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Reducing time headway in platoons under the MPF topology in the presence of communication delays

Elham Abolfazli * Bart Besselink ** Themistoklis Charalambous *

* Department of Electrical Engineering and Automation, School of Electrical Engineering, Aalto University, Espoo, Finland (e-mails: name.surname@aalto.fi)

** Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, Groningen, the Netherlands (e-mail: b.besselink@rug.nl)

Abstract: For platoons under the multiple-predecessor following (MPF) topology, communication delays can compromise both the internal stability and string stability. The most straightforward solution to guarantee stability is by increasing the time headway. On the other hand, time headway plays a significant role in road capacity and increasing its value is in contrast with the idea of platooning. In this study, internal stability and string stability of platoons suffering from communication delays are investigated and a lower bound for the time headway is proposed. Using this bound, platoons do not need to massively increase the time headway in order to compensate for the effects of communications delays. Finally, we evaluate the proposed lower bound on the time headway and the simulation results demonstrate its effectiveness.

Keywords: Platooning, multiple-predecessor following, communication delays, internal stability, string stability, time headway.

1. INTRODUCTION

In transportation, grouping some vehicles (cars or trucks) into a platoon, in which vehicles are close to each other, is a method to increase the capacity of roads. By using platooning, vehicles can brake almost simultaneously, which results in the improvement of traffic safety, even when the vehicles are packed tightly. In addition, vehicle platooning can help to reduce carbon dioxide emissions and fuel consumption, Alam et al. (2010); Tsugawa (2013).

Internal stability and string stability are the essential control objectives for a vehicle platoon. Internal stability refers to the individual stability defined for each vehicle and describes the ability to converge to given desired trajectories, Feng et al. (2019). Regarding string stability, although many definitions have been proposed in the literature, such as Besselink and Johansson (2017); Ploeg et al. (2014); Swaroop and Hedrick (1996), all entail the unique fact that disturbances must not amplify along the traffic flow.

It is known that the spacing policy, which defines the distance between two consecutive vehicles, can influence the ability of a platoon to attenuate the effects of disturbances and reach string stability. Two significant classes for spacing policy exist: Constant Spacing Policy (CSP) and Constant Time Headway Spacing policy (CTHP). In CSP, the desired inter-vehicle distance is constant, while in CTHP, a linear function of speed, with the proportional gain, named *Time headway* (h), dictates the desired inter-vehicle distance. A smaller time headway can increase the road throughput, but it could be dangerous for the platoon, since it could lead to internal and/or string instability and, hence, a collision. On the other hand, an increase in the time headway value leads to a larger inter-vehicle distance, which guarantees safety, but as a consequence, the capacity of the roads is decreased and the fuel consumption is increased; see for example, Nowakowski et al. (2016). In Xiao and Gao (2011), the minimum acceptable value for the time headway, in the presence of parasitic delays and lags is presented. Hence, by selecting a small time headway, which meets the requirements for string stability, a higher road throughput and fuel efficiency will be achieved.

Recently, with technological advancements, vehicles can receive information and be connected with multiple vehicles in the platoon by using vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications. The advantages of multiple connected vehicles have been widely studied in the literature. In Darbha et al. (2019), a platoon with a CTHP and time lag is considered, in which each vehicle can obtain information from multiple predecessor vehicles and a lower bound for the time headway, dependent on the number of connected vehicles, has been provided. This bound demonstrates that using V2V communications and increasing the number of connected vehicles, a higher road capacity can be achieved. In Bian et al. (2019), a new definition of desired inter-vehicle distance in a platoon

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with CTHP and time lag under MPF topology is proposed which avoids inconsistencies in the inter-vehicle distance. Under this definition, a lower bound that guarantees internal stability and string stability is provided, which implies that more connected cars can lead to a smaller time headway.

Despite the advantages of V2V and V2I communications, wireless communications inevitably introduce time delays in vehicle platoons and can be a big challenge for control design, since both the internal and string stability are compromised in the presence of communication delays. The effect of delays on platoons has been extensively studied in the literature. For example, for the kinematic model, it is shown in Xiangheng Liu et al. (2001) that the delays coming from communication links can be a cause of string instability. Also, in Zhang and Orosz (2016), the effects of information delays on the longitudinal dynamics of connected vehicles have been investigated. In Dileep et al. (2019), CTHP is considered and a controller that guarantees string stability for a platoon which has different types of delays, actuation, communication and sensor delay, and each vehicle is connected to one predecessor, is proposed. Moreover, in Salvi et al. (2017), a controller which guarantees asymptotic and exponential stability of the platoons with CTHP and suffering from time-varying delays, is proposed and it is shown that in the case of fixed delays, the system will be string stable, as well. For the dynamic model, in Xu et al. (2018), a controller for a platoon under the predecessor-leader following (PLF) topology and CSP is proposed which guarantees disturbance string stability while having sensor and communication delay.

However, none of the aforementioned works considered the effects of time delay on the minimum time headway. The first work that considered this problem in the scenario of one vehicle look-ahead is Xiao and Gao (2011). However, the results in Xiao and Gao (2011) is not an optimal value for the platoons with MPF topology, because it doesn't include the effects of connecting multiple vehicles. On the other hand, the lower bound proposed for platoons under MPF topology, such as Bian et al. (2019), doesn't include the effects of having communication delays.

For overcoming the problem of string instability in a platoon, where every vehicle is connected to *multiple* preceding vehicles via communication links suffering time delays, a lower bound on time headway is proposed in this paper for the case in which the communication delays are homogeneous. It has been shown that by selecting a larger time headway value than the lower bound, even in the presence of delayed information about the states of other vehicles, the disturbances acting on the lead vehicle do not propagate along the platoon and the knowledge of this lower bound allows the platoon to utilize its full potential, implying that the road capacity is maximized within the possibilities given by the platoon control system. Also, the proposed lower bound shows that by increasing the number of predecessors, a smaller time headway is achieved, which is in accordance to the results in Bian et al. (2019). In addition to string stability, the internal stability of the platoon with MPF topology, in the presence of delays, is considered by means of Nyquist stability theory.

