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Decision Support

Identifying and visualizing a diverse set of plausible scenarios for strategic planning

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ABSTRACT

When making long-term strategic decisions, organizations may benefit from characterizing their future operational environment with a set of scenarios. These scenarios can be built based on combinations of levels of uncertainty factors describing, e.g., alternative political or technological developments. However, the number of such combinations grows exponentially in the number of the uncertainty factors, whereby the selection of a few combinations to work as a basis for scenario development can be difficult. In this paper, we develop a method for identifying a small but diverse set of plausible combinations of uncertainty factor levels. The method filters an exponentially large set of possible combinations to a smaller set of most plausible combinations, as assessed by the consistencies of the pairs of uncertainty factor levels in the combinations. To support the selection of the final set of combinations from the most consistent ones, we formulate a weighted set cover problem, the solution to which gives the smallest number of maximally consistent combinations that together cover all uncertainty factor levels. Moreover, we develop an interactive software tool utilizing Multiple Correspondence Analysis to visualize the consistency and diversity of the combinations, thus improving the transparency and communicability of the methods. This paper also presents a real case in which our method was used to identify a set of plausible scenarios for the Finnish National Emergency Supply Organization to support their strategic decision making.

1. Introduction

Strategic decision makers have become increasingly aware of the high levels of uncertainty in their future operational environments (Wright, Cairns, O'Brien, & Goodwin, 2019), having recently faced several unforeseen but impactful events such as the subprime mortgage crisis, BREXIT vote, and COVID-19 pandemic. Being prepared for such regime shifts and discontinuous developments is vital to organizations so that they can retain their competitive edge by responding to these changes with high-quality strategic decisions (Massey & Wu, 2005; Vilkkumaa, Liesiö, Salo, & Ilmola-Sheppard, 2018). Especially in long-term strategy development, forecasting based on simple extrapolation of historical data may encourage organizations to downplay uncertainties, leaving them vulnerable to the impacts of the unprecedented when optimum strategies contingent on uncertain key assumptions lose their prescriptive value (Berger, Emmerline, & Tavoni, 2017; Lempert, Groves, Popper, & Bankes, 2006). For these reasons, forecasting has been complemented and even replaced by scenario-based methods (Schoemaker, 1995), where the future operational environment is characterized by a set of scenarios that can be used to test alternative strategies (e.g., Liesiö & Salo, 2012; O'Brien, 2004; Ram, Montibeller, & Morton, 2011; Vilkkumaa et al., 2018). In particular, strategy work benefits from the development of *explorative scenarios*, the purpose of which is to help prepare for a range of plausible (but not necessarily probable) futures by answering the question *What can happen?* (Börjeson, Höjer, Dreborg, Ekvall, & Finnveden, 2006; Kowalski, Stagl, Madlener, & Omann, 2009; Wiek, Withycombe, Schweizer, & Lang, 2013). This is in contrast to forecasts seeking to identify the most probable futures (*What is most likely to happen?*) and normative scenarios aiming for the most desirable futures (*How can a specific target be reached?*).

Explorative scenarios are typically elaborated with a long time-horizon to capture profound structural changes (Börjeson et al., 2006), which prohibits the objective assessment of future uncertainty through, e.g., probability distributions. Consequently, such scenarios are often built largely based on subjective, qualitative judgments elicited from various domain experts in a workshop setting. Ideally, the entire range of views and creativity of these experts could be incorporated in the scenario development process in

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a transparent and understandable way (McBride et al., 2017). On the other hand, systematic methods should be applied to ensure the quality of the final scenario set. Various criteria exist for assessing the quality of this set, but most authors agree that the scenarios should be (i) plausible depictions of the future in that they are internally consistent and (ii) sufficiently dissimilar in that they together span a diverse set of futures instead of being variations of the same theme (see, e.g., reviews by Börjeson et al., 2006; Bradfield, Wright, Burt, Cairns, & Heijden, 2005; Bunn & Salo, 1993). Furthermore, the number of scenarios should be relatively small, because people usually find it difficult to compare many qualitatively different scenarios (Heugens & van Oosterhout, 2001; Tietje, 2005). In this paper, we develop an analytic method to support the identification of a small but diverse set of plausible scenarios from exponentially many candidate scenarios representing a multitude of views held by the workshop participants. Moreover, to improve the transparency and communicability of this method, we develop an interactive software tool to visualize these scenarios and to guide the identification process.

In the literature, perhaps the most common approach (Bradfield et al., 2005; van der Heijden, 2005) to developing explorative scenarios is the *deductive method*, which falls into a wider category of intuitive logics approaches (Ramirez & Wilkinson, 2014). The deductive method begins by listing *uncertainty factors*[1] that reflect the most impactful and uncertain drivers of future change with respect to the focal issue of concern. Then, the two factors that are seen to be the most impactful and uncertain are selected as the *scenario axes*. Four skeletal scenarios are holistically developed by starting from the combinations of the extremes of these two axes, after which the outcomes for the remaining factors are inserted intuitively based on their interactions with the ends of the scenario axes (Schwartz, 1996). For example, McBride et al. (2017) present the New England Landscape Futures Project, where scenario axes 'Socioeconomic connectedness' (Local vs. Global) and 'Natural resource use' (Low vs. high innovation and planning) were used as the starting point for scenarios that describe possible consequences of the New England landscape changes over the next 50 years.

The popularity of the deductive method stems from its ability to produce diverging scenarios and its relative ease of use. However, building scenarios on the basis of two scenario axes only is controversial. On the one hand, limiting the focus on two axes only can result in a scenario set that is considered too obvious or simplistic (Ramirez & Wilkinson, 2014). On the other hand, focusing on the extremes of these axes may drive unnecessary polarization in thinking (Lord, Helfgott, & Vervoort, 2016; McBride et al., 2017; Wright, Bradfield, & Cairns, 2013). Finally, in larger scenario problems, the intuitive reasoning of the interplay between the remaining factors can be cognitively too demanding for the human brain that is limited in the number of concepts it can process simultaneously (Hogarth, 1987).

