + \sum_{j\in\Phi_n} g_{j,k} \left\|\mathbf{x}_j - \mathbf{r}_k\right\|^{-\beta} \left(1 - b^2(\|\mathbf{x}_j - \mathbf{r}_k\|)\right)}, \tag{1}$$

where $g_{j,k}$ is the shadow fading coefficient between BS j and UE k, $\mathbf{x}_j$ and $\mathbf{r}_k$ are the locations of BS j and UE k, respectively, Φ_n is the set of BSs caching file n, $\gamma_0 = \bar{p}_{\text{ref}}/(WN_0)$ is a reference SNR, given by the average received power $\bar{p}_{\text{ref}}$ of a BS transmission measured at a reference distance, N_0 is the noise spectral density and W is the total transmission bandwidth. A standard distance-dependent path-loss model is used in (1) with a path-loss exponent $\beta > 2$. Moreover, $b(x)$ is the *bias function* that for $x \geq 0$ is expressed as [12]:

$$b(x) = \begin{cases} 1 & \text{for} \quad 0 \leq \frac{x}{c} < T_{cp}, \\ 1 - \left(\frac{x}{cT_s} - \frac{T_{cp}}{T_s}\right) & \text{for} \quad T_{cp} \leq \frac{x}{c} < T_s + T_{cp} \\ 0 & \text{for} \quad \frac{x}{c} \geq T_s + T_{cp}, \end{cases} \tag{2}$$

where T_{cp} is the CP length in seconds, T_s is the OFDM symbol length in seconds and c is the speed of light, and the propagation delay is assumed to be directly proportional to the traveled distance.

III. The Overall Outage Probability

Assuming Gaussian codebooks, an upper bound of the rate of reliable transmission is given by AWGN channel capacity given the SINR, and the amount of resources that the network allocates for the file in request. The more resources allocated to a file, the lower transmission rate is needed, and the more likely it is that a user can successfully download the file. Consequently, there will be a rate threshold R_{Th} for successful reception such that a user experiences an outage when downloading file n if the maximum rate allowed by the SINR realization γ_n is less than this threshold. Normalized with total bandwidth, this becomes a spectral efficiency threshold α such that file n is successfully received by a user if the spectral efficiency in the resources dedicated for transmitting the file is sufficient. As all files have the same size B, the spectral efficiency threshold is equal for all files and is determined as $\alpha = BR_{\text{Th}}/W$. Given the amount of resources allocated to each file, the spectral efficiency threshold can, be translated to a file-specific channel gain threshold $\eta_n = (2^{\alpha/w_n} - 1)/\gamma_0$. Hence, the outage probability for UE k requesting file n is:

$$\mathcal{O}_{n,k} = \mathsf{Pr}\big(w_n \log_2(1+\gamma_{n,k}) \le \alpha\big). \tag{3}$$

The outage probability can be formulated in terms of a channel gain threshold η_n as:

$$\mathcal{O}_{n,k} = \mathsf{Pr}\left(\frac{\gamma_{n,k}}{\gamma_0} \le \eta_n\right). \tag{4}$$

According to Slivanyak-Mecke theorem [14], for stationary and homogeneous PPPs, it suffices to compute the SINR for a user located at the origin. For this user, without loss of generality we can set $g_j = g_{j,0}$, $r_0 = 0$ and $\mathcal{O}_n = \mathcal{O}_{n,0}$. Hence, taking into account that files are requested according to the file popularity f_n, the overall outage probability, as the system metric is:

$$\mathcal{O}^{(T)} = \sum_{n=1}^{N} f_n \mathcal{O}_n, \tag{5}$$

and we have:

$$\mathcal{O}_n = \mathsf{Pr}\Big(\underbrace{\sum_{j\in\Phi_n} g_j \left\|\mathbf{x}_j\right\|^{-\beta} k_n(\|\mathbf{x}_j\|)}_{z} \le \eta_n\Big), \tag{6}$$

where $k_n(x) = (1+\gamma_0\eta_n)b^2(x) - \gamma_0\eta_n$.

