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Published in:
2021 15th International Congress on Artificial Materials for Novel Wave Phenomena, Metamaterials 2021

DOI:
10.1109/Metamaterials52332.2021.9577064

Published: 20/09/2021

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Ptitcyn, G., Lamprianidis, A., Karamanos, T., Muller, M., Alaee, R., Asadchy, V. S., Mirmoosa, M. S., Albooyeh, M., Fan, S., Rockstuhl, C., & Tretyakov, S. A. (2021). Scattering of light by spheres made from a time-modulated and dispersive material. In *2021 15th International Congress on Artificial Materials for Novel Wave Phenomena, Metamaterials 2021* (pp. 347-349). IEEE. https://doi.org/10.1109/Metamaterials52332.2021.9577064

15^{th} *International Congress on Artificial Materials for Novel Wave Phenomena - Metamaterials 2021*
New York, USA, Aug. 2^{nd} - 7^{th} 2021

Scattering of light by spheres made from a time-modulated and dispersive material

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Abstract – **We derive the dispersion relation of eigenmodes propagating in a time-varying and dispersive medium. We use these eigenmodes to analytically study the scattering of light by a sphere made from a time-varying and dispersive medium. These results are compared to full-wave optical simulations and excellent agreement is observed. With that, we provide tools and outline a path towards further explorations of light scattering by time-varying finite particles.**

I. Introduction

Maxwell's equations govern interaction of light with matter. While known for more than 150 years, they continue to give rise to innovations when considering novel materials. Dispersive and anisotropic materials are nowadays subject of any course on electrodynamics. In recent times, slightly more sophisticated properties such as chirality or non-locality attracted increasing interest. The most recent addition constitutes the consideration of materials with time-modulated properties. This unlocks an additional degree of freedom in electromagnetic systems (e.g., [1]) that enables novel approaches to exceed conventional limitations [2] and design more efficient systems that realize conventional functionalities [3, 4]. However, the majority of prior contributions considered materials with no frequency dispersion; being synonymous to assuming instantaneous response. This assumption can be used only under severe limitations. Strictly speaking, this assumption is unphysical. Moreover, the theory remains valid only at a single frequency. In an extended spectral region it is applicable only in a frequency domain where the material has negligible dispersion. However, it is challenging to modulate properties of such materials. In contrast, material candidates which can be efficiently modulated in time, such as a plasma and graphene, are strongly dispersive. To our knowledge, only a few papers studied time-modulated *and* dispersive materials [5, 6, 7]. However, none of these studies numerically validated the theoretical results.

In this contribution, we develop a comprehensive theoretical description for the scattering of light by spheres of arbitrary size made from a general dispersive material with periodically varying permittivity. This scenario can be considered as referential for the field of scattering theory and many developments can be envisioned that capitalize on our understanding of how light scatters by such a sphere. We show how the dispersion relation of the eigenmodes of a homogeneous medium transforms in the presence of time modulation and provide the field Ansatz in spherical coordinates used to solve the scattering problem. During the presentation, all details concerning the solution for the generalized Mie problem will be presented. Simulation results will be discussed, also in comparison to results obtained from an alternative full-wave finite-element based solver to assess the accuracy of our analytical predictions. The contribution is made bold by discussing a selection of novel scattering phenomena that occur thanks to the temporal modulation.

II. Theoretical description

To solve Maxwell's equations, they have to be supplemented with dedicated constitutive relations. The latter express the relation between the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ and the corresponding electric and magnetic flux densities $\mathbf{D}$ and $\mathbf{B}$. Here, in the interest of space, we will show derivations for $\mathbf{D}$ only. For a dispersive, time-varying, and local material without magnetoelectric coupling, the expression for $\mathbf{D}$ reads [5]

$$\mathbf{D}(\mathbf{r},t) \quad = \quad \epsilon_0\mathbf{E}(\mathbf{r},t) + \epsilon_0 \int\limits_{-\infty}^{t} \chi(\mathbf{r},t,t-\tau)\mathbf{E}(\mathbf{r},\tau)\mathrm{d}\tau, \tag{1}$$

