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Downconversion of quantum fluctuations of photonic heat current in a circuit

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We discuss the nonzero frequency noise of heat current with the explicit example of energy carried by thermal photons in a circuit. Instead of the standard circuit modeling that gives a convenient way of predicting time-averaged heat current, we describe a setup composed of two resistors forming the heat baths by collections of bosonic oscillators. In terms of average heat transport this model leads to identical results with the conventional one, but besides this, it yields a convenient way of dealing with noise as well. The nonzero-frequency heat current noise does not vanish in equilibrium even at zero temperature, a result that is known for, e.g., electron tunneling. We present a modulation method that can convert the difficult-to-measure high-frequency quantum noise down to zero frequency.

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I. INTRODUCTION

The quantum noise of heat current, i.e., noise at nonzero frequencies, is an intriguing topic [1–7]. It has been demonstrated theoretically for several processes, including heat transport by electrons, phonons, and between the two, that this noise does not vanish even at zero temperature, thus jeopardizing the validity of the fluctuation-dissipation theorem. Energy can be transported by various mechanisms. In condensed matter systems the most common carriers of energy in the form of heat are electrons and phonons. Outside this domain radiation by photons in different frequency regimes provides an important channel of energy exchange and thermalization. But even within solid-state systems, radiation plays an important role, which is the topic of the current paper. In electrical circuits this mechanism corresponds to heat transmitted by thermal noise, which can also be interpreted as emission and absorption of thermal microwave photons [8–11]. In this context a straightforward way to treat the problem of average heat current in linear circuits is to employ a circuit theory where quantum noise emitted by dissipative elements induces Joule heating elsewhere in the said circuit [8,12,13]. To study noise of this heat current, various methods have been employed [14–16], including full-counting techniques with treatment of circuits by Keldysh Green functions but focusing on zero-frequency noise. Here we address the archetypal Johnson-Nyquist setup of two resistors coupled to each other [17,18] either directly or via a reactive element. We build up the resistors of bosonic oscillators [19–23], as shown in Fig. 1(a), this way obtaining a microscopic Hamiltonian amenable for investigating noise at arbitrary frequencies, specifically finding the noise spectrum of heat in these configurations. Observing the quantum heat current noise is an experimental challenge. To overcome it, we propose to shift the high-frequency noise to low or zero frequency by modulating the coupling between the two heat baths periodically. This mixing principle, familiar for electrical measurements [25–27], has been proposed by Averin for fermionic heat noise [5]. As pointed out by several authors [28], it is important to specify what quantity to measure in order to make precise theoretical predictions. In this respect heat current noise after the downconversion boils down to a measurement of the time-dependent temperature of the mesoscopic heat bath (resistor). Concretely we propose variation of a reactive element between the resistor baths to achieve this goal.

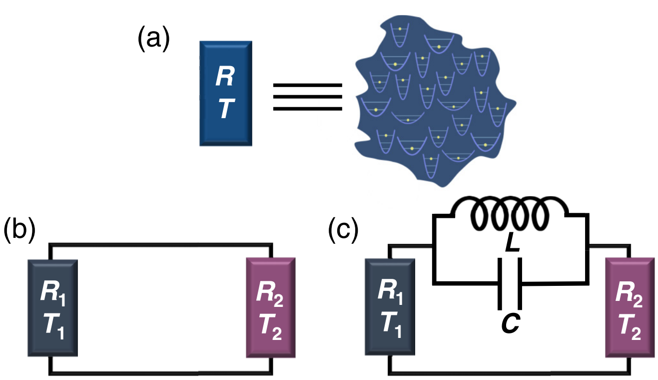

FIG. 1. Elements for radiative heat transport in a circuit. (a) We model the resistors by a collection of bosonic oscillators. (b) The basic setup of two resistors R_1 and R_2 at temperatures T_1 and T_2, respectively. (c) Same as (b) but with a parallel LC circuit with inductance L and capacitance C as frequency-dependent nondissipative elements between the resistors, representing, e.g., a classical superconducting quantum interference device (SQUID) [24].