Proposition 1. *Assume an OMPMC network with BSs distributed according to a homogenous PPP with intensity λ in an environment with path-loss exponent β, and an OFDM transmission scheme with symbol length T_s and CP length T_{cp}. If file n with popularity f_n is cached with probability q_n, and allocated bandwidth w_n, then the overall outage probability is:*

$$\mathcal{O}^{(T)} = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{N} f_n \times$$
$$\int_0^\infty \frac{1}{w}\sin\left(-w\eta_n + 2\pi\lambda q_n w\int_0^\infty \frac{r^{1-\beta}k_n(r)}{w^2 r^{-2\beta}k_n(r)^2+1}dr\right)$$
$$\exp\left(-2\pi\lambda q_n w^2 \int_0^\infty \frac{r^{1-2\beta}k_n(r)^2}{w^2 r^{-2\beta}k_n(r)^2+1}dr\right)dw. \tag{7}$$

Proof. Based on (5), we need to compute the outage probability $\mathcal{O}_n$. To do so, we resort to the characteristic function of random variable z denoted by $\phi_z(w)$. Based on (1) and taking into account that $\phi_z(w) = \mathbb{E}\left\{e^{iwz}\right\}$, we get:

$$\phi_z(w) = \mathbb{E}_{\Phi_n}\Big\{\mathbb{E}_{\{g_j\}_j}\Big\{\exp\Big(iw\sum_{j\in\Phi_n}\frac{g_j}{\|\mathbf{x}_j\|^{-\beta}}k_n(\|\mathbf{x}_j\|)\Big)\Big|\Phi_n\Big\}\Big\},$$

where $\mathbb{E}\{.|.\}$ is conditional expectation. By expanding the exponential function, we get:

$$\begin{aligned}\phi_z(w) &\overset{(a)}{=} \mathbb{E}_{\Phi_n}\Big\{\prod_{j\in\Phi_n}\mathbb{E}_{\{g_j\}_j}\Big\{\exp\left(iwg_j\|\mathbf{x}_j\|^{-\beta}k_n(\|\mathbf{x}_j\|)\right)\Big|\Phi_n\Big\}\Big\}\\ &\overset{(b)}{=} \mathbb{E}_{\Phi_n}\left\{\prod_{j\in\Phi_n}\frac{1+iw\|\mathbf{x}_j\|^{-\beta}k_n(\|\mathbf{x}_j\|)}{1+w^2\|\mathbf{x}_j\|^{-2\beta}k_n(\|\mathbf{x}_j\|)^2}\right\}\\ &\overset{(c)}{=} \exp\left(2\pi\lambda q_n\int_0^\infty\Big(\frac{1+iwr^{-\beta}k_n(r)}{1+w^2r^{-2\beta}k_n(r)^2}-1\Big)rdr\right),\end{aligned} \tag{8}$$

where (a) follows based on independence of channel coefficients, (b) follows from assuming exponentially distributed channel gains $g_{j,k}$. and (c) is obtained with regard to the Probability Generating Functional (PGFL) property of the point process Φ_n, and the thinning property [15]. Then, in order to find $\mathcal{O}_n$ from the characteristic function $\phi_z(w)$, the Gil-Pelaez theorem [16] is used:

$$\mathsf{Pr}\{z < \eta_n\} = \frac{1}{2} - \frac{1}{\pi}\int_0^{+\infty}\frac{1}{w}\Im\{e^{-iw\eta_n}\phi_z(w)\}dw, \tag{9}$$

where $\Im\{x\}$ is the imaginary part of x. By substituting (8) into (9) and using (5) the statement follows. □

Hereafter, we assume the path-loss exponent $\beta = 4$, which is typical for propagation at moderate and long distances [17].

IV. Optimal Caching Based on OMPMC

The outage probability of file n is a function of w_n and p_n. Hence, the optimal cache placement and delivery policy is formulated as:

$$P_0: \quad \begin{aligned}&\min_{\{p_n\},\{w_n\}} \sum_{n=1}^{N} f_n\mathcal{O}_n(w_n,p_n),\\ &\text{s.t.}\begin{cases}\sum_{n=1}^N w_n = 1, & \sum_{n=1}^N p_n = 1,\\ 0\le w_n\le 1, & \text{for } n=1,\cdots,N,\\ 0\le p_n\le \frac{1}{M}, & \text{for } n=1,\cdots,N.\end{cases}\end{aligned} \tag{10}$$