To overcome the above issues, many authors suggest the use of decomposition methods in which scenarios are developed based on combinations of outcomes on several uncertainty factors instead of just two (Ritchey, 2006; Tietje, 2005; Wright et al., 2019). Then, scenario narratives are written on the basis of a small number (e.g., 3–5) of plausible and diverse combinations of such levels to create tangible depictions of the future (Schwartz, 1996). In these methods, the outcomes of the uncertainty factors are captured by multiple different *levels*, which convey detailed descriptions of a broad range of relevant and plausible development paths for these factors. The key element of decomposition methods is that the tasks of assessing the plausibility of different level combinations are broken down into smaller and cognitively less demanding subtasks, after which analytical methods are used to recompose the responses to these subtasks into overall plausibility assessments (Ritchey, 2006; Salo & Bunn, 1995; Tietje, 2005). This process of decomposition and subsequent recomposition enables circumventing the limited processing capacity of the human brain, thereby allowing a wider range of issues and interactions to be addressed (Wright et al., 2019).

Two widely used classes of such decomposition methods are Cross-impact methods (Brauers & Weber, 1988; Salo & Bunn, 1995) and Consistency analysis (Lord et al., 2016; Ritchey, 2006; Tietje, 2005; Vilkkumaa et al., 2018). Cross-impact methods translate the knowledge of an expert panel into marginal probabilities of the levels of the uncertainty factors and conditional probabilities reflecting causal dependencies between level pairs. These probability assessments are aggregated into estimates about the joint probabilities of combinations of levels. In this way, Cross-impact analysis supports the development of likely future scenarios based on probable level combinations. Nevertheless, the purpose of explorative scenarios is not to predict the most likely future conditions but rather to prepare for a variety of plausible futures, some of which may be relatively improbable. The identification of such plausible futures can be supported by Consistency analysis, which assesses more generic compatibilities of factor levels describing whether two levels can logically coexist in the same scenario. Then, the consistency of a combination of levels is obtained as, e.g., the sum of the consistencies between each pair of levels included in that combination (Tietje, 2005). According to Wiek et al. (2013), a consistent combination of levels represents a plausible scenario given that each level by itself represents a plausible outcome for the given uncertainty factor.

A challenge with Consistency analysis (and other decomposition methods) is that the number of level combinations grows exponentially with the number of uncertainty factors. For example, if there are 10 uncertainty factors that each have 4 different levels, there are $4^{10} \approx 1{,}000{,}000$ possible combinations, which cannot all be inspected by hand. In particular, even the evaluation of the consistencies of all level combinations to support the elimination of implausible ones can become computationally challenging – especially in workshop settings, where computations will need to be carried out on the fly. This issue is typically circumvented by either (i) limiting the number of considered uncertainty factors to a small manageable amount (Brauers & Weber, 1988; Lord et al., 2016) or (ii) using an exclusive consistency measure that discards large amounts of level combinations based on discrepancies between only one pair of factors (Johansen, 2018; Tietje, 2005). However, these approaches are problematic because in the first approach, impactful uncertainty factors may be completely omitted, whereas in the latter approach, interesting level combinations may be discarded due to a perceived inconsistency between a single pair of levels only. In this paper, we address this gap in the literature by developing efficient computational algorithms for quickly evaluating the consistencies of exponentially many level combinations.

In addition to supporting the identification of internally consistent level combinations of which there can be hundreds or thousands, some decomposition methods support the selection of a diverse set of a few (3–5) final combinations on the basis of which the scenario narratives are written. In such methods, the level combinations are represented by vectors whose elements indicate which uncertainty factor levels are included in each. Such a vector representation enables the use of optimization methods to find the

[1] Various names exist in the literature for the same concept, e,g., key factors (Bunn & Salo, 1993; Lempert et al., 2006), key uncertainties (Vilkkumaa et al., 2018), critical uncertainties (van der Heijden, 2005), critical factors (Brauers & Weber, 1988), and impact factors (Tietje, 2005). Our term of choice emphasizes the unpredictability of these conditions (uncertainty) and that these conditions can be decomposed into parts (factors).

final set of combinations. For instance, Tietje (2005) and Lord et al. (2016) present optimization methods for maximizing the distances between the final level combination vectors. However, such methods may recommend a set of combinations in which some uncertainty factor levels are completely omitted, thereby overlooking developments that were deemed relevant and plausible by the process participants. This issue can be avoided through the approach by Jenkins (1997), who formulates a minimum set cover problem for identifying the smallest number of combinations that together cover all levels. Nevertheless, because the formulation does not consider the consistencies of the final combinations, it may result in numerous optimal solutions, some of which can contain relatively inconsistent level combinations. In this paper, we present a *weighted* set cover formulation to find the smallest number of combinations that together cover all factor levels such that the consistencies of the least consistent combinations are maximized. These combinations can then be used to develop a diverse set of internally consistent scenarios.

The use of optimization methods provides structure and methodological rigor into the scenario planning exercise. Yet, a major challenge with such methods is related to their transparency and accessibility. For instance, McBride et al. (2017) argue that complex opaque modeling can limit stakeholder understanding and engagement in participatory scenario development, further creating a lack of trust in the developed scenarios and reducing the effectiveness of the scenario planning intervention. Thus, the excessive introduction of technical sophistication can be counterproductive. Nevertheless, clear and meaningful visualizations can facilitate understanding and create trust in the model results, as has been demonstrated in, e.g., the field of Multi-Criteria Decision Analysis (Genest & Zhang, 1996).

A direct visualization of the vectors representing different uncertainty factor level combinations is not possible, because these vectors are multi(>2)-dimensional. One common approach to examining the important properties of a high-dimensional data set is to use *dimensionality reduction methods* (Greenacre & Blasius, 2006). These methods display data vectors in a much lower-dimensional (e.g., 2-d) space that captures the most important properties of the data sets. As a result, the high-dimensional point cloud of a data set can be inspected in a two-dimensional plane using a scatter plot, and inferences can be made based on the distances of points in the projected clouds. An effective dimensionality reduction method that handles multi-dimensional categorical data, such as vectors of factor-specific levels, is Multiple Correspondence Analysis (MCA; Greenacre & Blasius, 2006). Yet, to our knowledge, the possibilities of MCA in visualizing combinations of uncertainty factor levels to support scenario development have not been explored.

In this paper, we develop analytic methods to support the identification and visualization of a diverse set of a few plausible scenarios for strategic planning. In particular, we build scenarios based on combinations of uncertainty factor levels, and use Consistency analysis to assess scenario plausibility. Then, we develop an explicit enumeration algorithm to solve large consistency value evaluation problems efficiently so that the most consistent (e.g., 1000 or 10,000) combinations of factor levels can be filtered out of the exponentially large set of all possible combinations. To support the selection of a small diverse subset from the set of the most consistent combinations, we formulate an optimization problem whose solutions give the smallest number of combinations that together cover all uncertainty factor levels. Furthermore, we present an MCA-based method for visualizing the set of most consistent level combinations. This method can be used either for communicating the results of an optimization-based solution, or as a flexible, stand-alone scenario identification tool. Finally, we present a real case in which the developed methods were applied to identify a set of plausible scenarios for the Finnish National Emergency Supply Organization to aid their strategic decision-making.

