Ren, Xiaotao; Hannukainen, Antti; Belahcen, Anouar; Perriard, Yves

LOD-Homogenization of Multiscale Eddy Current Problem in Time Domain

Published in:
IEEE Transactions on Magnetics

DOI:
10.1109/TMAG.2021.3071813

Published: 01/06/2021

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Ren, X., Hannukainen, A., Belahcen, A., & Perriard, Y. (2021). LOD-Homogenization of Multiscale Eddy Current Problem in Time Domain. *IEEE Transactions on Magnetics*, *57*(6), Article 6302004. https://doi.org/10.1109/TMAG.2021.3071813

This is the author's version of an article that has been published in this journal. Changes were made to this version by the publisher prior to publication.
The final version of record is available at http://dx.doi.org/10.1109/TMAG.2021.3071813

LOD-Homogenization of Multiscale Eddy Current Problem in Time Domain

Xiaotao Ren[1], Antti Hannukainen[2], Anouar Belahcen[3*], and Yves Perriard[1]
[1]Integrated Actuators Laboratory (LAI), École Polytechnique Fédérale de Lausanne (EPFL), Neuchâtel, Switzerland
[2]Department of Mathematics and Systems Analysis, Aalto University, Espoo, Finland
[3]Department of Electrical Engineering and Automation, Aalto University, Espoo, Finland

Devices fabricated from Soft Magnetic Composites (SMCs) are gaining popularity in research and application. The multiscale characteristics require special attention. Solving the quasi-statics Maxwell's equations on such devices consumes huge time and memory if the granular scale of SMCs is resolved. We have proposed a Localized Orthogonal Decomposition (LOD) homogenization strategy which allows us to compute the problem on a middle scale while retrieving the material dimension. The LOD projector has a localization property so that it can be accurately approximated on a local patch. In this work, we explore the localization characteristic further to show that the projector can be reused at different time steps. The requirement for computational time and memory can be greatly reduced. A numerical example in two dimensions is provided to show the feasibility and advantage of this approach. This technique is applied to a domain of SMCs with randomly distributed polygon-shaped granules. Finally, error analysis is provided to show the validation of the LOD projector.

***Index Terms*—Eddy current problem, Finite Element Method, homogenization, multiscale, orthogonal function space.**

I. Introduction

MULTISCALE electromagnetic behavior is usually observed in electric devices. This is the case in a transformer manufactured from soft magnetic composite (SMCs) that is the motivation for this article. As the characteristic length scale of SMCs is much different from the device dimension, their modeling needs special attention. At low frequency regime the devices are modeled using the time-dependent eddy current approximation by vector potential together with the appropriate boundary and gauging conditions [1]. Generally, magnetic behavior and loss characteristics are the fundamental concerns. Devices are sometimes simplified into a two dimensional scenario for numerical calculations, which provides a satisfactory description. Commonly for laminated steel in motors or transformers, a perpendicular vector potential is formulated to determine the magnetic field in the studied domain. The vector potential becomes a scalar.

Homogenization studies on SMCs have been carried out analytically, semi-analytically, and numerically [2], [3], [4], [5]. Numerical homogenization has enjoyed a significant advancement in time-dependent magnetism [6], [7], [8]. The developments in the field of SMCs further boost its development [9], [10]. Most strategies assume that the lamination or SMCs have a periodic structure. To extend the numerical homogenization over the constraint of periodicity, a Localized Orthogonal Decomposition (LOD) method is proposed [11], [12]. The problem was solved on a coarse mesh with modified basis functions which contain information on micro-scale material properties. The LOD projector can be accurately approximated on a local patch. The LOD method has been under active study in the mathematics community, but it is seldom applied to homogenization problems in electromechanics. We have introduced LOD to solve multiscale eddy current problems in [13], where numerical tests on a domain of SMCs with spatially periodic inclusions are conducted. Another difficulty of modeling SMCs lies in the material property contrast between the two phases of the composite. The initial design of the LOD projector was affected heavily by the permeability contrast. We have improved the projector from [13] to rely trivially on the property contrast and carried out computational examples on SMCs with randomly distributed inclusions [14].