The remainder of this paper is structured as follows. Section 2 presents some notations and mathematical preliminaries. Section 3 gives the vehicle model and control structure. In Section 4, the control law for a platoon with homogeneous time delays is proposed and the objectives of the paper are formulated. Section 5 analyzes the internal stability and string stability of the system and proposes a lower bound for the time headway. Section 6 shows the simulation results verifying the suggested time headway. Finally, in Section 7, we draw conclusions and discuss future directions.

2. NOTATION AND MATHEMATICAL PRELIMINARIES

2.1 Notation

Vectors and matrices are denoted by lowercase and uppercase letters, respectively. Integer and natural numbers sets are denoted by $\mathbb{Z}$ and $\mathbb{N}$, respectively. $\mathbb{Z}_0 \triangleq \{0,1,2,\ldots\}$, $\mathbb{Z}_0^n \triangleq \{0,1,2,\ldots,n\}$, and $\mathbb{N}^n \triangleq \{1,2,\ldots,n\}$. Real and nonnegative real numbers sets are denoted by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. $m \times n$ real matrices are denoted by $\mathbb{R}^{m\times n}$. For any matrix $A \in \mathbb{R}^{m\times n}, (m,n) \in \mathbb{N}\times\mathbb{N}$, we denote its transpose by A^T and its entries by a_{ij}, $i \in \mathbb{N}^m$, $j \in \mathbb{N}^n$ (i.e., $A = [a_{ij}]$). The $n \times n$ identity matrix is represented by I_n.

2.2 Mathematical Preliminaries

Definition 1. We consider the vehicle platoon as a directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, ..., v_N\}$ is a set of nodes representing all the following vehicles and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is a set of edges representing the connections between each pair of following vehicles. The Laplacian matrix associated with $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]$, $i,j \in \mathbb{N}^N$, with

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{k=1}^{N} a_{ik}, & i = j, \end{cases} \tag{1}$$

where $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. Also, we assume a uni-directional communication structure, i.e., vehicles are able to receive information only from their predecessors, and hence $a_{ij} = 0$ if $j > i$. Moreover, the connections between the vehicles and the leader can be modeled by

$$\mathcal{P} = \mathrm{diag}\{p_{11}, p_{22}, \ldots, p_{NN}\}, \tag{2}$$

where $p_{ii} = 1$ when vehicle i is connected to the leader and $p_{ii} = 0$, otherwise. Then, a new information topology matrix can be defined as

$$\mathcal{L}_p := \mathcal{L} + \mathcal{P}. \tag{3}$$

It is easy to see that $\mathcal{L}_p$ is a lower triangular matrix.

3. VEHICLE MODEL AND CONTROL STRUCTURE

3.1 Vehicle Model

Consider a platoon of N vehicles with the following longitudinal model, as in, e.g., Ploeg et al. (2014)

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = a_i(t), \\ \tau_i \dot{a}_i(t) + a_i(t) = u_i(t), \end{cases} \tag{4}$$

where $p_i(t)$, $v_i(t)$, $a_i(t)$ and $u_i(t)$ are the position, velocity, acceleration and control input of ith vehicle, respectively. The time lag in the powertrain $\tau_i > 0$ is usually unknown but may be bounded, i.e., $\tau_i \leq \tau^*$, where $\tau^* \in \mathbb{R}_+$.

It is assumed that vehicle i can use information from multiple predecessor vehicles, as shown in Fig. 1, where vehicle i, $i \in \mathbb{Z}_3^N$, is connected to three predecessor vehicles. For this topology, the desired distance between vehicle i and the lth vehicle ahead of it is considered as Bian et al. (2019)

$$d_{i,i-l}(t) = \sum_{k=i-l+1}^{i} \left(h_k v_k(t) + d_k\right), \tag{5}$$

where $h_k \geq 0$ is the time headway of vehicle k and $d_k > 0$ is the desired standstill gap between vehicle k and $k-1$.

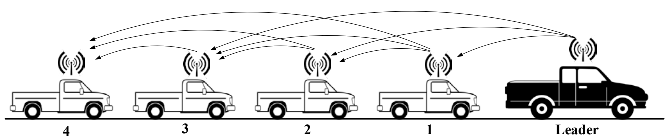


Fig. 1. A platoon under MPF topology.

3.2 Control Structure

The following linear feedback controller is used in Bian et al. (2019), for vehicle i when there are no time-delays:

$$u_i(t) = -\sum_{l=1}^{r_i}\Bigg(k_{pi}\Big(p_i - p_{i-l} + \sum_{k=i-l+1}^{i}(h_k v_k + d_k)\Big) + k_{vi}(v_i - v_{i-l}) + k_{ai}(a_i - a_{i-l})\Bigg), \tag{6}$$

where $r_i \leq i$ is the number of the vehicles directly ahead of vehicle i that send their information to it. The control parameters $(k_{pi}, k_{vi}, k_{ai}) \geq 0$ are tunable gains for feeding back distance, velocity and acceleration errors between vehicle i and the lth vehicle ahead.

4. PROBLEM FORMULATION

It is assumed that the controller of vehicle i has access to the difference between its own states and all the predecessors through wireless communication, which suffers from a homogeneous time-delay Δ. Then, based on the controller (6) proposed in Bian et al. (2019), the following control law is proposed:

$$\begin{aligned} u_i(t) = -\sum_{l=1}^{r_i}\Bigg(&k_{pi}\Big(p_i(t-\Delta) - p_{i-l}(t-\Delta) \\ &+ \sum_{k=i-l+1}^{i}(h_k v_k(t-\Delta) + d_k)\Big) \\ &+ k_{vi}\big(v_i(t-\Delta) - v_{i-l}(t-\Delta)\big) \\ &+ k_{ai}\big(a_i(t-\Delta) - a_{i-l}(t-\Delta)\big)\Bigg). \end{aligned} \tag{7}$$

The main goal is to coordinate the motion of vehicles so that they track the desired inter-vehicle distance and keep the desired velocity while preventing amplification of disturbances throughout the platoon. The first objective will be analyzed in terms of internal stability and the second one in terms of string stability.