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II. MODELING THE SYSTEM OF TWO RESISTORS

The basic systems to be described in this work are shown in Fig. 1. We consider resistor R_2 with phase operator $\hat{\varphi}_2$ across it and subject to current operator $\hat{i}_2$ injection, due to the current produced by resistor R_1 arising from its thermal noise at temperature T_1. Its effective Hamiltonian, describing the system and its coupling to the bath R_1, is then

$$\hat{H} = \hat{H}_2 - \hat{i}_2\hat{\Phi}_2 \equiv \hat{H}_2 + \hat{\mathbb{V}}, \tag{1}$$

where $\hat{H}_2 = \sum_k \hbar\omega_k \hat{c}_k^\dagger \hat{c}_k$ is the Hamiltonian of the bare resistor R_2 with oscillator energies $\hbar\omega_k$ and ladder operators $\hat{c}_k$, $\hat{c}_k^\dagger$, and $\hat{\Phi}_2 = \frac{\hbar}{e}\hat{\varphi}_2$. To find the properties of the circuits we write the charge operator of the oscillators forming resistor R_1 and phase operator of the oscillators of R_2 as a linear combination with coefficients $\mu_j^{(1)}$ and $\lambda_k^{(2)}$ in the interaction picture as

$$\hat{q}_1(t) = \sum_j \imath\mu_j^{(1)}(\hat{b}_j e^{-\imath\omega_j t} - \hat{b}_j^\dagger e^{\imath\omega_j t}), \tag{2}$$

$$\hat{\varphi}_2(t) = \sum_k \lambda_k^{(2)}(\hat{c}_k e^{-\imath\omega_k t} + \hat{c}_k^\dagger e^{\imath\omega_k t}). \tag{3}$$

Here the ladder operators for resistor R_1 are $\hat{b}_j$, $\hat{b}_j^\dagger$, and the superscript (i) refers to the resistor R_i.

Let us consider the basic setup [17,18] as shown in Fig. 1(b). We aim to calculate the photonic heat transport, its mean value and nonzero frequency noise, based on the model here. The current operator $\hat{i}_2$ through R_2 then reads $\hat{i}_2 = \frac{R_1}{R_1+R_2} d\hat{q}_1/dt$, i.e.,

$$\hat{i}_2(t) = \frac{R_1}{R_1+R_2}\sum_j \mu_j^{(1)}\omega_j(\hat{b}_j^\dagger e^{\imath\omega_j t} + \hat{b}_j e^{-\imath\omega_j t}). \tag{4}$$

As a result, the coupling Hamiltonian $\hat{\mathbb{V}}$ is given by

$$\hat{\mathbb{V}}(t) = -\frac{R_1}{R_1+R_2} \times \sum_{j,k}\lambda_k^{(2)}\mu_j^{(1)}\omega_j(\hat{b}_j^\dagger e^{\imath\omega_j t} + \hat{b}_j e^{-\imath\omega_j t})(\hat{c}_k^\dagger e^{\imath\omega_k t} + \hat{c}_k e^{-\imath\omega_k t}). \tag{5}$$

This is in the form that one obtains by scattering theory imposing energy conservation [2,29]. In order to establish consistency between the oscillator bath model and the actual circuit, we request the current $\hat{i} = d\hat{q}_1/dt$ noise of resistor R_1, $S_i(\omega) = \int_{-\infty}^{\infty} dt e^{\imath\omega t}\langle\hat{i}(t)\hat{i}(0)\rangle$, to be equal to the quantum current noise of that resistor, i.e., $S_i(\omega) = \frac{2}{R_1}\frac{\hbar\omega}{1-\exp(-\beta_1\hbar\omega)}$, where $\beta_1 = 1/(k_B T_1)$ is the inverse temperature of R_1. Similarly we set the voltage $\hat{v} = d\hat{\Phi}_2/dt$ noise of resistor R_2, $S_v(\omega) = \int_{-\infty}^{\infty} dt e^{\imath\omega t}\langle\hat{v}(t)\hat{v}(0)\rangle$, equal to $S_v(\omega) = 2R_2\frac{\hbar\omega}{1-\exp(-\beta_2\hbar\omega)}$ with similar notations. These conditions lead to the expressions