The LOD method has been applied to solve time-dependent problems in [15], where error analysis and initial numerical examples are given. The projector can be calculated as in a stationary problem and remains invariable. Thus, it can be recycled across different time steps. Recycling the projector causes an error whose propagation needs to be delicately handled. The upper bound for this error has been derived using perturbation analysis. We study the potential applicability of this method to the simulation of electrical machines by conducting numerical tests on simplified SMC geometry with realistic dimensions and parameter values.

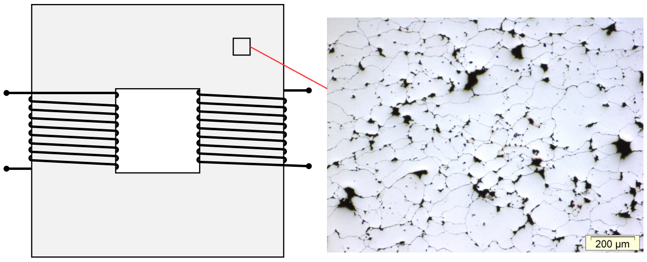
The paper is organized as follows. The LOD method is briefly explained in Section II. In Section III, a randomly distributed SMCs domain is calculated with the LOD strategy in the time domain. Error analyses with regard to the time are provided. Finally, conclusions are drawn for the efficiency of the LOD projector.

II. Localized orthogonal decomposition

We study the SMCs domain, denoted Ω, as shown in Fig. 1(b). The problem is solved by introducing a transient vector potential normal to the domain, $\mathbf{A} = A(x,y,t)\mathbf{e}_z$ where $\mathbf{e}_z$ represents the unit vector in the z direction. Under this assumption the eddy current problem reduces to a problem for $A(x,y,t)$ that we denote by u. On each time-step the

Manuscript received April 5, 2021; revised April 5, 2021. Corresponding author: Anouar Belahcen (email: anouar.belahcen@aalto.fi).

This is the author's version of an article that has been published in this journal. Changes were made to this version by the publisher prior to publication.
The final version of record is available at http://dx.doi.org/10.1109/TMAG.2021.3071813



(a) (b)

Fig. 1. (a) Sketch of a transformer composed of SMCs; (b) Micrograph of SMCs structure [16].

solution is searched in the Sobolev function space $W^{1,2}(\Omega)$ where the function and its first weak derivative are properly defined. Assuming there is no source of current density, the time-dependent eddy current problem writes in the weak form as follows:

$$\left.\begin{aligned} a\,(u,v) &= 0 \quad && \forall v \in W_0^{1,2}(\Omega) \\ u|_{t=0} &= 0 && \text{in } \Omega \\ u &= u_\mathrm{D}(x,y,t) && \text{on } \Gamma_\mathrm{D} \end{aligned}\right\} \tag{1}$$

with

$$a\,(u,v) := \left(\frac{1}{\mu}\nabla u, \nabla v\right) + (\sigma\partial_t u, v) \tag{2}$$

where Γ_D represents the Dirichlet boundary where the transient vector potential is imposed and u_D is a given time-dependent boundary data. The space $W_0^{1,2}$ is the subspace of $W^{1,2}$ with zero boundary trace on Γ_D. Observe that a homogeneous Neumann boundary condition is posed on the remaining part of the boundary $\Gamma_\mathbf{N} = \partial\Omega \setminus \Gamma_\mathrm{D}$. The brackets denote standard $L^2(\Omega)$ inner product, $(u,v) := \int uv \,\mathrm{d}\Omega$.

A. LOD in time domain

Let the domain Ω be triangulated into nested partitions $\mathcal{T}_\mathrm{H}$ and $\mathcal{T}_\mathrm{h}$, and denote the corresponding Finite Element (FE) spaces by V_H and V_h. As the two partitions are nested, $V_\mathrm{H} \subset V_\mathrm{h}$. The mesh parameter h is chosen to be sufficiently small so that the FE discretization can capture relevant features of the solution, whereas the triangulation $\mathcal{T}_\mathrm{H}$ is much coarser and has a similar scale with the domain Ω, i.e., $\mathrm{h} < \epsilon \ll \mathrm{H}$, where ϵ represents the characteristic length scale of the material. The parameter ϵ is a measure for the size of heterogeneity of the composite.