where ϵ_0 denotes the vacuum permittivity, $\mathbf{r}$ is the position vector, t denotes instantaneous time, and τ is the retardation time during which the system "remembers" its evolution. The material response is expressed in terms of the response function $\chi(\mathbf{r},t,t-\tau)$, also known as susceptibility, which becomes a function of both t and $t-\tau$ in dynamic time-varying media compared to the static case which is a function of t only. To transform Maxwell's equation into the frequency domain, one needs to take two Fourier transforms of the response function $\chi(\mathbf{r},t,t-\tau)$, with respect to the two time parameters t and τ. The new response function transforms into $\tilde{\chi}(\mathbf{r},\omega-\omega',\omega)$ as

$$\tilde{\chi}(\mathbf{r},\omega-\omega',\omega') \quad = \quad \frac{1}{2\pi}\iint\limits_{-\infty}^{+\infty} \chi(\mathbf{r},t,t-\tau)e^{\mathrm{i}(\omega t-\omega'\tau)}\mathrm{d}t\mathrm{d}\tau. \tag{2}$$

Plugging the susceptibility of this form into Eq. (1) and rewriting Ampère-Maxwell's equation in the frequency domain gives [5]:

$$\nabla\times\mathbf{H}(\mathbf{r},\omega) \quad = \quad -\mathrm{i}\omega\epsilon_0\mathbf{E}(\mathbf{r},\omega) - \mathrm{i}\omega\epsilon_0\int\limits_{-\infty}^{+\infty}\tilde{\chi}(\mathbf{r},\omega-\omega',\omega')\mathbf{E}(\mathbf{r},\omega')\mathrm{d}\omega'. \tag{3}$$

While our formulation is general, for this presentation we will specify at this point our assumptions on the material dispersion and the time modulation. We consider a material with a single-resonance Lorentz dispersion where the strength of the resonant term (the effective plasma frequency) depends on time. In particular, we assume that bulk electron density is varied periodically in time as $N(t) = N_0[1 + N_1\cos(\Omega t)]$. N_0 is the static electron density in the unmodulated case, N_1 is the modulation amplitude that can vary from 0 to 1, and Ω is the modulation frequency. The expression for the susceptibility of such a time-varying medium reads

$$\tilde{\chi}(\omega-\omega',\omega') \quad = \quad \frac{N_0e^2}{m\epsilon_0}\frac{\delta(\omega-\omega')+\frac{N_1}{2}\left[\delta(\omega-\omega'+\Omega)+\delta(\omega-\omega'-\Omega)\right]}{\omega_\mathrm{n}^2-\omega^2-i\gamma_\mathrm{n}\omega}, \tag{4}$$

where m is the electron mass, ω_n is the natural frequency of the plasma, and γ_n stands for the damping coefficient of the unmodulated plasma. We stress that while a few assumptions found their way into this derivation, none of them is essential and our approach remains valid in general.

To study scattering of light from a time-varying sphere, it is desirable to find a solution to Maxwell's equations in such a material in the form of spherical functions:

$$\mathbf{E}(\mathbf{r},\omega) \quad = \quad \int\limits_{\mathbf{C}}\sum_{\nu\mu,\alpha}\mathcal{C}^{\mathrm{e}}_{\alpha,\mu\nu}(k,\omega)\mathbf{F}^{(1)}_{\alpha,\mu\nu}(k\mathbf{r})\mathrm{d}k, \tag{5}$$

where $\mathbf{F}^{(1)}_{\alpha,\mu\nu}$ are vector spherical harmonics of Bessel type and $\mathbf{C}^{\mathrm{e}}_{\alpha,\mu\nu}$ are the corresponding coefficients. Indicator α acquires the names M, N for the TE (magnetic), TM (electric) vector spherical harmonics, respectively, index ν stands for the angular momentum quantum number that takes the values 1,2,... and corresponds to dipoles, quadrupoles, etc., and index μ stands for the azimuthal index number that takes the values $-\nu,..,-1,0,1,..,\nu$. Superscript e indicates the coefficient for electric field, and integration is performed over all complex valued wavenumbers k.