To find the optimal solution, we consider the Karush-Kuhn-Tucker (KKT) conditions of P_0. The KKT analysis dictates that there exists a number k_a of active files for which the cache of BSs and network resources are utilized, i.e., $w_n = 0$, and $p_n = 0$ for all $n > k_a$. However, the objective of P_0 lacks a closed-form expression based on (2) and (7). This makes it difficult to find the optimal solution by directly analyzing KKT condition. Therefore, we are motivated to resort to another mechanism. We follow an approach in the line with Homotopy Continuation (HC) methods [18]. The HC pursues a smooth path whose all points are solutions of a homotopy problem. For this, the predictor-corrector methods are mainly exploited. Accordingly, a directional search and an optimization method are leveraged to achieve a sufficient close solution [19]. However, the existence of the homotopy path is not guaranteed in general [18]. Therefore, the target solution may not be found by the path-following method provided by HC. Here, we devise a predictor-corrector approach such that the optimal solution of the optimization problem is guaranteed to be found.

Accordingly, we parameterize the popularity distribution by θ using functions $\{a_n(\theta)\}_n^N$ such that $\lim_{\theta\to\tau} a_n(\theta) = f_n$ and $\lim_{\theta\to 0} a_n(\theta) = \frac{1}{N}$ for all $n \in \{1,\ldots,N\}$. The KKT conditions of P_0 for parameters θ and k_a are:

$$K_0(k_a,\theta): \begin{cases}v_1 = a_n(\theta)D_w\mathcal{O}_n(w_n,p_n),\\ v_2 = a_n(\theta)D_p\mathcal{O}_n(w_n,p_n),\\ 0<p_n<\frac{1}{M},\ 0<w_n<1,\\ \sum_{n=1}^{k_a} w_n = 1,\ \sum_{n=1}^{k_a} p_n = 1,\end{cases} \tag{11}$$

for all $n \in \{1,\ldots,k_a\}$, where $D_z g(.)$ indicates the derivative of function $g(.)$ with respect to (w.r.t.) z, and v_1 and v_2 are the Lagrange multipliers related to the equality constraints of (10). Some remarks are pointed out.

- Assume the solution of $K_0(k_a,\tau)$ exists denoted by $\{w_n^*\}_n^{k_a}$ and $\{p_n^*\}_n^{k_a}$. Then, solving $K_0(k_a,\theta)$ with $\theta<\tau$ leads to a solution lying in $(0,1)$ and $(0,\frac{1}{M})$. At the extreme case when $\theta=0$, the solution will be $w_n^*=\frac{1}{k_a}$ and $p_n^*=\frac{1}{k_a}$ for $n\in\{1,\ldots,k_a\}$ which still lie in the interval $(0,1)$ and $(0,\frac{1}{M})$.
- To obtain a solution for a skewness τ, we follow a stepwise path from $\theta=0$ to target skewness $\theta=\tau$. We find the solution corresponding to $K_0(k_a,\theta)$ at each step, based on the previous solution.
- Taking k_a as the number of active files along the path from $\theta=0$ to $\theta=\tau$, we do not need to enforce the inequality constraints $0<p_n\leq\frac{1}{M}$ and $0<w_n\leq 1$ at each step, as they implicitly hold given k_a. If no solution exists for a given k_a along the path, the value of k_a should be changed for the target $\theta=\tau$.

For the stepwise transitioning, we have

Proposition 2. *Given k_a and a solution of $K_0(k_a,\theta)$, the solution of $K_0(k_a,\theta+d\theta)$ is given by:*

$$\begin{aligned}&F_nD_\theta a_n(\theta)d\theta+a_n(\theta)\nabla_{w,p}F_n^T\ d\boldsymbol{x}_n=c_0,\\&G_nD_\theta a_n(\theta)d\theta+a_n(\theta)\nabla_{w,p}G_n^T\ d\boldsymbol{x}_n=c_1,\\&\sum_{n=1}^{k_a}dw_n=0,\ \ \sum_{n=1}^{k_a}dp_n=0,\end{aligned}\tag{12}$$

where $d\boldsymbol{x}_n=[dw_n,dp_n]^T$, $F_n=D_{w_n}\mathcal{O}_n(w_n,p_n)$ and $G_n=D_{p_n}\mathcal{O}_n(w_n,p_n)$. Note that w_n, dw_n, p_n and dp_n are solutions at θ. Further, c_0 and c_1 are some constants, and $\nabla_{w,p}g(.)$ indicates the Gradient of $g(.)$ w.r.t. w and p.