The LOD method solves (1) without the excessive computational cost related to the direct FE simulation. This is achieved by splitting the solution into large- and micro-scale components and posing a problem only for the large-scale component. In LOD, such a splitting is expressed by decomposing the FE space V_h as $V_\mathrm{h} := V_\mathrm{H} \oplus V_\mathrm{f}$ where V_f is the space of *rapidly-oscillating* functions containing micro-scale solution component concerning the characteristic length scale of the material that can not be observed in V_H.

We assume that the Dirichlet data u_D has only large-scale features so that the condition $u = u_\mathrm{D}$ on Γ_D can be imposed to the coarse solution component u_H. We use notation V_D and V_0 when boundary condition $u = u_\mathrm{D}$ or $u = 0$ on Γ_D is imposed to space V, respectively.

a) LOD in stationary case: First, consider application of LOD to a stationary problem: find $u \in V_\mathrm{hD}$ satisfying $(\frac{1}{\mu}\nabla u, \nabla v) = (f, v)$ for all $v \in V_\mathrm{h0}$. Decompose $u = u_\mathrm{HD} + u_\mathrm{f0}$ for $u_\mathrm{HD} \in V_\mathrm{HD}$ and $u_\mathrm{f0} \in V_\mathrm{f0}$. The problem for the large-scale solution component u_H is posed using the orthogonal projection operator $\mathrm{P} : V_\mathrm{h} \to V_\mathrm{f0}$ so that for given $w \in V_\mathrm{h}$, P satisfies the orthogonality relationship $\left(\frac{1}{\mu}\nabla(\mathrm{I}-\mathrm{P})w, \nabla v\right) = 0 \;\forall v \in V_\mathrm{f0}$, where I is the identity matrix. Define $\mathrm{Q} := \mathrm{I} - \mathrm{P}$. Thus, choosing $\mathrm{Q}v_\mathrm{H0}$ for some $v_\mathrm{H0} \in V_\mathrm{H0}$ as a test function gives

$$(\frac{1}{\mu}\nabla u_\mathrm{HD}, \nabla \mathrm{Q} v_\mathrm{H0}) = (f, \mathrm{Q}v_\mathrm{H0}). \tag{3}$$

The micro-scale solution component is recovered as $u_\mathrm{f} \approx -\mathrm{P}u_\mathrm{HD}$ leading to the LOD approximation $u \approx \mathrm{Q}u_\mathrm{HD}$. The error in this approximation depends on the loading function f. If $f \in V_\mathrm{H}$, the LOD solution coincides with the exact one. Otherwise, the LOD approximation error is related to the mesh size H.

b) LOD in time domain: Decompose the solution to (1) as $u = u_\mathrm{HD} + u_\mathrm{f0}$ for $u_\mathrm{HD}(\cdot,t) \in V_\mathrm{HD}$ and $u_\mathrm{f0}(\cdot,t) \in V_\mathrm{f0}$. Choosing the test function $\mathrm{Q}v_\mathrm{H0}$ in (1) and using orthogonality of P gives

$$(\frac{1}{\mu}\nabla u_\mathrm{HD}, \nabla \mathrm{Q}v_\mathrm{H0}) + (\sigma\partial_t u, \mathrm{Q}v_\mathrm{H0}) = 0 \tag{4}$$

Observe that the orthogonality of P cannot be used to eliminate u_f0 from the term $(\sigma\partial_t u, v)$. However, intuitively speaking, the amplitude of the micro-scale solution component u_f0 tends to zero when $\epsilon \to 0$. This motivates the approximation $\partial_t u \approx \partial_t \mathrm{Q}u_\mathrm{HD}$ leading to the perturbed problem

$$(\frac{1}{\mu}\nabla \hat{u}_\mathrm{HD}, \nabla \mathrm{Q}v_\mathrm{H0}) + (\sigma\partial_t \mathrm{Q}\hat{u}_\mathrm{HD}, \mathrm{Q}v_\mathrm{H0}) = 0. \tag{5}$$