By using Eqs. (3), (4), and (5) one can write a dispersion relation for a time-varying and dispersive medium as

$$\left(k^2-k_0^2(\omega)\left[1+\chi(\omega)\right]\right)\mathcal{C}^{\mathrm{e}}_{\alpha,\mu\nu}(k,\omega) \quad = \quad \frac{N_1}{2}\chi_e(\omega)k_0^2(\omega)\left[\mathcal{C}^{\mathrm{e}}_{\alpha,\mu\nu}(k,\omega+\Omega)+\mathcal{C}^{\mathrm{e}}_{\alpha,\mu\nu}(k,\omega-\Omega)\right], \tag{6}$$

where $\chi(\omega)$ is the susceptibility of the unmodulated material, ω and k are sets of frequencies and wavenumbers, respectively. In the case of vanishing modulation, the right hand side of Eq. (6) becomes 0 and we resort to the usual dispersion relation $k^2 = k_0^2(\omega)(1 + \chi(\omega))$. In such a conventional material, a single wavenumber k is related to a single frequency ω in a bijective correspondence. This is evidenced in Fig. 1 (a) that shows the dispersion relation in the unmodulated case for the considered material in some selected frequency range. The figure shows the frequency associated with each mode (the mode number is shown on the x-axis, and the color codes the associated wavenumber). It is a diagonal matrix.

In contrast, the dispersion relation for a time-varying medium is expressed in Eq. (6) with all the terms on the right hand side now being non-zero. By discretizing the frequency, the solution of Eq. (6) results in a matrix where the eigenvectors express the dispersion relation of the modes without a bijective correspondence. Any solution to Maxwell's equations can be written as a superposition of these modes, and each mode consists of multiple frequencies ω (again plotted on the y-axis) where each frequency is associated with a specific wavenumber k (color coded). Note that the discrete nature of coupling in our case is a result of periodicity of time modulation. For a more general time modulation, every frequency is possibly coupled to every other frequency!

Using this expansion, we can solve for the scattered light by a spherical time-modulated dispersive particle for a given incident field. As a static material we take silicon with natural frequency $w_{\mathrm{n}} = 967$ THz, decay factor $\gamma_{\mathrm{n}} = 120$ THz, and static permittivity $\epsilon_{\mathrm{s}} = 12$. The validity of our approach is assessed by comparing predictions to full-wave simulations using a finite-element based method (see Fig. 1(c)), where a silicon is modulated with frequency $\Omega = 64$ THz and modulation amplitude $N_1 = 0.9$. The structure is excited by a pulse of 3 fs width and is centered at 290 THz. During the presentation we will provide an overview of physical effects supported in such a scattering problem.

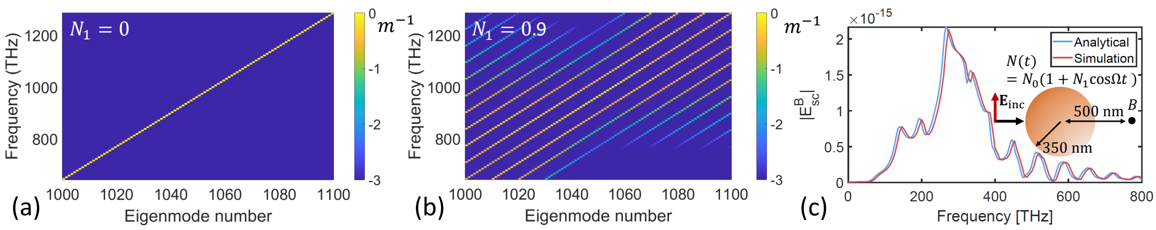

Fig. 1: (a) Eigenvectors of dispersion relation in unmodulated case. (b) Eigenvectors in modulated case. Both figures (a) and (b) are plotted in logarithmic scale. (c) Scattered field amplitude obtained using theory and simulations at point B, as sketched in the inset.

III. Conclusion

Here, we have presented and discussed the dispersion relation for waves propagating in a general time-varying material. This is the foundation for many further theoretical developments which we will show during the presentation. This canonical problem constitutes an ideal playground to identify and elaborate on many peculiarities that are expected to emerge when considering scattering of light in time-varying and dispersive media.

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