5. STABILITY ANALYSIS

First, we define the following errors

$$\begin{cases} \bar{p}_i(t) = p_i(t) - p_0(t) + \sum_{k=1}^{i}(h_k v_k(t) + d_k), \\ \bar{v}_i(t) = v_i(t) - v_0(t), \\ \bar{a}_i(t) = a_i(t) - a_0(t). \end{cases} \tag{8}$$

It is assumed that the lead vehicle moves at a constant speed, i.e., $u_0(t) = 0$ and $a_0(t) = 0$. From (4), the dynamics of the error variables can be written as

$$\begin{cases} \dot{\bar{p}}_i(t) = \bar{v}_i(t) + \sum_{k=1}^{i} h_k \bar{a}_k(t), \\ \dot{\bar{v}}_i(t) = \bar{a}_i(t), \\ \dot{\bar{a}}_i(t) = -\frac{1}{\tau_i}\bar{a}_i(t) + \frac{1}{\tau_i}u_i(t). \end{cases} \tag{9}$$

Using (8) and after some algebraic manipulations, the control law (7) becomes:

$$\begin{aligned} u_i(t) = -\sum_{l=1}^{r_i}\Bigg(&k_{pi}\Big(\bar{p}_i(t-\Delta) - \bar{p}_{i-l}(t-\Delta)\Big) \\ &+ k_{vi}\Big(\bar{v}_i(t-\Delta) - \bar{v}_{i-l}(t-\Delta)\Big) \\ &+ k_{ai}\Big(\bar{a}_i(t-\Delta) - \bar{a}_{i-l}(t-\Delta)\Big)\Bigg). \end{aligned} \tag{10}$$

By substituting (10) into (9) and defining augmented errors $\bar{p} = [\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_N]^\top$, $\bar{v} = [\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_N]^\top$ and $\bar{a} = [\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_N]^\top$, the dynamics model of the closed loop network can be recast as

$$\begin{aligned} &\dot{\xi}(t) = A\xi(t) + A_\Delta \xi(t-\Delta), \\ &\xi(t) = \Phi(t), \qquad t \in [-\Delta, 0], \end{aligned} \tag{11}$$

where $\xi = [\bar{p}^\top, \bar{v}^\top, \bar{a}^\top]^\top$ is the lumped state vector, $\Phi(\cdot) \in \mathcal{C}([-\Delta, 0], \mathbb{R}^\nu)$ represents the initial state of the system and A and $A_\Delta \in \mathbb{R}^{\nu\times\nu}$, $\nu = 3N$, are as follows

$$A = \begin{bmatrix} 0 & I_N & H \\ 0 & 0 & I_N \\ 0 & 0 & -\mathrm{T} \end{bmatrix}, \tag{12}$$

$$A_\Delta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\mathrm{T}K_p\mathcal{L}_p & -\mathrm{T}K_v\mathcal{L}_p & -\mathrm{T}K_a\mathcal{L}_p \end{bmatrix}, \tag{13}$$

with

$$K_m = \mathrm{diag}\{k_{m1}, \ldots k_{mN}\}, \qquad m \in \{p, v, a\}, \tag{14}$$

$$\mathrm{T} = \mathrm{diag}\{1/\tau_1, \ldots 1/\tau_N\}, \tag{15}$$

and

$$H = \begin{bmatrix} h_1 & 0 & \ldots & 0 \\ h_1 & h_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ h_1 & h_2 & \ldots & h_N \end{bmatrix}. \tag{16}$$

5.1 Internal Stability Analysis

In this section, we will find a sufficient condition in which, the vehicle platoon (11) will be asymptotically stable.

Theorem 1. By selecting the control gains (k_{pi}, k_{vi}, k_{ai}) in order that all the following conditions hold

$$k_{pi} \neq 0, \tag{17a}$$
$$k_{ai} - \tau_i(k_{vi} + k_{pi}h_i) + {\tau_i}^2 k_{pi} \neq 0, \tag{17b}$$
$$k_{vi} + k_{pi}(h - \tau) \geq 0, \tag{17c}$$

the closed loop system (11) is asymptotically stable for any time delay Δ that satisfies the following inequality

$$\Delta r_i(k_{vi} + k_{pi}h_i) < 1, \quad i \in \mathbb{N}. \tag{18}$$

Proof. See Appendix 1.

Remark 1. Note that the condition is sufficient and not necessary. As a result, there might be delays for which (18) is not satisfied, but the system is internally stable.

Remark 2. While the conditions seems to be counter-intuitive, having a large time headway h in a system with delays may lead to instability. A possible reason is that the error in the speed difference is amplified. Additionally, the higher the number of hops that the information is transferred, the smaller the delay that the network can tolerate.

5.2 Homogeneous String Stability Analysis

For the string stability analysis, we assume that the platoon is homogeneous, which means that $\tau_i = \tau > 0$, $r_i = r$, $h_i = h$, $k_{pi} = k_p$, $k_{vi} = k_v$ and $k_{ai} = k_a$, $\forall i \in \mathbb{Z}_0^n$. Then from (4) and (7) we have

$$\begin{aligned}\tau \dddot{p}_i(t) + \ddot{p}_i(t) = &-\sum_{l=1}^{r}\Bigg(k_p\Big(p_i(t-\Delta) - p_{i-l}(t-\Delta)\\&+\sum_{k=i-l+1}^{i}(hv_k(t-\Delta)+d_k)\Big) + k_v\Big(v_i(t-\Delta)\\&-v_{i-l}(t-\Delta)\Big) + k_a\Big(a_i(t-\Delta) - a_{i-l}(t-\Delta)\Big)\Bigg),\end{aligned} \tag{19}$$

and

$$\begin{aligned}\tau \dddot{p}_{i-1}(t) + \ddot{p}_{i-1}(t) = &-\sum_{l=1}^{r}\Bigg(k_p\Big(p_{i-1}(t-\Delta)\\&-p_{i-1-l}(t-\Delta) + \sum_{k=i-l}^{i-1}(hv_k(t-\Delta)+d_k)\Big)\\&+k_v\Big(v_{i-1}(t-\Delta) - v_{i-1-l}(t-\Delta)\Big)\\&+k_a\Big(a_{i-1}(t-\Delta) - a_{i-1-l}(t-\Delta)\Big)\Bigg).\end{aligned} \tag{20}$$