The error $u_\mathrm{HD} - \hat{u}_\mathrm{HD}$ resulting from this can be bounded by the perturbation argument. Observe that the problem for $\hat{u}_\mathrm{HD}$ is posed on V_HD and its solution does not require the recovery of the solution u. The solution u is obtained as $u \approx \mathrm{Q}\hat{u}_\mathrm{HD}$. The accuracy of this approximation is studied in [12].

c) Implementation: Solution of (5) using Backward Euler method requires assembly of matrices B_H and M_H, with entries $(B_\mathrm{H})_{ij} = (\frac{1}{\mu}\nabla\varphi_i^\mathrm{H}, \nabla \mathrm{Q}\varphi_j^\mathrm{H})$ and $(M_\mathrm{H})_{ij} = (\sigma \mathrm{Q}\varphi_i^\mathrm{H}, \mathrm{Q}\varphi_j^\mathrm{H})$. Here φ_i^H are the basis functions of V_H0. The exact projection $\mathrm{P}\varphi_i^\mathrm{H}$ is expensive to calculate. It has been proven in [12] that $\mathrm{P}\varphi_i^\mathrm{H}$ decays rapidly when moving away from the node associated with φ_i^H, as illustrated in Fig. 2. This property is called as localization, and it allows us to accurately approximate $\mathrm{P}\varphi_i^\mathrm{H}$ while keeping the computational cost small. Let $\omega_i \subset \Omega$ be a local patch associated to the basis function φ_i^H, and $V_\mathrm{f0}(\omega_i)$ the space of rapidly oscillating functions on ω_i with zero Dirichlet boundary value on $\partial\omega_i \setminus \Gamma_\mathrm{N}$. Different size local patches constructed from layers of elements as shown in Fig. 3 can be used. The function $\mathrm{P}\varphi_i^\mathrm{H}$ is approximated as $\mathrm{P}\varphi_i^\mathrm{H} \approx z_i$, where $z_i \in V_\mathrm{f0}(\omega_i)$ satisfies

$$\left(\frac{1}{\mu}\nabla z_i, \nabla v\right) = \left(\frac{1}{\mu}\nabla\varphi_i^\mathrm{H}, \nabla v\right) \;\forall v \in V_\mathrm{f0}(\omega_i). \tag{6}$$

There exists another, more accurate method for approximating $\mathrm{P}\varphi_i^{\mathrm{H}}$, see [12].

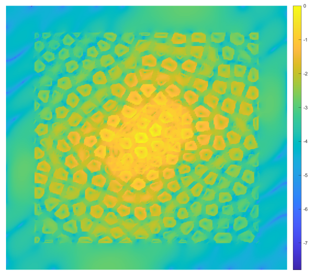


Fig. 2. The normalized function $\log \frac{(\nabla \mathrm{P}\varphi^{\mathrm{H}})^T(\nabla \mathrm{P}\varphi^{\mathrm{H}})}{\max\left[(\nabla \mathrm{P}\varphi^{\mathrm{H}})^T(\nabla \mathrm{P}\varphi^{\mathrm{H}})\right]}$. The basis function φ^{H} is related to the center node of the square.

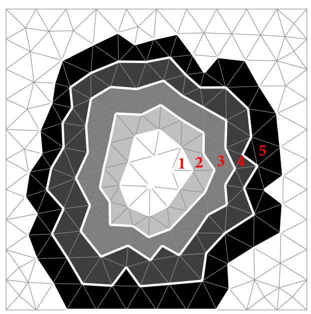


Fig. 3. An illustration of the patch size related to triangles in the center of the square used for computation of P. The k-patch is obtained by finding all triangles sharing a node with $(k-1)$-patch.