$$\mu_j^{(1)} = \left(\frac{\hbar}{\pi\nu_1(\omega_j)\omega_j R_1}\right)^{1/2}, \quad \lambda_k^{(2)} = \left(\frac{\hbar R_2}{\pi\nu_2(\omega_k)\omega_k}\right)^{1/2}, \tag{6}$$

where $\nu_i(\omega)$ is the oscillator density of states in R_i with $i = 1, 2$. The operator of heat current to R_2, $\dot{\hat{H}}_2 = \frac{\imath}{\hbar}[\hat{H}, \hat{H}_2] = \frac{\imath}{\hbar}[\hat{\mathbb{V}}, \hat{H}_2]$, reads then

$$\dot{\hat{H}}_2 = \imath\frac{\hbar\sqrt{r_0}}{2\pi}\sum_{j,k}\frac{\sqrt{\omega_j\omega_k}}{\sqrt{\nu_1(\omega_j)\nu_2(\omega_k)}} \times(\hat{b}_j^\dagger e^{\imath\omega_j t} + \hat{b}_j e^{-\imath\omega_j t})(\hat{c}_k^\dagger e^{\imath\omega_k t} - \hat{c}_k e^{-\imath\omega_k t}), \tag{7}$$

with $r_0 = 4R_1R_2/(R_1+R_2)^2$.

A. Quantum of thermal conductance

According to the Kubo formula, the expectation value of heat current $\dot{Q} \equiv \langle\dot{\hat{H}}_2\rangle$ to R_2 under stationary conditions is

$$\dot{Q} = -\frac{\imath}{\hbar}\int_{-\infty}^{0} dt'\langle[\dot{\hat{H}}_2(0), \hat{\mathbb{V}}(t')]\rangle_0, \tag{8}$$

where $\langle\cdot\rangle_0$ denotes the expectation value of a quantity for the noninteracting resistor at a given temperature. Since $\langle\hat{b}_j^\dagger\hat{b}_j\rangle_0 = n_1(\omega_j)$ with $n_1(\omega) = 1/(e^{\beta_1\hbar\omega}-1)$ the Bose-Einstein distribution of resistor R_1, and similarly $\langle\hat{c}_k^\dagger\hat{c}_k\rangle_0 = n_2(\omega_k)$ for resistor R_2, the heat current is given by

$$\dot{Q} = r_0\frac{\pi k_B^2}{12\hbar}(T_1^2 - T_2^2), \tag{9}$$

which is equal to the expression obtained by standard circuit theory [8,12,13], thus providing a sanity check of our model. Equation (9) yields heat conductance $G_{th} = d\dot{Q}/dT_1|_T$ in the form $G_{th} = r_0 G_Q$, where $G_Q = \pi k_B^2 T/(6\hbar)$ is the thermal conductance quantum at temperature T [30].

B. Equilibrium quantum heat current noise

The focus in this paper is the equilibrium noise [31] of heat current [15,16]. With the same operators, this noise at (angular) frequency ω, $S_{\dot{Q}}(\omega) = \int_{-\infty}^{\infty} e^{\imath\omega t}\langle\dot{\hat{H}}_2(t)\dot{\hat{H}}_2(0)\rangle$, is given by

$$S_{\dot{Q}}(\omega) = \frac{\hbar^2 r_0}{2\pi}\int_{-\infty}^{\infty} d\Omega\,\Omega(\Omega+\omega)n_1(\Omega)[1+n_2(\Omega+\omega)]. \tag{10}$$

The symmetrized heat current noise $S_{\dot{Q}}^{\text{symm}}(\omega) \equiv (S_{\dot{Q}}(\omega) + S_{\dot{Q}}(-\omega))/2$ at equal temperatures $T_1 = T_2 \equiv T$ for two resistors reads

$$S_{\dot{Q}}^{\text{symm}}(\omega) = \frac{r_0}{24\pi\hbar}[(2\pi k_B T)^2 + \hbar^2\omega^2]\hbar\omega\coth\left(\frac{\hbar\omega}{2k_B T}\right). \tag{11}$$

This result exhibits nonvanishing noise at $\omega \neq 0$ even at $T = 0$, a result known for some other systems [2–6] but not for the present one previously. At $\omega = 0$ it reproduces the fluctuation-dissipation theorem [1,31]. We note that identical results here and in what follows for the physical quantities $\dot{Q}$ and $S_{\dot{Q}}$ can be obtained with a similar strategy by replacing $\hat{i}_2\hat{\Phi}_2$ in Eq. (1) with $\hat{v}_2\hat{q}_2$, where $\hat{v}_2$ is the operator of voltage induced by R_1 on R_2, and $\hat{q}_2$ the charge operator for oscillators forming R_2.