The contribution of this paper compared to existing literature is threefold. First, the method developed in the paper is the first to enable the efficient evaluation of the consistencies of all combinations of uncertainty factor levels without the need to limit the number of uncertainty factors to a small amount. This makes it possible to harness a wide variety of different perspectives and viewpoints represented by the participants in the scenario process. Thus, in contrast to critiques of analytical strategy methods as oversimplifying and lacking in creative imagination (e.g., March, 2006), our method can actually utilize the creative input of the process participants to a fuller extent than what would be possible without analytical tools. Second, this paper presents a novel optimization method for selecting a small but diverse set of consistent level combinations to be used as a basis for scenario development. This method enables the automation of the selection phase by guaranteeing that (i) all uncertainty factor levels are covered by the final set of level combinations and (ii) the least consistent combinations are as consistent as possible. Finally, this paper is the first to use MCA for visualizing the set of consistent scenarios, thereby increasing the transparency and accessibility of the methods to stakeholders with no background in mathematical fields. Together, these three contributions help bridge the gap between the structure and rigor introduced by analytic methods, and the creativity and critical thinking of human experts in scenario development.

2. Method for identifying a diverse set of plausible scenarios

Scenarios are developed based on combinations of plausible levels of different uncertainty factors representing, e.g., political, technological, or economic developments. These uncertainty factors are denoted by indices $j = 1, \dots, J$. The level of uncertainty factor j is represented by a *categorical variable* $m_j \in \mathcal{M}_j = \{1, \dots, M_j\}$, where $\mathcal{M}_j$ is the set of plausible levels and M_j is the number of such levels for uncertainty factor j. A combination of uncertainty factor levels is represented by vector $\mathbf{s} = \begin{bmatrix} m_1 & \cdots & m_J \end{bmatrix} \in \mathcal{S}$, where $\mathcal{S} = \mathcal{M}_1 \times \cdots \times \mathcal{M}_J$ is the set of all possible level combinations with cardinality $I = |\mathcal{S}| = \prod_{j=1}^{J} M_j$.

As an example, consider a hypothetical case on building scenarios for a Finnish electricity sales company. The company has recognized six relevant uncertainty factors affecting its future business environment: Focus of energy regulation, Electricity price, Competitive field, Customer churn rate, Technology & digitalization, and Finnish economy. Each of these six uncertainty factors has three to four plausible levels, which are shown in Table 1 in a format referred to as the *morphological field* (Ritchey, 2006). There are altogether $I = \prod_{j=1}^{6} M_j = 4^3 \cdot 3^3 = 1{,}728$ possible level combinations in set $\mathcal{S} = \bigtimes_{j=1}^{3} \{1, 2, 3, 4\} \times \bigtimes_{j=4}^{6} \{1, 2, 3\}$. The highlighted cells in Table 1 illustrate one possible combination $\mathbf{s} = \begin{bmatrix} 4 & 3 & 4 & 3 & 2 & 3 \end{bmatrix}$.

The set $\mathcal{S}$ can be represented by matrix $\mathbf{S}$, the rows of which are the elements of $\mathcal{S}$ ordered lexicographically. In a lexicographic order, $\mathbf{s} = \begin{bmatrix} m_1 & \cdots & m_J \end{bmatrix} <^{\text{lex}} \mathbf{s}' = \begin{bmatrix} m_1' & \cdots & m_J' \end{bmatrix}$ if and only if the first element in which the two vectors differ is smaller in $\mathbf{s}$ than in $\mathbf{s}'$.

Definition 1. Let $\mathcal{S} = \{\mathbf{s}_1, \dots, \mathbf{s}_I\}$ be the set of all possible level combinations, where $\mathbf{s}_1 <^{\text{lex}} \cdots <^{\text{lex}} \mathbf{s}_I$ are ordered lexicographically. Then, matrix $\mathbf{S}$ is defined as

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_I \end{bmatrix} = \begin{bmatrix} m_{ij} \end{bmatrix} \in \mathbb{N}^{I \times J},$$

Table 1
Example of uncertainty factors and their levels. Levels of one combination are highlighted.

A. Energy regulation's focus	B. Electricity price	C. Competitive field	D. Customer churn rate	E. Technology & digitalization	F. Finnish economy
1. Environment & renewable energy	1. Low, under 30 e/MWh	1. Traditional: private & municipal	1. Low, under 8 %/year	1. Digital evolution	1. Deep recession
2. Energy security & reliability	2. Moderate, 30-45 e/MWh	2. Consolidation	2. Moderate, 9-14 %/year	2. Fast digitalization	2. Zero growth
3. Market-based energy industry	3. High, over 45 e/MWh	3. International competitive field	3. High, over 15 %/year	3. Digital revolution	3. Strong growth
4. Consumer-oriented & decentralized	4. Turbulent, 0-200 e/MWh	4. New external players			

where m_{ij} is the level of uncertainty factor j in combination i.

For instance, this matrix for the electricity sales company example is

$$\mathbf{S} = \begin{array}{cc} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 2 & 4 & 4 & 3 & 3 & 2 \\ 2 & 4 & 4 & 3 & 3 & 3 \\ 3 & 1 & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 4 & 4 & 4 & 3 & 3 & 2 \\ 4 & 4 & 4 & 3 & 3 & 3 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ 863 \\ 864 \\ 865 \\ \vdots \\ 1{,}727 \\ 1{,}728 \end{array} \end{array}. \quad (1)$$

The goal of the scenario identification method of this paper is to find a small set $\mathcal{S}^{\mathrm{f}} \subset \mathcal{S}$ of K (typically $K = 4$ or 5) final level combinations which are (i) plausible in that the levels in each combination are consistent with one another and (ii) diverse in that taken together, these combinations attain all factor levels. In Section 2.1, we show how to evaluate the plausibility of level combinations by using Consistency analysis, and in Section 2.2 we formulate an optimization problem for finding the smallest possible subset of the most consistent combinations such that this set covers all factor levels. Efficient algorithms for identifying the most consistent combinations are presented in Section 3.