III. Numerical study

The numerical studies are carried out on a square SMCs domain of size $L = 720$ μm, as shown in Fig. 4. Inclusions are modeled by randomly distributed polygonal shapes.

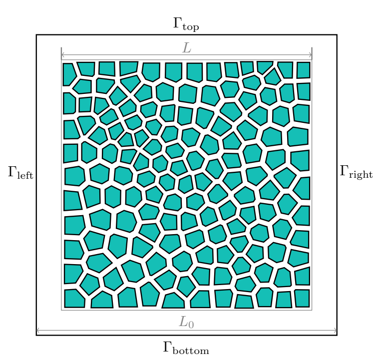


Fig. 4. SMCs structure with randomly distributed polygon-shaped inclusions.

The magnetic and highly conductive ferromagnetic inclusions in Fig. 1 have a typical dimension of approximately 60 μm. The coating matrix between these inclusions is electrically insulating which ensures a high global electrical resistance in SMCs. Nevertheless, the coating matrix is usually non-magnetic. The material properties of SMCs components used in our numerical tests are listed in Tab. I. The working frequency is fixed at $f = 50$ Hz. The time step for the backward Euler method is chosen as $\Delta t = \frac{1}{100f} = 0.2$ ms. To avoid the material interpolation over the boundaries, an air margin is augmented over the SMCs domain. The study domain becomes a square of length L_0, as illustrated in Fig. 4. A homogeneous Dirichlet boundary condition is set to Γ_{bottom}. On Γ_{top}, the harmonic vector potential is imposed as $u_{\mathrm{D}} = \alpha \sin(2\pi f t)$, where α is the crest value. The value of α is chosen such that the magnetic flux density averaged over the geometric domain is $B_0 = 1$ T along the horizontal direction. The corresponding vector potential magnitude is $\alpha = B_0 L_0$. We impose the homogeneous Neumann boundary condition to the left and right edges, $\Gamma_{\mathrm{N}} := \Gamma_{\text{left}} \cup \Gamma_{\text{right}}$.

TABLE I
Material properties of SMCs

	Inclusion	Coating matrix
Electric conductivity	$1 \times 10^7 \mathrm{S/m}$	0
Magnetic permeability	$50\,\mu_0$	μ_0
Volume fraction	56.25%	-

To investigate the effect of LOD method parameters on the accuracy of the LOD solution, we generate the computational meshes as follows. First, an initial triangulation $\mathcal{T}_{\mathrm{H}}^0$ is generated for the domain Ω. The coarse mesh is obtained from $\mathcal{T}_{\mathrm{H}}^0$ by n-refinements, and the fine mesh $\mathcal{T}_{\mathrm{h}}$ by 5-refinements. In this way, the fine mesh stays always the same and the parameter H can be varied by choosing different n.

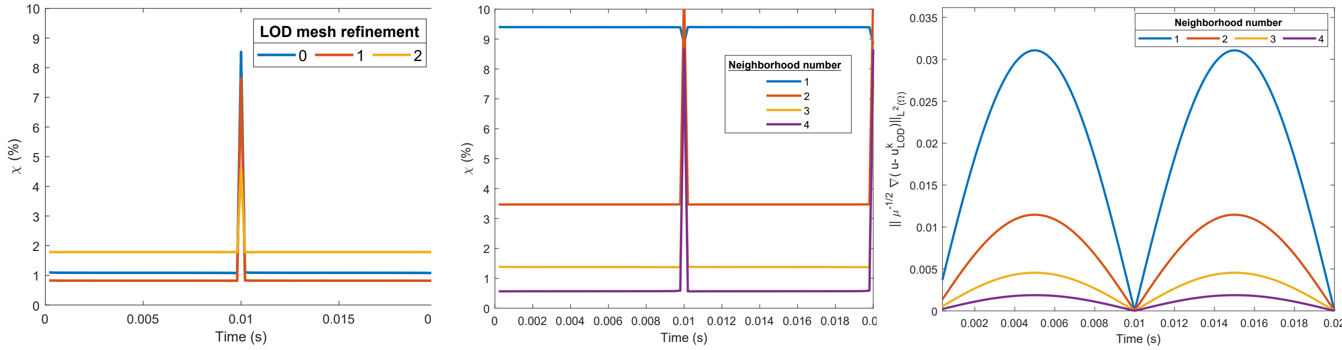
We use the FEM solution computed on the fine grid, $\mathbf{u}$, as the reference solution. On the other hand, LOD homogenization results are solved on the coarse grid, $\mathbf{u}_{\text{LOD}}^k$, using an approximation for LOD-projector P computed on k-neighborhood patches.