III. ADDING A REACTIVE COUPLING ELEMENT

Next we investigate the influence of frequency-dependent dissipationless impedance in the circuit. This element can then filter the heat current and it will be used as one of the examples in illustrating the noise downconversion toward the end of the paper. Taking the circuit in Fig. 1(c) with parallel inductance L and capacitance C, we find the differential equation for the current $\hat{i}_{\rm L}$ through the inductor given with the help of $\hat{i} = d\hat{q}_1/dt$ of the noise source with $\hat{q}_1$ from Eq. (2) as $\frac{d^2 i_{\rm L}}{dt^2} + \frac{1+\frac{C\dot{L}}{L}(R_1+R_2)}{C(R_1+R_2)}\frac{di_{\rm L}}{dt} + \frac{1}{LC}i_{\rm L} = \frac{1}{LC}\frac{R_1}{R_1+R_2}i(t)$. Here $\dot{L}$ is the rate of change of the inductance due to, e.g., variation of the magnetic flux through the superconducting quantum interference device (SQUID). For periodic sinusoidal variation of L at driving frequency $\omega_0/2\pi$, we have $|\dot{L}| \lesssim \omega_0 L$. Thus we may ignore the direct influence of $\dot{L}$ in this equation if $C \ll 1/(\omega_0(R_1+R_2))$. Since, as it will turn out later, the modulation frequency will be of the order of $k_{\rm B}T/(2\pi\hbar)$ in order to obtain meaningful downconversion, we have the condition $C \ll 2\pi\hbar/(k_{\rm B}T(R_1+R_2))$. This yields $C \ll 1$ pF at $T = 10$ mK and for $R_1 + R_2 = 100\,\Omega$, which is a forgiving bound since the typical junction capacitances are of the order of 1–10 fF. Then ignoring the direct influence of $\dot{L}$, the solution of the circuit equation for $\hat{i}_2(t)$ is, with the help of Eq. (3),

$$i_2(t) = \frac{R_1}{R_1+R_2}\sum_j \mu_j^{(1)}\omega_j \times\left\{\frac{1-LC\omega_j^2}{(1-LC\omega_j^2)+\imath\frac{\omega_j L}{R_1+R_2}}\hat{b}_j^\dagger e^{\imath\omega_j t} + \text{H.c.}\right\}, \tag{12}$$

which is exact for a stationary LC circuit and approximately correct for sufficiently slowly varying L as described above. Since the relevant angular frequencies ω_j are again of the order of $k_{\rm B}T/(2\pi\hbar)$, it turns out that this equation is well approximated by

$$i_2(t) = \frac{R_1}{R_1+R_2}\sum_j \mu_j^{(1)}\omega_j\left\{\frac{1}{1+\imath\omega_j/\omega_{\rm L}}\hat{b}_j^\dagger e^{\imath\omega_j t} + \text{H.c.}\right\}, \tag{13}$$

with the same condition for the capacitance C as above. Here $\omega_{\rm L} = (R_1+R_2)/L$. This is the equation for a pure inductance L between the two resistors, and exact for that case at arbitrary frequencies of modulation as well; thus at the end we only need to consider this simplified circuit both in the stationary and modulated cases as long as the condition given for C is satisfied. Using Eqs. (13) and (3), the coupling Hamiltonian is given by

$$\hat{\mathbb{V}} = -\frac{R_1}{R_1+R_2}\sum_{j,k}(\hat{c}_k^\dagger e^{\imath\omega_k t} + \hat{c}_k e^{-\imath\omega_k t}) \times\left(\frac{\mu_j^{(1)}\omega_j}{1+\imath\omega_j/\omega_{\rm L}}\hat{b}_j^\dagger e^{\imath\omega_j t} + \frac{\mu_j^{(1)}\omega_j}{1-\imath\omega_j/\omega_{\rm L}}\hat{b}_j e^{-\imath\omega_j t}\right). \tag{14}$$