Proof. According to the conditions $K_0(k_a,\theta+d\theta)$, we have:

$$\begin{aligned}v_1'&=a_n(\theta+d\theta)D_{w+dw}\mathcal{O}\Big(w_n+dw_n,p_n+dp_n\Big)\\&=a_n(\theta)F_n+D_\theta\Big\{a_n(\theta)F_n\Big\}d\theta\\&=v_1+F_nD_\theta a_n(\theta)d\theta+a_n(\theta)\nabla_{w,p}F_n^T[dw_n,dp_n]^T,\end{aligned}\tag{13}$$

where, here, v_1 and v_1' are the Lagrange multipliers corresponding to the conditions $K_0(k_a,\theta)$ and $K_0(k_a,\theta+d\theta)$, respectively. Accordingly, for the other Lagrange multiplier, i.e. v_2, we have:

$$v_2'=v_2+G_nD_\theta a_n(\theta)d\theta+a_n(\theta)\nabla_{w,p}G_n^T[dw_n,dp_n]^T,\tag{14}$$

where v_2' is the corresponding Lagrange multiplier of $K_0(k_a,\theta+d\theta)$. Note that v_1 and v_1' are constant w.r.t. n dictating that $F_nD_\theta a_n(\theta)d\theta+a_n(\theta)\nabla_{w,p}F_n^T[dw_n,dp_n]^T$ should be constant. Furthermore, since the previous solutions satisfy $\sum_{n=1}^{k_a}w_n=1$ and $\sum_{n=1}^{k_a}p_n=1$ and we want to enforce these conditions for (w_n+dw_n,p_n+dp_n), we have:

$$\sum_{n=1}^{k_a}dw_n=0,\ \ \sum_{n=1}^{k_a}dp_n=0.\tag{15}$$

Considering (13), (14) and (15), (12) holds. □

Solving (12) leads to the following Ordinary-Differential-Equations (ODEs) related to the KKT equations $K_0(k_a,\theta)$:

$$\frac{d\boldsymbol{x}_n}{d\theta}=\mathbf{H}_n^{-1}\big(\mathbf{c}-D_\theta a_n(\theta)\nabla\mathcal{O}_n(w_n,p_n)\big),\ n=1,\ldots,k_a,\tag{16}$$

where

$$\mathbf{c}=\Big(\sum_{n=1}^{k_a}\mathbf{H}_n^{-1}\Big)^{-1}\sum_{n=1}^{k_a}\Big(\mathbf{H}_n^{-1}D_\theta a_n(\theta)\nabla\mathcal{O}_n(w_n,p_n)\Big),\tag{17}$$

and

$$\mathbf{H}_n=a_n(\theta)\nabla^2\mathcal{O}_n(w_n,p_n)\ .\tag{18}$$

Here $\nabla\mathcal{O}_n(w_n,p_n)$ and $\nabla^2\mathcal{O}_n(w_n,p_n)$ indicate the Gradient vector and Hessian matrix of $\mathcal{O}_n(w_n,p_n)$ w.r.t. w_n and p_n.

By solving the ODEs (16) for the initial point $\theta=0,\{w_n\}_n=1/k_a,\{p_n\}_n=1/k_a$ and the target point $\theta=\tau,\ \{w_n\}_n,\{p_n\}_n$, the optimal solution of P_0 is obtained, assuming k_a. Hence, we associate a set of ODEs with problem P_0 such that its optimal solution can be found assuming existence of $\mathbf{H}_n^{-1}$ and $\Big(\sum_{n=1}^{k_a}\mathbf{H}_n^{-1}\Big)^{-1}$. However, we use a numerical method to obtain the solution. We start from the parameter $\theta=0$, and increment it by $\Delta\theta$. At each step $\theta'=\theta+\Delta\theta$ we find $(w_n+\Delta w_n,p_n+\Delta p_n)$ by numerically solving the ODEs (16). We repeat this incremental process, until reaching the target skewness τ. We call $\boldsymbol{\Delta}_n=(\Delta p_n,\Delta w_n)$ the predictor step and the corresponding updated solution $(w_n',p_n')=(w_n+\Delta w_n,p_n+\Delta p_n)$ the predicted points.