We are interested in the relative error between $\mathbf{u}_{\text{LOD}}^k$ and the reference solution in the norm,

$$\chi = \left[\frac{\left(\frac{1}{\mu}\nabla\left(\mathbf{u}-\mathbf{u}_{\text{LOD}}^k\right), \nabla\left(\mathbf{u}-\mathbf{u}_{\text{LOD}}^k\right)\right)}{\left(\frac{1}{\mu}\nabla\mathbf{u}, \nabla\mathbf{u}\right)} \right]^{1/2} \times 100\%, \quad (7)$$

which is a function of time.

Series of numerical experiments are carried out by means of control variable while the material properties are invariable. First, we investigate the effect of mesh size H by varying the number of coarse grid mesh refinements as $n = 0, 1, 2$. The neighborhood size $k = 3$ is used. It is worth pointing out that the refinement of $\mathcal{T}_{\mathrm{H}}$ does not attain the resolution of the fine grid $\mathcal{T}_{\mathrm{h}}$. Second, the patch size k alters while material properties are fixed as in Tab. I and coarse mesh with one refinement is kept fixed. In both tests, there are 100 temporal samples for each magnetic wave period, and one period is computed. All computations are performed using a Linux workstation with an Intel i7 8-core processor and 16G memory. Computing P constitutes the main computational cost of LOD, for $\mathcal{T}_{\mathrm{H}} = \mathcal{T}_{\mathrm{H}}^0$ and $k = 3$, the evaluation of P took approximately 75 seconds and computing 100-time steps using standard first order finite element method approximately 1.7 seconds. Nevertheless, taking one time step using LOD with these parameters was in average approximately 50 times



(a) LOD mesh refinement (b) Patch size (c) Patch size (abs. error)

Fig. 5. In sub-figures (a) and (b), the relative error as a function of time with regard to mesh refinement and patch size. In sub-figure (c), the absolute error with regard to patch size.

faster compared to direct FE-solution. We conclude that LOD becomes attractive when the calculation is repeated sufficiently many times. The relative error as a function of time with respect to the number of coarse mesh refinements and neighborhood sizes are drawn in Fig. 5.

We observe from Fig. 5 that the relative errors are mainly below 3%, a level that is usually considered negligible in the engineering community. There are certain peaks in the error curves because, at certain time steps, the magnetic field is trivial when the imposing vector potential is zero. In Fig. 5 (a) the error increases after 2 refinements, but remains still small. We believe that this phenomenon is related to the approximation of P. Observe that the local patch consists of element layers, as depicted in Fig. 3, hence, the diameter of ω_i decreases with coarse grid mesh size. For large H, several local patches intersect with the boundary $\partial\Omega$. When this happens, the auxiliary problems (6) have accurate boundary conditions on a large part of boundary $\partial\omega$ that may cause the localization error to be unrealistically small. This phenomenon vanishes for small H, possibly causing the total error to increase. The effect of neighborhood size to error is depicted in Fig. 5 (b) and Fig. 5 (c). The errors converge to zero when k grows, also supporting our hypothesis for the unexpected error behavior in Fig.5 (a). The values $k = 1, 2$ do not yield acceptable accuracy. For $k = 3, 4$ the error, neglecting the peaks, is under 2%. We regard the value $k = 3$ as a good choice, as it yields reasonable error with small computational cost compared to larger k.