The time derivative of (19) is

$$\begin{aligned}\tau \dddot{v}_i(t) + \ddot{v}_i(t) = &-\sum_{l=1}^{r}\Bigg(k_p\Big(v_i(t-\Delta) - v_{i-l}(t-\Delta)\\&+\sum_{k=i-l+1}^{i} ha_k(t-\Delta)\Big) + k_v\Big(a_i(t-\Delta) - a_{i-l}(t-\Delta)\Big)\\&+k_a\Big(\dot{a}_i(t-\Delta) - \dot{a}_{i-l}(t-\Delta)\Big)\Bigg).\end{aligned} \tag{21}$$

After defining the spacing error $e_i = p_i - p_{i-1} + hv_i + d_i$, calculating (19) − (20) + h × (21) and doing some algebraic manipulations, by taking Laplace transform, we obtain

$$E_i(s) = \sum_{l=1}^{r} H_l(s) E_{i-l}(s), \tag{22}$$

where $E_i(s)$ is the Laplace transformation of $e_i(t)$ and

$$H_l(s) = \frac{k_a s^2 e^{-\Delta s} + (k_v - k_p h(r-l))\, s e^{-\Delta s} + k_p e^{-\Delta s}}{\tau s^3 + s^2 + r k_a s^2 e^{-\Delta s} + r\,(k_v + k_p h)\, s e^{-\Delta s} + r k_p e^{-\Delta s}}. \tag{23}$$

It can be shown that for homogeneous platoons, $e_i/e_{i-l} = v_i/v_{i-l}$. In a platoon under the MPF topology, the spacing error of the vehicles are affected by their multiple predecessors. Therefore, in order to have string stability, in addition to $H_1(s)$, all the string stability functions $H_l(s)$ for $l \leq r$ must be examined. Since (23) is identical for all vehicles, string stability of the platoon can be guaranteed if, Bian et al. (2019)

$$\|H_l(j\omega)\|_\infty \leq \frac{1}{r}, \quad \forall 1 \leq l \leq r, \tag{24}$$

where $H_l(j\omega)$ can be derived from (23) by substituting $s = j\omega$.

Theorem 2. Consider system (4) with input (7) that is internally stable. Then, the string stability specification (24) holds if all the following conditions are satisfied:

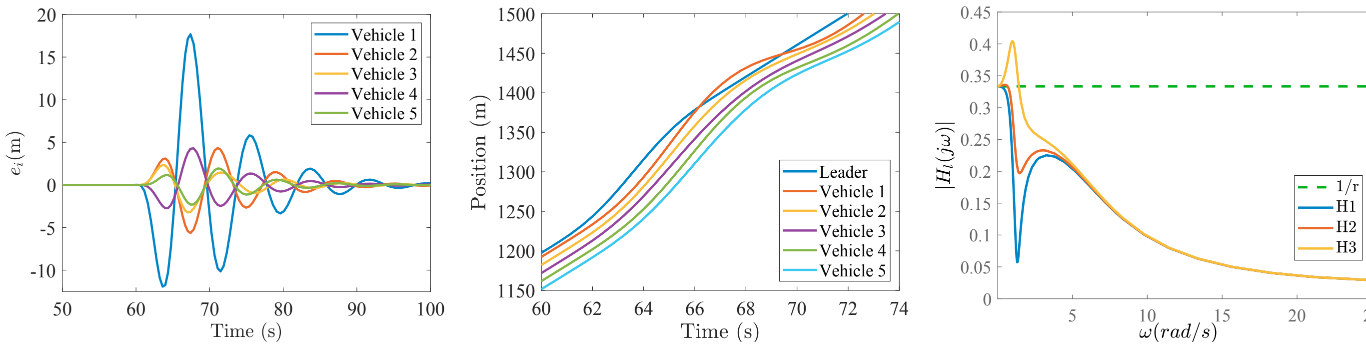
$$k_v + k_p(h - \tau) \geq 0, \tag{25a}$$
$$2\tau\Delta - \Delta h - \tau h \leq 0, \tag{25b}$$
$$k_a - \tau(k_v + k_p h) \leq 0, \tag{25c}$$
$$\tau - 2rk_a\Delta \geq 0, \tag{25d}$$
$$1 + 2r\Big(k_a - \tau(k_v + k_p h)\Big) + 2r\Delta\Big(k_p(\tau - h) - k_v\Big) \geq 0, \tag{25e}$$
$$\begin{aligned}&r^2k_p^2h^2(1-(r-l)^2) + 2r^2k_pk_vh(1+r-l)\\&-2rk_p \geq 0, \quad 1 \leq l \leq r.\end{aligned} \tag{25f}$$

For the region defined by (25), string stability specification (24) holds if

$$h \geq h_{\min} = \frac{2(\tau + \Delta)}{2rk_a + 1}. \tag{26}$$

Proof. See Appendix 2.

Remark 3. Theorem 2 proves the conjecture made in Bian et al. (2019). As it is shown, the larger the delay Δ, the larger the minimum time headway h. This comes into contrast with the internal stability condition obtained in Theorem 1. Nevertheless, one can tune the triplet of control gains (k_p, k_v, k_a), such that both conditions can be satisfied.



(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 2. Platoon response, in the presence of sinusoidal disturbance acting on the leader, with $h < h_{\min}$ and $r = 3$.