After a number of prediction iterations, we may have deviated from the optimal solution due to numerical imprecision. Hence, we correct the predicted points by searching for the corrector step $(\Delta w_n',\Delta p_n')$ such that the outage probability is minimized:

$$\begin{aligned}&\min_{\{\Delta w_n'\},\{\Delta p_n'\}}\sum_{n=1}^{k_a}a_n(\tau')\mathcal{O}_n(w_n'+\Delta w_n',p_n'+\Delta p_n'),\\&\quad\text{s.t.}\sum_{n=1}^{k_a}\Delta w_n'=0,\ \ \sum_{n=1}^{k_a}\Delta p_n'=0.\end{aligned}\tag{19}$$

Using a second-order expansion, the corrector step of (19) can be obtained as:

$$\boldsymbol{\Delta}_n'=\mathbf{H}_n'^{-1}\big(\mathbf{c}'-a_n(\theta')\nabla\mathcal{O}_n(w_n',p_n')\big),\ n=1,\ldots,k_a,\tag{20}$$

where $\boldsymbol{\Delta}_n'=[\Delta w_n',\Delta p_n']^T$, and

$$\mathbf{c}'=\Big(\sum_{n=1}^{k_a}\mathbf{H}_n'^{-1}\Big)^{-1}\sum_{n=1}^{k_a}\Big(\mathbf{H}_n'^{-1}a_n(\theta')\nabla\mathcal{O}_n(w_n',p_n')\Big),\tag{21}$$

and

$$\mathbf{H}_n'=a_n(\theta')\nabla^2\mathcal{O}_n(w_n',p_n').\tag{22}$$

Some remarks about the predictor-corrector method are worth pointing out. We use a numerical method to obtain the Gradient vector and Hessian matrix related to predictor and corrector steps. In practice the corrected points are obtained as $w_n''=w_n'+\mu_c\Delta w_n'$, $p_n''=p_n'+\mu_c\Delta p_n'$, where $\mu_c\in[0.1,1]$, and we update the solution using corrected points only if $\sum_{n=1}^{k_a}a_n(\theta')\mathcal{O}_n(w_n'',p_n'')<\sum_{n=1}^{k_a}a_n(\theta')\mathcal{O}_n(w_n',p_n')$. For the popularity parameterization, $\{a_n(\theta)\}_{n=1}^{N}$ is chosen such that the need for corrector step is minimized, for this we set $a_n(\theta) = \frac{1}{N}(N f_n)^{\theta/\tau}$, for $n \in \{1, \ldots, N\}$.

A pseudo code for the predictor-corrector method to find the solution of P_0 is given in Algorithm 1. Note that we perform a one-dimensional search over a candidate set $\mathcal{K}$ to find the optimum value for k_a.

V. Simulation Results and Discussion

We consider an Urban Microcell NLOS scenario from 5G NR [20], slightly modified to have path-loss exponent $\beta = 4$. For UE height 1.5 m, BS height 10 m, and carrier frequency 3.5 GHz, path-loss in dB is then modeled as $L = 139.889 + 40\log_{10}(r)$, with r is measured in km. We assume a homogenous PPP for the BSs deployment with intensity ranging from 2 to 300 per km^2. As $M = 10$, the effective intensity λ_{eff} correspondingly changes from 20 to 3×10^3 per km^2. The BS transmission power is assumed to be 33 dBm, the antenna gain at the BS is 5 dBi, the antenna gain at the UE is 0 dBi, the UE noise-figure is 9 dB, the noise spectrum density is -174 dBm and the bandwidth is 20 MHz. These result in $\gamma_0 = 0.105$. An OFDM modulation scheme is considered; the symbol duration $T_s = 66.6\mu s$, and the variable CP length ranging up to a so-called extended CP of $16.6\mu s$, which corresponds to about $1/4$ of the symbol duration.

A library with $N = 100$ files is considered. File popularity is modeled using a Zipf distribution, with skewness values $\tau = 0.6$ and $\tau = 2.6$. The spectral efficiency threshold is $\alpha = 0.1$. This means that for a library of size N, there are $10/N$ time-frequency resources available for the transmission of each bit in the library. The incremental step in the predictor-corrector algorithm is taken to be $\Delta\theta = 3 \times 10^{-3}$.