2.1. Consistency analysis for evaluating the plausibility of level combinations

In Consistency analysis, the plausibility of a combination of uncertainty factor levels is assessed by rating the consistencies between each pair of levels. Here, the consistencies are rated on a linear 7-point scale $\mathcal{C} = \{-3, -2, -1, 0, 1, 2, 3\}$: the higher the consistency value, the more plausibly these factor levels are seen to coexist in the same scenario (see, e.g., Scholz & Tietje 2001 or Brauers & Weber 1988 about consistency estimation). For cases in which all factors have only two or three levels, the more commonly used 5-point scale $\mathcal{C}' = \{-2, -1, 0, 1, 2\}$ is sufficient (see, e.g., Tietje 2005; Wiek, Gasser, & Siegrist 2009). A linear scale is compensating in that an inconsistent pair of levels with a negative consistency value may be compensated by several consistent level pairs with positive consistency values. If an exclusive rating is deemed more appropriate (in that a single or few pairs of inconsistent uncertainty factor levels should render the entire combination inconsistent), then a non-linear scale may be used, e.g., $\mathcal{C}' = \{-\infty, -1, 0, 1, 1.5\}$.

More formally, let $c_{j_1 j_2}(m_{j_1}, m_{j_2}) \in \mathcal{C}$ denote the consistency between levels (m_{j_1}, m_{j_2}) of uncertainty factors $j_1 = 1, \ldots, J-1$ and $j_2 = 2, \ldots, J$, $j_1 < j_2$. Each uncertainty factor pair j_1, j_2 is associated with a matrix $\mathbf{C}_{j_1 j_2} = \left[c_{j_1 j_2}(m_{j_1}, m_{j_2})\right] \in \mathcal{C}^{M_{j_1} \times M_{j_2}}$ of pairwise consistency values, where m_{j_1} and m_{j_2} refer to the row and column indices of matrix $\mathbf{C}_{j_1 j_2}$, respectively. Then, the pairwise consistency matrices can be aggregated into a *consistency table* $\mathbf{C}$, which due to the symmetry of the consistency indicator can be reduced to an upper triangular matrix of $J(J-1)/2$ blocks $\mathbf{C} = \left[\mathbf{C}_{j_1 j_2}\right] \in \mathcal{C}^{(M-M_J) \times (M-M_1)}$, $j_1 < j_2$.

Table 2 shows the consistency table for our example on building scenarios for an electricity sales company. For uncertainty factors $j_1 = 3$ (C. Competitive field) and $j_2 = 4$ (D. Customer churn rate), levels $m_3 = 4$ (New players from different industries) and $m_4 = 3$ (High, over 15%/year) are found to be strongly consistent, as reflected by a high pairwise consistency value $c_{34}(4, 3) = 3$. On the other hand, levels $m_3 = 3$ (International competitive field) and $m_4 = 1$ (Low, under 8%/year) are seen as strongly inconsistent, whereby $c_{34}(3, 1) = -3$.

The overall consistency $\bar{c}(\mathbf{s})$ of level combination $\mathbf{s}$ is the arithmetic mean[2] of the pairwise consistencies between all factor levels in the combination:

$$\bar{c}(\mathbf{s}) = \frac{2}{J(J-1)} \sum_{j_1=1}^{J-1} \sum_{j_2=j_1+1}^{J} c_{j_1 j_2}(m_{j_1}, m_{j_2}). \quad (2)$$

Let $\mathbf{c}_{j_1 j_2} = [c_{j_1 j_2}(m_{1 j_1}, m_{1 j_2}) \; c_{j_1 j_2}(m_{2 j_1}, m_{2 j_2}) \; \cdots \; c_{j_1 j_2}(m_{I j_1}, m_{I j_2})]^{\mathrm{T}} \in \mathcal{C}^I$ be the vector of pairwise consistencies between columns j_1, j_2 of matrix $\mathbf{S} = \left[m_{ij}\right]$ in Definition 1. Then, the vector of all overall consistencies $\bar{\mathbf{c}}$ can be obtained as

$$\bar{\mathbf{c}} = \left[\bar{c}_1 \cdots \quad \bar{c}_I\right]^{\mathrm{T}} = \left[\bar{c}(\mathbf{s}_1) \cdots \quad \bar{c}(\mathbf{s}_I)\right]^{\mathrm{T}} = \frac{2}{J(J-1)} \sum_{j_1=1}^{J-1} \sum_{j_2=j_1+1}^{J} \mathbf{c}_{j_1 j_2}. \quad (3)$$

To give a simple illustration of the calculation of overall consistencies, we focus on three uncertainty factors only: C. Competitive field, D. Customer churn rate, and E. Digitalization and technology. The consistencies between these factors are bordered by a blue rectangle in Table 2. Fig. 1 shows the corresponding matrix $\mathbf{S}$ and vector of overall consistencies $\bar{\mathbf{c}}$. For example, the overall consistency of level combination $\mathbf{s}_1 = \left[1 \quad 1 \quad 1\right]$ is $\bar{c}_1 = 2/[3(3-1)](3+3+3) = 3$, indicating that this combination (traditional competitive field with a low customer churn rate and digital evolution; cf. Table 1) consists of mutually compatible levels. On the other hand, the consistency of combination $\mathbf{s}_{28} = \left[4 \quad 1 \quad 1\right]$ is $\bar{c}_{28} = 2/[3(3-1)](-3-3+3) = -1$, implying that the factor levels in this combination (where new external players enter the market but both the customer churn rate and rate of digitalization are relatively low) are in conflict with one another.