6. NUMERICAL RESULTS

In this section, numerical simulations are presented to verify the results in our theorems. The controller (7) is considered for a five-vehicle platoon. The model used for each car is the nominal linear model (4) and the number of predecessors is $r = 3$. Vehicle i starts at the point $-id$ and moves to reach the desired distance as well as the desired velocity, which is 20 m/s, same as the leader's velocity. After 60 seconds, when the platoon reaches to a stable state, a sinusoidal perturbation $u_0(t) = A_0 \sin(\omega_0 t)$ acts on the leader for the duration of one cycle ($\frac{2\pi}{\omega_0} s$). The numerical values for system parameters are given in Table 1. By selecting control parameters (k_p, k_v, k_a) as (0.7, 0.5, 0.4), conditions (25) are satisfied and the minimum value for time headway will be $h_{\min} = 0.41$ s. Also, internal stability specification (18) holds. Then, to verify the proposed criterion for the time headway, we simulate the platoon in two cases, i.e., when $h < h_{min}$ and also when $h > h_{\min}$.

Table 1. Model Parameters

N	r	d	τ	Δ	v_0	A_0	ω_0
5	3	5 m	0.5 s	0.2 s	20 m/s	10 m/s^2	1 rad/s

Fig. 2 demonstrates the platoon response after adding the disturbance in the case of $h < h_{\min}$. In Fig. 2(a), after the disturbance, large overshoots for spacing error between vehicles can be seen, which is a sign of string instability. Fig. 2(b) is a more accurate analysis of the platoon, that shows the position of each vehicle. We observe that after the disturbance, there would be a collision between the leader and vehicle 1 and also, the space between other vehicles decreases in an unsafe way. Also, the magnitude-frequency diagram of $H_l(j\omega)$ is shown in Fig. 2(c), where it is shown that the magnitudes surpass $1/r$ and thus, (24) doesn't hold. According to the simulation results, by considering $h < h_{\min}$, the platoon will be string unstable.

Fig. 3 depicts the platoon response for $h > h_{\min}$. It can be seen from Fig. 3(a), the height of the peaks in spacing errors after adding the disturbance is lower than the previous case. Besides, Fig. 3(b) shows that although the space between the leader and vehicle 1 decreases, there isn't any collision between them. Finally, the magnitude-frequency diagram of $H_l(j\omega)$ when $h > h_{\min}$ is shown in Fig. 3(c). As can be seen in the figure, $|H_l(j\omega)|$ does not surpass the maximum acceptable value for string stability, i.e., $1/r$, which means (24) holds, and hence the platoon is string stable.

7. CONCLUSION AND FUTURE DIRECTIONS

7.1 Conclusions

In this paper, we have investigated the internal stability as well as the string stability conditions for platoons under MPF topology, which are affected by communication delays. More specifically, we provide a sufficient condition that guarantees internal stability and, additionally, based on string stability, we have formulated a lower bound for the time headway. The simulation results corroborate the importance of this lower bound.

7.2 Future directions

Part of ongoing work studies the effect of packet drops in the (internal and string) stability of platoons. Part of future work includes the analysis of platoons under MPF topology with bidirectional connectivity. We plan to investigate the effect of receiving information from vehicles in the back on time headway. Additionally, we plan to find conditions that guarantee string stability in platoons with time-varying heterogeneous delays.

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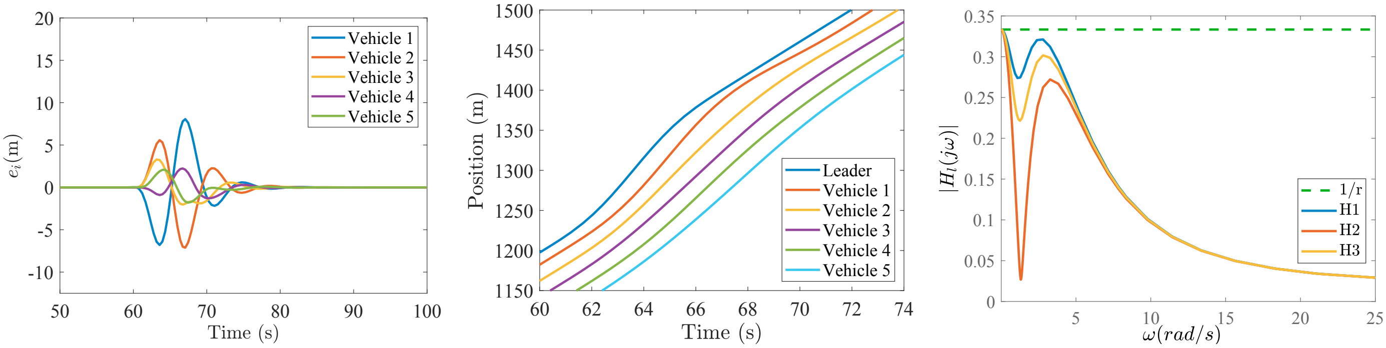

(a) Spacing Error (b) Position of vehicles (c) String stability transfer functions

Fig. 3. Platoon response, in the presence of sinusoidal disturbance acting on the leader, with $h > h_{\min}$ and $r = 3$.

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Appendix A. PROOF OF THEOREM 1

Taking Laplace transform of (11), we have

$$\Xi(s) = (sI - A - A_\Delta e^{-\Delta s})^{-1}\xi(0). \tag{A.1}$$

By finding $\det(sI - A - A_\Delta e^{-\Delta s})$, the characteristic equation of (11) will be obtained as (A.2).