The results of Predictor-Corrector Method for the Interference-Aware policy, with legend $\mathtt{PCM-IA}$, are compared to three schemes. An Interference-Ignoring solution is considered, where the caching policy in [8] is directly used despite the presence of ICI/IBI. For this, the overall outage optimization problem in [8] is solved using the Predictor-Corrector Method. We denote the obtained solution by $\mathtt{PCM-II}$. Note that, the obtained solution is evaluated in the presence of ICI/IBI. In addition, we consider simplified Threshold-Based (TB) policies, where resources are allocated equally for files more popular than a threshold file $\nu \in \{1, \ldots, N\}$, while no resources are allocated for other files. More specifically, the $\mathtt{TB-IA}$ policy is the Threshold-Based Interference-Aware policy, and $\mathtt{TB-II}$ is the Threshold-Based Interference-Ignoring solution from [8] that is applied despite the presence of ICI/IBI.

Algorithm 1 The predictor-corrector method for cache policy.

for $k_a \in \mathcal{K}$ **do**
 Initialization:
  Set $\theta = 0$ and $(w_n, p_n) = (\frac{1}{k_a}, \frac{1}{k_a}), \forall n \in \{1, \ldots, k_a\}$.
 while $\theta' \leq \tau$ **do**
  Increment the popularity distribution, $\theta' = \theta + \Delta\theta$.
  Predictor step:
   Find $(\Delta w_n, \Delta p_n)$ from (16) and (17) and obtain $w_n' = w_n + \Delta w_n,\ p_n' = p_n + \Delta p_n$.
  Corrector step (if necessary):
   Find $(\Delta w_n', \Delta p_n')$ from (20) and (21).
  if $\sum_{n=1}^{k_a} a_n(\theta')\mathcal{O}_n(w_n'', p_n'') < \sum_{n=1}^{k_a} a_n(\theta')\mathcal{O}_n(w_n', p_n')$ **then**
   Update the solution by:
    $w_n'' = w_n' + \mu_c \Delta w_n',\ p_n'' = p_n' + \mu_c \Delta p_n'$.
  else
    $w_n'' = w_n',\ p_n'' = p_n'$.
  end if
  Set $\theta = \theta'$, $w_n = w_n''$ and $p_n = p_n''$.
 end while
 Save $\{p_n\}$, $\{w_n\}$ and the overall outage probability for k_a.
end for
Search over k_a to find the optimum caching policy.

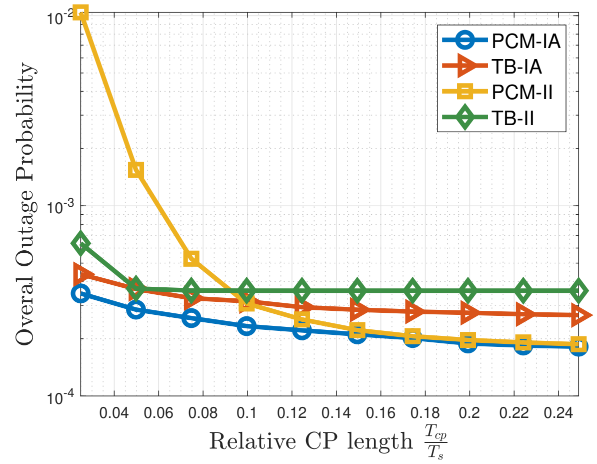

Fig. 1: The overall outage probability as a function of ratio of CP length to the symbol length for skewness $\tau = 2.6$ and effective intensity $\lambda_{\text{eff}} = 3 \times 10^3$.

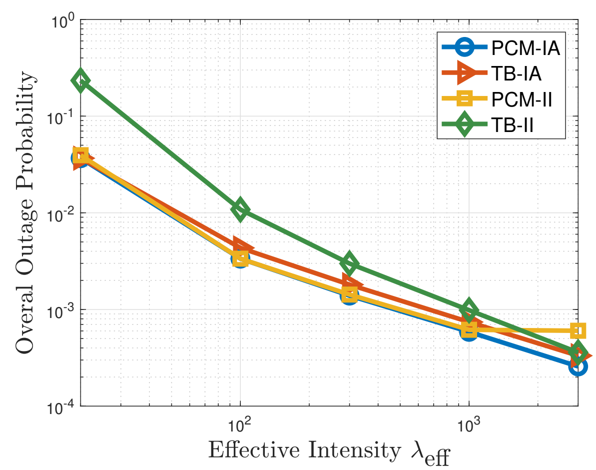

Fig. 2: The overall outage probability as a function of the effective intensity λ_{eff} for skewness $\tau = 2.6$, and relative CP length 7%.