[2] Some authors (e.g., Tietje 2005) advocate using multiplicative aggregation; yet, a multiplicative consistency measure can be transformed into an additive one by simply applying the logarithm.

$$
\mathbf{S}=\begin{bmatrix}1&1&1\\1&1&2\\1&1&3\\1&2&1\\1&2&2\\1&2&3\\1&3&1\\1&3&2\\1&3&3\\2&1&1\\2&1&2\\2&1&3\\2&2&1\\2&2&2\\2&2&3\\2&3&1\\2&3&2\\2&3&3\\3&1&1\\3&1&2\\3&1&3\\3&2&1\\3&2&2\\3&2&3\\3&3&1\\3&3&2\\3&3&3\\4&1&1\\4&1&2\\4&1&3\\4&2&1\\4&2&2\\4&2&3\\4&3&1\\4&3&2\\4&3&3\end{bmatrix},\qquad
\bar{\mathbf{c}}=\frac{1}{3}\left(\mathbf{c}_{12}+\mathbf{c}_{13}+\mathbf{c}_{23}\right)=\frac{1}{3}\left(\begin{bmatrix}3\\3\\3\\2\\2\\2\\-1\\-1\\-1\\2\\2\\2\\2\\2\\2\\2\\2\\2\\-3\\-3\\-3\\1\\1\\1\\2\\2\\2\\-3\\-3\\-3\\1\\1\\1\\3\\3\\3\end{bmatrix}+\begin{bmatrix}3\\1\\1\\3\\1\\1\\3\\1\\1\\2\\1\\1\\2\\1\\1\\2\\1\\1\\-1\\2\\3\\-1\\2\\3\\-1\\2\\3\\-3\\2\\3\\-3\\2\\3\\-3\\2\\3\end{bmatrix}+\begin{bmatrix}3\\2\\-2\\2\\3\\1\\-2\\2\\2\\3\\2\\-2\\2\\3\\1\\-2\\2\\2\\3\\2\\-2\\2\\3\\1\\-2\\2\\2\\3\\2\\-2\\2\\3\\1\\-2\\2\\2\end{bmatrix}\right)=\begin{array}{cc}\begin{bmatrix}3\\2\\0.67\\2.33\\2\\1.33\\0\\0.67\\0.67\\2.33\\1.67\\0.33\\2\\2\\1.33\\0.67\\1.67\\1.67\\-0.33\\0.33\\-0.67\\0.67\\2\\1.67\\-0.33\\2\\2.33\\-1\\0.33\\-0.67\\0\\2\\1.67\\-0.67\\2.33\\2.67\end{bmatrix} & \begin{matrix}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\\19\\20\\21\\22\\23\\24\\25\\26\\27\\28\\29\\30\\31\\32\\33\\34\\35\\36\end{matrix}\end{array}
$$

Fig. 1. Matrix **S** of all level combinations and the calculation of overall consistencies $\bar{\mathbf{c}}$ for factors C, D, and E of Table 2.

Table 2
Consistency indicators of the electricity sales example.

		B				C				D			E			F		
		1	2	3	4	1	2	3	4	1	2	3	1	2	3	1	2	3
A	1	1	0	0	1	0	1	0	1	-1	-1	0	-2	1	2	-2	0	3
	2	0	-1	-1	1	2	1	-2	-2	1	-1	-1	0	0	0	1	0	1
	3	0	-1	0	1	1	0	2	1	-1	0	1	-2	1	2	2	0	3
	4	-1	-1	1	1	1	0	0	1	0	0	1	0	2	1	2	0	2
B	1					-2	2	-2	1	3	1	-2	1	2	3	3	2	-2
	2					1	1	1	1	1	1	1	2	1	1	1	1	2
	3					1	-2	2	1	-2	2	3	1	1	-3	-1	-1	3
	4					0	0	0	1	-2	1	2	3	2	-2	0	0	0
C	1									3	2	-1	3	1	1	-2	1	2
	2									2	2	2	2	1	1	3	2	1
	3									-3	1	2	-1	2	3	-2	1	2
	4									-3	1	3	-3	2	3	-1	1	2
D	1												3	2	-2	0	0	0
	2												2	3	1	0	0	0
	3												-2	2	2	2	0	0
E	1															3	2	-2
	2															-1	2	2
	3															-3	-1	2

A level combination represents a plausible scenario if (i) each level included in the combination represents a plausible outcome for the given uncertainty factor and (ii) the overall consistency of the combination is relatively high (Wiek et al., 2013). Assuming that the first condition is satisfied by how the levels have been defined, plausible scenarios can be developed by focusing on the set $\mathcal{S}^*$ of $N \ll I$ (e.g., $N = 1{,}000$ or $10{,}000$, depending on the number I of all combinations) of most consistent level combinations, defined as follows.

Definition 2. Let $\mathcal{S}$ be the set of all possible level combinations and $\bar{c}(\mathbf{s})$ be the consistency of combination $\mathbf{s} \in \mathcal{S}$ as in (2). Set $\mathcal{S}^*$ of the N most consistent level combinations is such that $\bar{c}(\mathbf{s}^*) \geq \bar{c}(\mathbf{s}) \forall \mathbf{s}^* \in \mathcal{S}^*, \mathbf{s} \in \mathcal{S} \setminus \mathcal{S}^*$ and $|\mathcal{S}^*| = N$. The elements $\mathbf{s}_n^* \in \mathcal{S}^*$ are indexed such that $\bar{c}(\mathbf{s}_1^*) \geq \cdots \geq \bar{c}(\mathbf{s}_N^*)$.

The vector of consistencies of the most consistent level combinations is denoted by

$$\bar{\mathbf{c}}^* = \begin{bmatrix}\bar{c}_1^* & \cdots & \bar{c}_N^*\end{bmatrix}^{\mathrm{T}} = \begin{bmatrix}\bar{c}(\mathbf{s}_1^*) & \cdots & \bar{c}(\mathbf{s}_N^*)\end{bmatrix}^{\mathrm{T}}. \tag{4}$$

For example, the set $\mathcal{S}^*$ of the $N = 6$ most consistent combinations among those shown in Fig. 1 and the vector $\bar{\mathbf{c}}^*$ of the consistencies of these combinations are

$$\begin{aligned}\mathcal{S}^* &= \Big\{\begin{bmatrix}1&1&1\end{bmatrix},\begin{bmatrix}4&3&3\end{bmatrix},\begin{bmatrix}1&2&1\end{bmatrix},\begin{bmatrix}2&1&1\end{bmatrix},\\ &\qquad \begin{bmatrix}3&3&3\end{bmatrix},\begin{bmatrix}4&3&2\end{bmatrix}\Big\} \text{ and}\\ \bar{\mathbf{c}}^* &= \begin{bmatrix}3&2.67&2.33&2.33&2.33&2.33\end{bmatrix}^{\mathrm{T}}.\end{aligned} \tag{5}$$

Set $\mathcal{S}^*$ of the most consistent level combinations contains attractive candidates to be used as a basis for the final scenarios, because the combinations in this set facilitate the development of plausible scenario narratives. Importantly, no efforts are made at this point to assess the probabilities of these combinations to mitigate the risk of gravitating towards business-as-usual scenarios. In fact, if the participants assessing the pairwise consistencies between factor levels are encouraged to explore even unusual relationships between these levels, set $\mathcal{S}^*$ may contain combinations that, prior to the process, would have seemed surprising or highly unusual.