After decoupling (A.2) to N subsystems, we define

$$\begin{aligned} p_i'(s) =& s^3 + \frac{1}{\tau_i}s^2 + s^2\frac{1}{\tau_i}(k_{ai}r_i e^{-\Delta s}) \\ &+ s\frac{1}{\tau_i}r_i\left(k_{vi} + k_{pi}h_i\right)e^{-\Delta s} + \frac{1}{\tau_i}r_i k_{pi}e^{-\Delta s}. \end{aligned} \tag{A.3}$$

Note that for $\Delta = 0$, (A.3) can be written as

$$p_i'(s) = s^3 + \frac{1 + k_{ai}r_i}{\tau_i}s^2 + \frac{r_i\left(k_{vi} + k_{pi}h_i\right)}{\tau_i}s + \frac{r_i k_{pi}}{\tau_i}. \tag{A.4}$$

It is easy to see that if $k_{pi} \neq 0$, $s = 0$ cannot be a solution to (A.4). Thus, (A.4) can be written as

$$1 + \frac{1 + k_{ai}r_i}{\tau_i}\frac{1}{s} + \frac{r_i\left(k_{vi} + k_{pi}h_i\right)}{\tau_i}\frac{1}{s^2} + \frac{r_i k_{pi}}{\tau_i}\frac{1}{s^3} = 0, \tag{A.5}$$

and by invoking the Nyquist stability criterion, one can find the same conditions for internal stability as the one given in Bian et al. (2019) via Routh-Hurwitz stability criterion (omitted due to space limitations).

If the roots of (A.3) lie in the complex left-half plane, then the system is stable. It is easy to see that $s = 0$ and $s = -1/\tau_i$ cannot be the solution to (A.3), if

$$k_{pi} \neq 0, \tag{A.6a}$$

$$k_{ai} - \tau_i(k_{vi} + k_{pi}h_i) + {\tau_i}^2 k_{pi} \neq 0. \tag{A.6b}$$

Thus, after dividing (A.3) by $s^3 + \frac{1}{\tau_i}s^2$, the characteristic equation can be rewritten as

$$1 + \frac{be^{-\Delta s}}{s + a} + \frac{ce^{-\Delta s}}{s(s + a)} + \frac{de^{-\Delta s}}{s^2(s + a)} = 0, \tag{A.7}$$

where

$$a = \frac{1}{\tau_i},\ b = \frac{k_{ai} r_i}{\tau_i},\ c = \frac{r_i\left(k_{vi} + k_{pi} h_i\right)}{\tau_i},\ d = \frac{r_i k_{pi}}{\tau_i}. \quad \text{(A.8)}$$

Define the open loop transfer function, denoted by $G(s)$,

$$G(s) = \frac{be^{-\Delta s}}{s+a} + \frac{ce^{-\Delta s}}{s(s+a)} + \frac{de^{-\Delta s}}{s^2(s+a)} = \frac{bs^2 + cs + d}{s^2(s+a)} e^{-\Delta s}. \quad \text{(A.9)}$$

After replacing s by $j\omega$ in (A.9), we obtain

$$G(j\omega) = \frac{d - b\omega^2 + jc\omega}{-\omega^2(j\omega + a)} e^{-j\omega\Delta}. \quad \text{(A.10)}$$

From (A.10), the phase of $G(j\omega)$ will be

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\left(\frac{\omega}{a}\right) + \tan^{-1}\left(\frac{c\omega}{d - b\omega^2}\right) - \omega\Delta. \quad \text{(A.11)}$$

If $\angle G(j\omega) > -180^\circ$ as $\omega \to 0^+$, then it can be shown that the Nyquist plot of $G(j\omega)$ does not encircle the point $-1 + j0$ as $\omega \to 0$. Hence, we investigate what happens as $\omega \to 0^+$. From (A.11), we know that $\angle G(j0^+) > -180^\circ$ if

$$\tan^{-1}\left(\frac{c\omega}{d - b\omega^2}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) - \omega\Delta > 0. \quad \text{(A.12)}$$

Then, if

$$\tan^{-1}\left(\frac{c\omega}{d - b\omega^2}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) > 0 \quad \text{(A.13)}$$

there exist a time delay Δ such that (A.12) holds. If

$$\frac{c\omega}{d - b\omega^2} - \frac{\omega}{a} = \frac{\omega(ac - d + b\omega^2)}{(d - b\omega^2)} > 0, \quad \text{(A.14)}$$

then, inequality (A.13) holds. As $\omega \to 0^+$, then $\omega > 0$, $d - b\omega^2 > 0$ and by assuming

$$k_{vi} + k_{pi}(h - \tau) \geq 0, \quad \text{(A.15)}$$

then $ac - d > 0$. As a result, inequality (A.14) holds and, thus, there will be no encirclement around point $-1+j0$ for the Nyquist plot. In Fig. A.1, a Nyquist plot is provided for some given parameters (same as the numerical example), depicting that as long as (A.6) and (A.15) hold, there is no encirclement of the point $-1 + j0$ as $\omega \to 0$.

Then, we should investigate if there is any encirclement as $\omega \neq 0$. After algebraic manipulation, (A.10) becomes

$$G(j\omega) = -\frac{\left(c\omega - a\omega(b - \frac{d}{\omega^2})\right) + j\left(ac + \omega^2(b - \frac{d}{\omega^2})\right)}{\omega(a^2 + \omega^2)} e^{-j\Delta\omega}.$$

By substituting $e^{-j\Delta\omega} = \cos(\Delta\omega) - j\sin(\Delta\omega)$ and then defining

$$Y(\omega) \triangleq c\omega - a\omega\left(b - \frac{d}{\omega^2}\right),\ Z(\omega) \triangleq ac + \omega^2\left(b - \frac{d}{\omega^2}\right), \quad \text{(A.16)}$$

the imaginary part of $G(j\omega)$ is as follows

$$\mathrm{Im}\{G(j\omega)\} = -\frac{Z(\omega)\cos(\Delta\omega) - Y(\omega)\sin(\Delta\omega)}{\omega(a^2 + \omega^2)}. \quad \text{(A.17)}$$