Figure 1 shows the overall outage probability obtained with different caching policies as a function of the CP length for skewness $\tau = 2.6$ and effective intensity $\lambda_{\text{eff}} = 3 \times 10^3$. The $\mathtt{PCM-IA}$ policy outperforms other policies for all values of CP length. There is a remarkable performance gap between Interference-Ignoring policies especially for short CPs. As the CP length increases, the performance gap between $\mathtt{PCM-IA}$, and TB policies grows. Note that the TB policies develop an error floor despite increasing CP length. Overall, these results

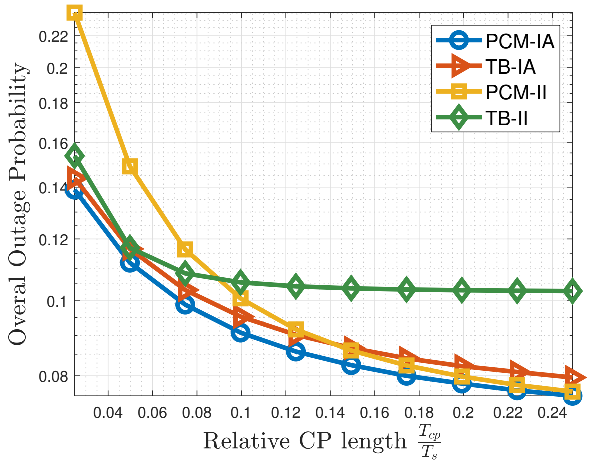

Fig. 3: The overall outage probability as a function of ratio of CP length to the symbol length for skewness $\tau = 0.6$ and effective intensity $\lambda_{\text{eff}} = 4 \times 10^3$.

portray that the effect of ICI/IBI has to be considered in optimization and show the merit of $\texttt{PCM}-\texttt{IA}$ when designing an optimal caching policy. Interestingly, the $\texttt{TB}-\texttt{II}$ policy is more robust against interference than the $\texttt{PCM}-\texttt{II}$ policy, when CP is short and the interference is accordingly strong.

Figure 2 shows the overall outage probability as a function of the effective cache intensity λ_{eff} for skewness $\tau = 2.6$ and normal CP length $T_{cp} = 4.7\mu s$, corresponding to the CP-length ratio 7%. The $\texttt{PCM}-\texttt{IA}$ method outperforms the TB policies. This is expected, since with $\texttt{PCM}-\texttt{IA}$, resources are optimally allocated, in contrast to equal allocation in the TB policies. For high cache intensities, the Interference-Ignoring solution $\texttt{PCM}-\texttt{II}$ becomes unstable, and exhibits cross-over to an interference limited regime.

Figure 3 shows the outage probability results for skewness $\tau = 0.6$ and effective intensity $\lambda_{\text{eff}} = 4 \times 10^3$. As more caching is needed with a smaller skewness, a larger value of effective intensity is used as compared to Figure 1. We again find that the Interference-Ignoring solutions are less reliable than Interference-Aware policies. As the CP length increases, the reliability of $\texttt{TB}-\texttt{II}$ considerably decreases and becomes saturated.

Overall, with the considered OFDM parametrizations and network densities, the effect of ICI/IBI has a considerable role for higher caching densities λ_{eff}. This can be understood as follows: If there is no ICI/IBI, the statistics of SINR experienced by UEs improve indefinitely with caching densification. As a consequence, with increasing λ_{eff} one can reduce the outage of the cached files, or use less resources per file, and cache more files; both methods reduce the overall outage. When there is ICI/IBI, the network becomes interference limited; the statistics of SINR become asymptotically independent of λ_{eff}. As a consequence, an outage floor develops asymptotically in λ_{eff}. In the plots, we see these floors starting to have an effect.

VI. Conclusion

In this paper, we considered OMPMC caching in OFDM cellular networks. The optimal cache placement policy was obtained considering ICI and IBI. The outage probability was analyzed in a micro-cellular simulation scenario. A predictor-corrector method was devised to find the optimal cache policy considering ICI and IBI. We found that taking the ICI/IBI into account in cache placement and delivery optimization provides systematically better results than neglecting it. Furthermore, a simple ICI/IBI-aware threshold-based algorithm was found to perform near to the optimal strategy. In future work we shall extend the analysis to optimization of CP length and spectral efficiency target in caching networks.

Acknowledgment

This work was funded in part by the Academy of Finland (grant 319058).

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