2.2. Identification of a diverse set of consistent level combinations

Once the set $\mathcal{S}^*$ of the N most consistent level combinations has been obtained, the goal is to identify a small subset $\mathcal{S}^{\mathrm{f}}$ of K final combinations which is diverse in that these combinations cover all factor levels (excluding those levels that are not included in any of the N most consistent combinations). In this section, we formulate the selection of this final set as a weighted set cover problem with a lexicographic weighting function.

For this purpose, we first define M'_j as the number of levels that are attained for uncertainty factor j by at least one of the N most consistent level combinations. Then, we map the levels $m^*_{nj} \in \{1,\dots,M_j\}$ of each combination $\mathbf{s}^*_n = \begin{bmatrix} m^*_{n1} & \cdots & m^*_{nJ} \end{bmatrix}$ to $\{1,\dots,M'_j\}$. Finally, denoting the total number of such levels by $M = \sum_{j=1}^{J} M'_j$, we construct an $N \times M$ matrix $\mathbf{Z}$ with elements $z_{nm} \in \{0,1\}$ that indicate whether the nth consistent combination attains the level corresponding to column index m of this matrix. For instance, the indicator matrix corresponding to set $\mathcal{S}^*$ in (5) is

$$\mathbf{Z} = \left[\begin{array}{cccc|ccc|ccc} 1&0&0&0&1&0&0&1&0&0\\ 0&0&0&1&0&0&1&0&0&1\\ 1&0&0&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&0&1&0&0\\ 0&0&1&0&0&0&1&0&0&1\\ 0&0&0&1&0&0&1&0&1&0 \end{array}\right]. \tag{6}$$

Formally, the indicator matrix $\mathbf{Z}$ is defined as follows.

Definition 3. Let $\mathbf{s}^*_n = \begin{bmatrix} m^*_{n1} & \cdots & m^*_{nJ} \end{bmatrix}$ be the elements of $\mathcal{S}^*$ as in Definition 2. Let $M'_j = \sum_{m=1}^{M_j} [\exists n' \in \{1,\dots,N\} : m^*_{n'j} = m]$, where $[P] \in \{0,1\}$ is the Iverson bracket the value of which is 1 if and only if P is true. Let $M = \sum_{j=1}^{J} M'_j$ and let $m'_{nj} = \sum_{m=1}^{m^*_{nj}} [\exists n' \in \{1,\dots,N\} : m^*_{n'j} = m] \in \{1,\dots,M'_j\}$. The *indicator matrix* of the N most consistent level combinations is $\mathbf{Z} = \begin{bmatrix} z_{nm} \end{bmatrix} \in \{0,1\}^{N\times M}$, where element $z_{nm} = 1$ for all $m = m'_{nm} + \sum_{u=1}^{j-1} M'_u$, $j = 1,\dots,J$, and $z_{nm} = 0$ otherwise.

In the common and more convenient case where all factor levels are attained by at least one consistent combination, we have $M'_j = M_j$, whereby $z_{nm} = 1$ for all $m = m^*_{nj} + \sum_{u=1}^{j-1} M_u$, $j = 1,\dots,J$. For notational convenience, we henceforth only consider such cases.

Using matrix $\mathbf{Z}$, we may formulate the weighted set cover problem for finding the final set $\mathcal{S}^{\mathrm{f}}$ of level combinations as:

$$\min_{\mathbf{x}\in\{0,1\}^N} \mathbf{w}(\bar{\mathbf{c}}^*)^{\mathrm{T}}\mathbf{x} \qquad \text{s.t.} \quad \mathbf{Z}^{\mathrm{T}}\mathbf{x} \geq \mathbf{1}, \tag{7}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & \cdots & x_N \end{bmatrix}^{\mathrm{T}} \in \{0,1\}^I$ is a vector of decision variables representing the final set of combinations such that $x_n = 1$ if and only if combination $\mathbf{s}^*_n$ is included in $\mathcal{S}^{\mathrm{f}}$, and $\mathbf{w}(\bar{\mathbf{c}}^*) = \begin{bmatrix} w(\bar{c}^*_1) & \cdots & w(\bar{c}^*_N) \end{bmatrix}^{\mathrm{T}}$ is a weighting function in which the weight $w(\bar{c}^*_n)$ of combination $\mathbf{s}^*_n$ depends on its consistency $\bar{c}^*_n$.

In problem (7), constraint $\mathbf{Z}^{\mathrm{T}}\mathbf{x} \geq \mathbf{1}$ ensures that in the final set of level combinations corresponding to solution $\mathbf{x}$, each uncertainty factor level is represented. Preferences between feasible solutions $\mathbf{x}$ satisfying this constraint are captured by the weighting function $\mathbf{w}(\bar{\mathbf{c}}^*)$. Here, we specifically look for a solution in which (i) there are as few combinations as possible and (ii) the consistencies of the least consistent combinations are as high as possible. Let us denote the minimum number of combinations that must be included in a feasible solution by $\underline{K} = \max_{j=1,\dots,J} M_j$. To satisfy condition (i), function $\mathbf{w}(\bar{\mathbf{c}}^*)$ should be such that if there exists a feasible solution to problem (7) with $\underline{K}$ combinations, then the optimal solution has $\underline{K}$ combinations. Consider two feasible solutions $\mathbf{x}$ and $\mathbf{x}'$, both with the lowest possible number $\underline{K}$ of combinations. To reflect condition (ii), we assume that preference between solutions $\mathbf{x}$ and $\mathbf{x}'$ is determined by a lexicographic ordering between vectors $\bar{\mathbf{c}}^{\mathrm{f}}$ and $\bar{\mathbf{c}}^{\mathrm{f}\prime}$ of overall consistencies corresponding to these solutions. In particular, solution $\mathbf{x}'$ is preferred to solution $\mathbf{x}$ if the lowest consistency in $\bar{\mathbf{c}}^{\mathrm{f}\prime}$ is higher than the lowest consistency in $\bar{\mathbf{c}}^{\mathrm{f}}$; or, if there is a tie, the second lowest consistency in $\bar{\mathbf{c}}^{\mathrm{f}\prime}$ is higher than that in $\bar{\mathbf{c}}^{\mathrm{f}}$, and so on. More formally, this preference is defined as follows.