Suppose that $G(j\omega)$ crosses the real line when $\omega = \omega_0$. From (A.17), we have

$$\tan(\Delta\omega_0) = \frac{Z(\omega_0)}{Y(\omega_0)}. \quad \text{(A.18)}$$

Now, we just need to find the conditions for which $\mathrm{Re}\{G(j\omega_0)\} > -1$. For the sake of simplicity, we write Y and Z for $Y(\omega_0)$ and $Z(\omega_0)$, respectively. We have

$$\begin{aligned}\mathrm{Re}\{G(j\omega_0)\} &= -\frac{Y\cos(\Delta\omega_0) + Z\sin(\Delta\omega_0)}{\omega_0(a^2 + \omega_0{}^2)} \\ &= -\frac{Y\frac{\cos(\Delta\omega_0)}{\sin(\Delta\omega_0)}\sin(\Delta\omega_0) + Z\sin(\Delta\omega_0)}{\omega_0(a^2 + \omega_0{}^2)} \\ &= -\frac{(Y^2 + Z^2)\sin(\Delta\omega_0)}{(a^2 + \omega_0{}^2)Z\omega_0}. \end{aligned} \quad \text{(A.19)}$$

Considering (A.15), we have $ac - d > 0$ and accordingly $Z > 0$. Then, using the fact that $\sin(\Delta\omega) \leq \Delta\omega$ for $\omega \geq 0$, we can continue analyzing $\mathrm{Re}\{G(j\omega_0)\}$ as follows

$$\mathrm{Re}\{G(j\omega_0)\} \geq -\frac{\Delta(Y^2 + Z^2)}{(a^2 + \omega_0{}^2)Z}. \quad \text{(A.20)}$$

Defining

$$m \triangleq \frac{\Delta(Y^2 + Z^2)}{(a^2 + \omega_0{}^2)Z}, \quad \text{(A.21)}$$

a sufficient condition for stability is having $m < 1$. By substituting from (A.16) into (A.21) and simplifying, we obtain

$$m = \Delta\left(\frac{\omega_0^2(b - \frac{d}{\omega_0{}^2})^2 + c^2}{ac + \omega_0{}^2(b - \frac{d}{\omega_0{}^2})}\right). \quad \text{(A.22)}$$

Let $X \triangleq b - \frac{d}{\omega_0{}^2}$. Then from (A.22) we have that

$$\frac{\Delta(c^2 + \omega_0^2 X^2)}{ac + \omega_0^2 X} < 1. \quad \text{(A.23)}$$

Since $ac - d > 0$, we can easily deduce that $ac + \omega_0^2 X > 0$, and inequality (A.23) can be written as

$$\Delta(c^2 + \omega_0{}^2 X^2) < ac + \omega_0{}^2 X. \quad \text{(A.24)}$$

By multiplying both sides of (A.24) by Δ, and rearranging we obtain

$$(\Delta X)^2 - (\Delta X) + \frac{(\Delta c)^2 - a(\Delta c)}{\omega_0{}^2} < 0. \quad \text{(A.25)}$$

Since (A.25) is a quadratic inequality, for having $(\Delta X) \in \mathbb{R}$, we need a positive discriminant, i.e.,

$$1 - 4\frac{(\Delta c)^2 - a(\Delta c)}{\omega_0{}^2} > 0, \quad \text{(A.26)}$$

which can be rewritten as

$$(\Delta c)^2 - a(\Delta c) - \frac{\omega_0{}^2}{4} < 0. \quad \text{(A.27)}$$

Obviously, if (Δc) lies between the roots of (A.27), then (A.27) holds, i.e.,

$$\frac{a - \sqrt{a^2 + \omega_0{}^2}}{2} < \Delta c < \frac{a + \sqrt{a^2 + \omega_0{}^2}}{2}. \quad \text{(A.28)}$$

$$\begin{aligned}\left|sI_{3N} - A - A_\Delta e^{-\Delta s}\right| &= \begin{vmatrix} sI_N & -I_N & -H \\ 0 & sI_N & -I_N \\ \mathrm{T}K_p\mathcal{L}_p e^{-\Delta s} & \mathrm{T}K_v\mathcal{L}_p e^{-\Delta s} & sI_N + \mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s} \end{vmatrix} \\ &= \left|s^3 I + s^2(\mathrm{T} + \mathrm{T}K_a\mathcal{L}_p e^{-\Delta s}) + s(\mathrm{T}K_v\mathcal{L}_p + \mathrm{T}K_p\mathcal{L}_p H)e^{-\Delta s} + \mathrm{T}K_p\mathcal{L}_p e^{-\Delta s}\right| \\ &= \prod_{i=1}^{N}\left(s^3 + s^2\frac{1}{\tau_i}\left(1 + k_{ai}r_i e^{-\Delta s}\right) + s\frac{1}{\tau_i}r_i\left(k_{vi} + k_{pi}h_i\right)e^{-\Delta s} + \frac{1}{\tau_i}r_i k_{pi} e^{-\Delta s}\right). \end{aligned} \quad \text{(A.2)}$$

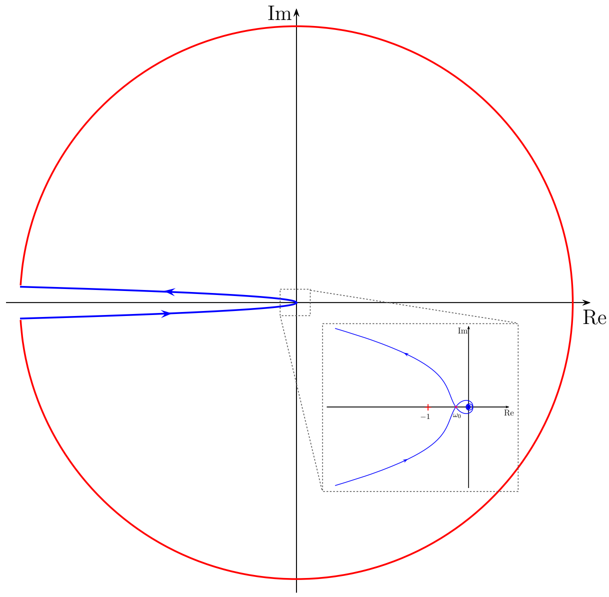

Fig. A.1. Nyquist plot for some given parameters.