IV. Conclusion

A multiscale time-dependent eddy current problem is homogenized by a localized orthogonal decomposition method. Although the projector takes time to calculate, it can be approximated on a local patch. The LOD homogenization algorithm is applied to a domain of SMCs. The recyclable characteristics of the LOD confirm its advantages in the time domain. After the projector has been computed, time is saved for each time-step in transient problems. Since the projector is highly coupled with material properties, the proposed approach is currently feasible for linear materials only. Its application with nonlinear materials will be investigated in future work.

References

[1] R. Touzani and J. Rappaz, *Mathematical Models for Eddy Currents and Magnetostatics: With Selected Applications*, ser. Scientific Computation. Springer Netherlands, 2014.
[2] O. de la Barrière, M. LoBue, and F. Mazaleyrat, "Semianalytical and analytical formulas for the classical loss in granular materials with rectangular and elliptical grain shapes," *IEEE Transactions on Magnetics*, vol. 50, no. 10, p. 1–8, Oct 2014.
[3] X. Ren, R. Corcolle, and L. Daniel, "Bounds and estimates on eddy current losses in soft magnetic composites," *Journal of Applied Physics*, vol. 123, no. 23, p. 235109, jun 2018.
[4] A. Maruo and H. Igarashi, "Analysis of magnetic properties of soft magnetic composite using discrete element method," *IEEE Transactions on Magnetics*, vol. 55, no. 6, pp. 1–5, Jun 2019.
[5] J. Vesa and P. Rasilo, "Permeability estimations of smc material particles," *IEEE Transactions on Magnetics*, vol. 56, no. 9, pp. 1–7, Sep 2020.
[6] P. Henning and A. Persson, "Computational homogenization of time-harmonic maxwell's equations," *SIAM Journal on Scientific Computing*, vol. 42, no. 3, pp. B581–B607, Jan 2020.
[7] K. Hollaus and M. Schöbinger, "A mixed multiscale FEM for the eddy-current problem with T, Φ–Φ in laminated conducting media," *IEEE Transactions on Magnetics*, vol. 56, no. 4, pp. 1–4, Apr 2020.
[8] C. A. Valdivieso, G. Meunier, B. Ramdane, J. Gyselinck, C. Guerin, and R. V. Sabariego, "Time-domain finite-element eddy-current homogenization of windings using foster networks and recursive convolution," *IEEE Transactions on Magnetics*, pp. 1–1, 2020.
[9] I. Niyonzima, R. V. Sabariego, P. Dular, and C. Geuzaine, "Nonlinear computational homogenization method for the evaluation of eddy currents in soft magnetic composites," *IEEE Transactions on Magnetics*, vol. 50, no. 2, pp. 61–64, 2014.
[10] Y. Sato and H. Igarashi, "Time-domain analysis of soft magnetic composite using equivalent circuit obtained via homogenization," *IEEE Transactions on Magnetics*, vol. 53, no. 6, pp. 1–4, Jun 2017.
[11] A. Abdulle and P. Henning, "Localized orthogonal decomposition method for the wave equation with a continuum of scales," *Mathematics of Computation*, vol. 86, no. 304, pp. 549–587, 2017.
[12] A. Målqvist and D. Peterseim, *Numerical Homogenization by Localized Orthogonal Decomposition*, ser. SIAM Spotlights. Society for Industrial and Applied Mathematics, 2020.
[13] X. Ren, A. Hannukainen, and A. Belahcen, "Homogenization of multiscale eddy current problem by localized orthogonal decomposition method," *IEEE Transactions on Magnetics*, vol. 55, no. 9, pp. 1–4, 2019.
[14] ——, "LOD-homogenization of high contrast eddy current problem," in *22nd International Conference on the Computation of Electromagnetic Fields (COMPUMAG)*. Paris, France: IEEE, July 2019.
[15] A. Målqvist and A. Persson, "Multiscale techniques for parabolic equations," *Numerische Mathematik*, vol. 138, no. 1, p. 191–217, Jan 2018.
[16] A. Schoppa and P. Delarbre, "Soft magnetic powder composites and potential applications in modern electric machines and devices," *IEEE Transactions on Magnetics*, vol. 50, no. 4, p. 1–4, Apr 2014.