The operator of heat current to R_2 for the circuit of Fig. 1(c) reads then

$$\dot{\hat{H}}_2 = \imath\frac{\hbar\sqrt{r_0}}{2\pi}\sum_{j,k}\frac{\sqrt{\omega_j\omega_k}}{\sqrt{\nu_1(\omega_j)\nu_2(\omega_k)}}(\hat{c}_k^\dagger e^{\imath\omega_k t} - \hat{c}_k e^{-\imath\omega_k t}) \times\left(\frac{1}{1+\imath\frac{\omega_j}{\omega_{\rm L}}}\hat{b}_j^\dagger e^{\imath\omega_j t} + \frac{1}{1-\imath\frac{\omega_j}{\omega_{\rm L}}}\hat{b}_j e^{-\imath\omega_j t}\right). \tag{15}$$

A. Average heat current

The expectation value of the above operator which gives the heat current to R_2 is given by

$$\dot{Q} = \int_0^\infty \frac{d\omega}{2\pi}\frac{4R_1R_2}{|R_1+R_2+\imath\omega L|^2}\hbar\omega[n_1(\omega)-n_2(\omega)], \tag{16}$$

again obtained by Eq. (8). This is again the same result as that from the circuit theory, and it shows that the presence of nonvanishing inductance decreases the heat current, as expected. In order to solve the above integral analytically, we assume a small inductance such that $k_{\rm B}TL/\hbar \ll (R_1+R_2)$. Using the Taylor expansion for the integrand $4R_1R_2/|R_1+R_2+\imath\omega L|^2 \simeq r_0(1-(L/(R_1+R_2))^2\omega^2)$, the heat current between the two resistors via an inductor reads

$$\dot{Q} = r_0\frac{\pi k_{\rm B}^2}{12\hbar}(T_1^2-T_2^2) - r_0\frac{\pi^3 k_{\rm B}^4}{30\hbar^3}\frac{L^2}{(R_1+R_2)^2}(T_1^4-T_2^4). \tag{17}$$

The first part is the heat current between two bare resistors of Eq. (9). The thermal conductance then reads

$$G_{\rm th}^{(\rm L)} = r_0 G_{\rm Q}\left\{1-\frac{1}{5}\left(\frac{2\pi k_{\rm B}}{\hbar}\frac{L}{R_1+R_2}\right)^2 T^2\right\}. \tag{18}$$

Here $G_{\rm th}^{(\rm L)}$ denotes the thermal conductance of this particular circuit.

B. Heat current noise

With the same procedure as before, we obtain the symmetrized noise of heat current. In this circuit, we find that the lowest-order correction to the result of Eq. (11) is $-\delta S_{\dot{Q}}^{\rm symm}(\omega)$, where

$$\delta S_{\dot{Q}}^{\rm symm}(\omega) = \frac{\hbar r_0}{4\pi}\frac{L^2}{(R_1+R_2)^2} \times\left\{\frac{1}{30}\left(\frac{2\pi k_{\rm B}T}{\hbar}\right)^4 + \frac{1}{12}\left(\frac{2\pi k_{\rm B}T}{\hbar}\right)^2\omega^2 + \frac{1}{20}\omega^4\right\} \times\hbar\omega\coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right). \tag{19}$$

Like the average thermal conductance, also the noise is suppressed by the inductive filter in between.