The left term of (A.28) is negative, while we know that Δ and c are positive. Then we just need to have

$$\Delta c < \frac{a + \sqrt{a^2 + {\omega_0}^2}}{2}. \tag{A.29}$$

Since we do not know ω_0 (which changes for different Δ), we provide a conservative bound, that is, since

$$\frac{a + \sqrt{a^2 + {\omega_0}^2}}{2} > a,$$

then if $\Delta c < a$, then inequality (A.29) also holds. Therefore, if

$$\Delta \frac{c}{a} < 1, \tag{A.30}$$

then, the Nyquist plot of $G(j\omega)$ does not encircle the point $-1 + j0$ when $\omega \neq 0$. Therefore, there is no encirclement for all frequencies and the system is stable. By substituting from (A.8) into (A.30), condition (18) is obtained.

Appendix B. PROOF OF THEOREM 2

The proof of Theorem 2 follows *mutatis mutandis* that of Xiao and Gao (2011).

Inequality (24) is equivalent to

$$\max_{1\le l\le r} \|H_l(j\omega)\|_\infty^2 = \max_{1\le l\le r} \sup_{\omega\ge 0} |H_l(j\omega)|^2 \le \frac{1}{r^2}. \tag{B.1}$$

Define

$$|H_l(j\omega)|^2 \triangleq \frac{N_l}{D_l}, \tag{B.2}$$

where

$$N_l = (k_p - k_a\omega^2)^2 + \big(k_v - k_p h(r-l)\big)^2\omega^2, \tag{B.3a}$$

$$\begin{aligned} D_l =& \big(-\omega^2 - rk_a\omega^2\cos(\Delta\omega) \\ &+ r(k_v + k_p h)\omega\sin(\Delta\omega) + rk_p\cos(\Delta\omega)\big)^2 \\ &+\big(-\tau\omega^3 + rk_a\omega^2\sin(\Delta\omega) \\ &+ r(k_v + k_p h)\omega\cos(\Delta\omega) - rk_p\sin(\Delta\omega)\big)^2. \end{aligned} \tag{B.3b}$$

After some simplifications, we obtain that the inequality in (B.1) holds for $l \in \{1, r\}$ if

$$D_l - r^2 N_l = M_6\omega^6 + M_5\omega^5 + M_4\omega^4 + M_3\omega^3 + M_2\omega^2 \ge 0, \tag{B.4}$$

where

$$M_6 = \tau^2, \tag{B.5a}$$

$$M_5 = -2\tau r k_a \sin(\Delta\omega), \tag{B.5b}$$

$$M_4 = 1 + 2r\Big(k_a - \tau(k_v + k_p h)\Big)\cos(\Delta\omega), \tag{B.5c}$$

$$M_3 = 2r\Big(k_p(\tau - h) - k_v\Big)\sin(\Delta\omega), \tag{B.5d}$$

$$\begin{aligned} M_2 =& r^2k_p^2h^2(1-(r-l)^2) + 2r^2k_pk_vh(1+r-l) \\ &- 2rk_p\cos(\Delta\omega). \end{aligned} \tag{B.5e}$$

Considering the fact that $\sin(\Delta\omega) \le \Delta\omega$ for $\omega \ge 0$ and $\cos(\Delta\omega) \le 1$ and also considering

$$k_p(\tau - h) - k_v \le 0, \tag{B.6}$$

if

$$k_a - \tau(k_v + k_p h) \le 0, \tag{B.7}$$

then we need to have

$$D_l - r^2N_l \ge \omega^2(\overline{M}_4\omega^4 + \overline{M}_2\omega^2 + \overline{M}_0), \tag{B.8}$$

where

$$\overline{M}_4 = \tau^2 - 2\tau r k_a\Delta, \tag{B.9a}$$

$$\overline{M}_2 = 1 + 2r\Big(k_a - \tau(k_v + k_p h)\Big) + 2r\Delta\Big(k_p(\tau - h) - k_v\Big), \tag{B.9b}$$

$$\overline{M}_0 = r^2k_p^2h^2(1-(r-l)^2) + 2r^2k_pk_vh(1+r-l) - 2rk_p. \tag{B.9c}$$

Having $\overline{M}_4, \overline{M}_2, \overline{M}_0 \ge 0$ results in string stability. Hence, there are three cases as follows

$$\overline{M}_4 \ge 0 \Longleftrightarrow k_a \le \frac{\tau}{2r\Delta} \tag{B.10a}$$

$$\overline{M}_2 \ge 0 \Longleftrightarrow k_v \le \frac{1 + 2rk_a + 2rk_p(\tau\Delta - \Delta h - \tau h)}{2r(\tau + \Delta)} \tag{B.10b}$$

$$\overline{M}_0 \ge 0 \Longleftrightarrow k_v \ge \frac{1}{rh} - \frac{k_p h}{2}, \quad \text{if } \ l = r. \tag{B.10c}$$

By combining (B.10b) and (B.10c), we obtain

$$\frac{1}{rh} - \frac{k_p h}{2} \le k_v \le \frac{1 + 2rk_a + 2rk_p(\tau\Delta - \Delta h - \tau h)}{2r(\tau + \Delta)}. \tag{B.11}$$

The above inequality implies that

$$\frac{1 + 2rk_a + 2rk_p(\tau\Delta - \Delta h - \tau h)}{2r(\tau + \Delta)} - \left(\frac{1}{rh} - \frac{k_p h}{2}\right) \ge 0. \tag{B.12}$$

After some simplifications, we obtain

$$\frac{h(2rk_a + 1) - 2(\tau + \Delta) + rhk_p(2\tau\Delta - \Delta h - \tau h)}{2rh(\tau + \Delta)} \ge 0. \tag{B.13}$$

Then, by assuming $2\tau\Delta - \Delta h - \tau h \le 0$ and knowing that the control gains are positive, obviously we have

$$h(2rk_a + 1) - 2(\tau + \Delta) \ge 0. \tag{B.14}$$

Therefore, the lower bound for the time headway can be obtained as

$$h \ge h_{\min} = \frac{2(\tau + \Delta)}{2rk_a + 1}. \tag{B.15}$$