Definition 4. Let $\mathbf{x}$ and $\mathbf{x}'$ be feasible solutions to problem (7) such that $\sum_{n=1}^{N} x_n = \sum_{n=1}^{N} x'_n = \underline{K}$. Let $\bar{\mathbf{c}}^{\mathrm{f}} = \begin{bmatrix} \bar{c}^{\mathrm{f}}_1 & \cdots & \bar{c}^{\mathrm{f}}_K \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \bar{c}^*_{n^{\mathrm{f}}_1} & \cdots & \bar{c}^*_{n^{\mathrm{f}}_K} \end{bmatrix}^{\mathrm{T}}$ be the vector of consistencies of the final level combinations in an increasing order, where $\bar{c}^*_n$ are as in (4) and $\{n^{\mathrm{f}}_1,\dots,n^{\mathrm{f}}_K\} = \{n \in \{1,\dots,N\} \mid x_n = 1\}$ such that $n^{\mathrm{f}}_1 \geq \cdots \geq n^{\mathrm{f}}_K$. Let $\bar{\mathbf{c}}^{\mathrm{f}\prime}$ be defined similarly for $\mathbf{x}'$. Then, $\mathbf{x}'$ is (strictly) preferred to $\mathbf{x}$, denoted by $\mathbf{x}' \succ \mathbf{x}$, if and only if $\bar{\mathbf{c}}^{\mathrm{f}\prime} >^{\mathrm{lex}} \bar{\mathbf{c}}^{\mathrm{f}}$.

The lexicographic preference ordering of Definition 4 together with the requirement that there are $K = \underline{K}$ combinations in the final set can be captured by defining the weight for combination $\mathbf{s}^*_n$ in (7) as $w(\bar{c}^*_n) = \underline{K} + \underline{K}^{1-q(\bar{c}^*_n)}$, where $q(\bar{c}^*_n)$ is equal to q given that combination $\mathbf{s}^*_n$ has the qth lowest unique consistency value among $\mathcal{S}^*$. This result is formalized in the following Theorem. All proofs are in Appendix A in the supplementary material.

Theorem 1. *Let* $\bar{\mathbf{c}}^* = \begin{bmatrix} \bar{c}^*_1 & \cdots & \bar{c}^*_N \end{bmatrix}^{\mathrm{T}}$ *as in (4), let* $\bar{c}^*_{(q)}$ *be the qth order statistic of the set* $\{\bar{c}^*_n\}_{n=1}^{N}$ *of unique overall consistencies, and let* $q(\bar{c}^*_n) = q \in \mathbb{N} \mid \bar{c}^*_{(q)} = \bar{c}^*_n$. *Let* $\underline{K} = \max_{j=1,\dots,J} M_j$. *Let us define the lexicographic weighting function in problem (7) as*

$$w : \{\bar{c}^*_n\}_{n=1}^{N} \to (\underline{K}, \underline{K}+1], \quad w(\bar{c}^*_n) = \underline{K} + \underline{K}^{1-q(\bar{c}^*_n)}.$$

If there is a feasible solution $\mathbf{x}$ *to problem (7) such that* $\sum_{n=1}^{N} x_n = \underline{K}$, *then (i) the optimal solution* $\underline{\mathbf{x}}$ *to problem (7) has* $\sum_{n=1}^{N} \underline{x}_n = \underline{K}$ *combinations, and (ii) there does not exist another feasible solution* $\mathbf{x}$ *with* $\sum_{n=1}^{N} x_n = \underline{K}$ *such that* $\mathbf{x} \succ \underline{\mathbf{x}}$, *where* $\succ$ *is as in Definition 4.*

As an example, consider again the electricity sales company case with a focus on three uncertainty factors only (cf. Eqs. (5) and (6)). Here, the smallest number of combinations that can together attain all the uncertainty factor levels is $\underline{K} = \max_{j\in\{1,2,3\}} M_j = \max\{4,3,3\} = 4$. Consequently, the lexicographic weights of Theorem 1 are $w(\bar{c}^*_1) = \underline{K} + \underline{K}^{1-q(\bar{c}^*_1)} = 4 + 4^{1-3} = 4.125$, $w(\bar{c}^*_2) = 4 + 4^{1-2} = 4.25$, and $w(\bar{c}^*_n) = 4 + 4^{1-1} = 5$ for $n = 3,\dots,6$. The optimal solution $\underline{\mathbf{x}}$ to problem (7) gives a minimum set cover of $\underline{K} = 4$ combinations $\mathbf{s}^{\mathrm{f}}_1 = \begin{bmatrix} 4 & 3 & 2 \end{bmatrix}$, $\mathbf{s}^{\mathrm{f}}_2 = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix}$, $\mathbf{s}^{\mathrm{f}}_3 = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}$ and $\mathbf{s}^{\mathrm{f}}_4 = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ with consistencies $\bar{\mathbf{c}}^{\mathrm{f}} = \begin{bmatrix} 2.33 & 2.33 & 2.33 & 2.33 \end{bmatrix}^{\mathrm{T}}$.

An attractive feature of selecting the weighting function as in Theorem 1 is that the optimal solution to the weighted set cover problem is stable with respect to increasing the size of the set of the most consistent combinations. More formally, the optimal solution $\underline{\mathbf{x}}$ for set $\mathcal{S}^*$ of size N is also optimal[3] for sets $\mathcal{S}^{*+}$ of size $N^+ = N+1, N+2,\dots,I$. This is because $\underline{\mathbf{x}}$ maximizes the consistency of the least consistent combination selected from $\mathcal{S}^*$ to be

[3] There is one possible exception to this stability: Suppose that the least consistent combinations in $\mathcal{S}^{\mathrm{f}}$ and $\mathcal{S}^*$ have equal consistency $\bar{c}'$ and that there is a combination $\mathbf{s}' \notin \mathcal{S}^*$ with $\bar{c}(\mathbf{s}') = \bar{c}'$. Then $\mathbf{s}'$ might belong to a subset of all combinations $\mathcal{S}^{\mathrm{f}\prime} \not\subset \mathcal{S}^*$ which would be preferred to $\mathcal{S}^{\mathrm{f}}$ in the lexicographic sense. This exception is easy to avoid by having a sufficiently high N so that the least consistent combinations in $\mathcal{S}^{\mathrm{f}}$ have a higher consistency than those in $\mathcal{S}^*$.