IV. EXPERIMENTAL ASPECTS

We have shown that, in both the configurations that we studied here, the noise of heat current at nonzero frequencies does not vanish even at zero temperature. A question arises whether these setups are realistic for experiment. The answer

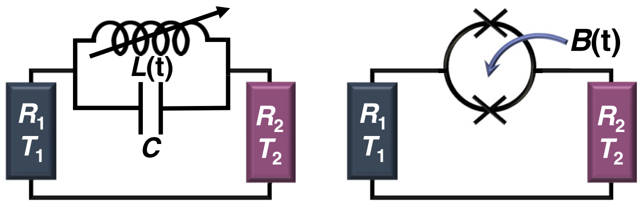

FIG. 2. Illustration of the principle and a potential practical realization of quantum heat noise downconversion. On the principal diagram on the left, the resistors are connected by a parallel LC circuit, with variable inductor. This configuration can be realized as a SQUID, which is modulated by magnetic field $B(t)$.

is positive: these configurations were proven to provide adequate description of the circuit in the experiments [10,11] without including extra reactive elements modeling it. This is confirmed by the following estimates. The geometric inductance of a line of length 100 μm yields an inductance leading to impedance of $\sim$0.1 Ω at thermal frequencies at 100 mK. This impedance is well below a typical series resistance of $R_i = 100\ \Omega$. On the other hand, the parallel shunting capacitance for a similar circuit leads to an impedance of order 100 k$\Omega \gg R_i$. Therefore, such parasitic impedances can be neglected in our analysis and in the basic experiments on microcircuits at subkelvin temperatures. The same applies to the downconversion measurements to be proposed below, since modulation frequencies are of the same order as temperature. Another question is how to observe the quantum heat current noise at high frequencies. This is a most challenging experiment; here we propose a mixing method to shift the high-frequency noise to low or zero frequency [5], where measuring such noise would be easier.

V. DOWNCONVERSION OF HEAT CURRENT

According to the setup shown in Fig. 2, i.e., coupling two resistors via an inductor, the inductance is varied such that $L(t) = L_0[1 + \eta\cos(\omega_0 t + \phi)]$, where η is the amplitude of the modulation and ϕ is the random phase of the drive with respect to system dynamics. In this case, we have the contribution from the ac drive as well. After averaging over ϕ the symmetrized heat current noise of the inductance modulation at frequency $\omega = 0$, $S_{\dot{Q}}(0)$, at equal temperatures of the two resistors reads

$$S_{\dot{Q}}(0) = 2k_{\mathrm{B}}T^2G_{\mathrm{th}}^{(\mathrm{L})} + \frac{\eta^2}{2}\delta S_{\dot{Q}}^{\mathrm{symm}}(\omega_0), \tag{20}$$

where $\delta S_{\dot{Q}}^{\mathrm{symm}}(\omega_0)$ is given in Eq. (19) with $L = L_0$. Specifically for $T \to 0$, we find

$$S_{\dot{Q}}(0) = \frac{\eta^2\hbar^2 r_0}{160\pi}\frac{L_0^2}{(R_1 + R_2)^2}\omega_0^5. \tag{21}$$

VI. DISCUSSION

The results obtained here for high-frequency noise of heat current are in line with those derived for electron and phonon transport elsewhere. All these systems demonstrate nonvanishing noise at zero temperature, which is an intriguing property not easily accountable with the standard fluctuation-dissipation theorem. Not dwelling further on this last point, we focus finally on the experimental feasibility of observing these nonzero frequency fluctuations discussed above. The direct measurement of heat current noise is a challenging task even at low frequency, but much more so at high frequencies. Even the measurement principle for noise at these frequencies is not obvious: one needs to do it at gigahertz frequencies to make the frequency-dependent contribution dominant over the thermal one [see Eq. (11)] even at a temperature of 10 mK achievable by standard techniques. The downconversion of this noise, summarized by Eqs. (20) and (21), can, however, be achieved in present-day experimental circuits operating at low temperatures. Varying the reactive impedance in between fixed resistors (Fig. 2) could be realized by a standard technique, by varying the magnetic flux through a SQUID that acts as a tunable inductor. Another option, not analyzed here, would be to vary the resistance of one of the two resistors. Since the electrical conductance of two-dimensional materials can be easily controlled by gate voltage due to Coulomb repulsion, the resistors could be formed either out of semiconducting two-dimensional electron gas [32] or out of graphene. Finally, the measurement of heat current noise can in practice be realized as in Ref. [33], by detecting the variations of the effective temperature of a nanocalorimeter using a fast tunnel junction thermometer.

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