included in the final set $\mathcal{S}^{\mathrm{f}}$, and because the consistency of any combination not included in $\mathcal{S}^*$ can be at most as high as that of a combination in $\mathcal{S}^*$. Consider, for instance, a problem of selecting the final set of level combinations for the electricity sales company example from a set of $N^+ = 13$ most consistent combinations. Because the set of the N^+ most consistent combinations $\mathcal{S}^{*+}$ is obtained by adding seven strictly less consistent combinations with $\bar{c}_i = 2.00$ to set $\mathcal{S}^*$, the optimal set of combinations from $\mathcal{S}^{*+}$ is equal to that obtained from set $\mathcal{S}^*$. Another feasible solution to problem (7) for $N^+ = 13$ comprises combinations $\mathbf{s}_1^{\mathrm{f}} = \begin{bmatrix}3 & 3 & 2\end{bmatrix}, \mathbf{s}_2^{\mathrm{f}} = \begin{bmatrix}2 & 2 & 1\end{bmatrix}, \mathbf{s}_3^{\mathrm{f}} = \begin{bmatrix}4 & 3 & 3\end{bmatrix}$, and $\mathbf{s}_1^{\mathrm{f}} = \begin{bmatrix}1 & 1 & 1\end{bmatrix}$ with $\bar{\mathbf{c}}^{\mathrm{f}} = \begin{bmatrix}2 & 2 & 2.67 & 3\end{bmatrix}^{\mathrm{T}}$. This set of combinations has a higher mean consistency (2.42) than the optimal set (2.33) but, on the other hand, a lower consistency value for the least consistent combination (2 compared to 2.33). Preference towards maximizing the minimum instead of the mean consistency is reflected by a lower objective function value:

$$q(\bar{c}_n^*) = \begin{cases} 4, & \text{if } \bar{c}_n^* = 3, \\ 3, & \text{if } \bar{c}_n^* = 2.67, \\ 2, & \text{if } \bar{c}_n^* = 2.33, \\ 1, & \text{if } \bar{c}_n^* = 2. \end{cases} \Rightarrow \mathbf{w}(\bar{\mathbf{c}}^*)^{\mathrm{T}}\mathbf{x} = \begin{cases} 4\left(\underline{K} + \underline{K}^{1-2}\right) = 17, & \text{for } \mathcal{S}^{\mathrm{f}} = \left\{\begin{matrix}[4\ 3\ 2], & [3\ 3\ 3], \\ [2\ 1\ 1], & [1\ 2\ 1]\end{matrix}\right\}, \\ 4\underline{K} + \underline{K}^{-3} + \underline{K}^{-2} + 2 = 18.08, & \text{for } \mathcal{S}^{\mathrm{f}} = \left\{\begin{matrix}[3\ 3\ 2], & [2\ 2\ 1], \\ [4\ 3\ 3], & [1\ 1\ 1]\end{matrix}\right\}. \end{cases}$$

Due to this stability of the optimal solution, the optimization problem may first be solved for a relatively small set $\mathcal{S}^*$, after which the size N of this set can be increased if no solution with $K = \underline{K}$ combinations can be found. Once a solution with $K = \underline{K}$ combinations is obtained, we can be certain of its optimality with respect to the entire set of all possible level combinations without the need to examine any more combinations.

3. Computation of the most consistent level combinations

Obtaining the set $\mathcal{S}^*$ of the N most consistent level combinations requires evaluating all I overall consistencies $\bar{\mathbf{c}}$. In a large scenario planning case, this evaluation may be computationally intensive: for example, if there are $J = 10$ uncertainty factors with $M_j = 4$ factor levels each, then $[J(J-1)/2]\prod_{j=1}^{J} M_j = 47{,}185{,}920$ pairwise consistency values need to be evaluated. Without proper algorithms for carrying out these evaluations, it is not possible to find the final set of level combinations in a workshop setting, nor to iteratively search for a suitable number N of the most consistent combinations.

In this section, we develop an explicit enumeration algorithm that can find the set $\mathcal{S}^*$ of the N most consistent level combinations efficiently. In particular, we first show how to implement the vectorized computation of the overall consistencies $\bar{\mathbf{c}}$ in (3). Then, we divide the complete consistency value enumeration problem into subproblems of smaller size to help mitigate memory bandwidth issues related to processing large arrays. Third, we show how the row indices of matrix $\mathbf{S}$ corresponding to the N most consistent combinations can be determined from the indices of the N most consistent combinations in each subproblem. Finally, we show how these row indices can be used to determine the set $\mathcal{S}^*$.

The recursive algorithm for evaluating overall consistencies for the level combinations is presented in Algorithm 1, where $\mathbf{M} = \begin{bmatrix}M_1 & \cdots & M_J\end{bmatrix}$. In this algorithm, the consistency value repetitions are presented using vector concatenation for illustrative purposes; in programmatic implementations, however, these should be done by using arrays with preallocated memory.

In large scenario planning cases, vectors of pairwise and overall consistencies do not fit the cache of conventional computers, which may cause memory bandwidth issues in the application of Algorithm 1. To mitigate these issues, we partition the complete consistency value enumeration problem into smaller cache fitting subproblems. This partition is done by fixing the levels of the first $\eta \in \{0, \ldots, J-1\}$ factors, which can be done in $H = \prod_{j=1}^{\eta} M_j$ different ways. In each subproblem $h = 1, \ldots, H$, the number of uncertainty factor level combinations for the remaining $J - \eta$ factors is $L = \prod_{j=\eta+1}^{J} M_j$. Thus, instead of running Algorithm 1 to evaluate the consistencies of all $I = \prod_{j=1}^{J} M_j$ level combinations at once, we may run it in parallel for H subproblems that comprise L combinations each.

To formalize this approach, we note that matrix $\mathbf{S}$ can be expressed with the help of matrices $\mathbf{S}^{j\le\eta}$ and $\mathbf{S}^{j>\eta}$ corresponding to the first η and the remaining $J - \eta$ factors.

Lemma 1. *Let $\eta \in \{1, \ldots, J-1\}$, and let $\mathbf{S}^{j\le\eta} = \left[m_{hj}^{j\le\eta}\right] \in \mathbb{N}^{H\times\eta}$ and $\mathbf{S}^{j>\eta} = \left[m_{\ell j}^{j>\eta}\right] \in \mathbb{N}^{L\times(J-\eta)}$ be matrices constructed from sets of combinations $\mathcal{S}^{j\le\eta} = \times_{j=1}^{\eta}\mathcal{M}_j$ and $\mathcal{S}^{j>\eta} = \times_{j=\eta+1}^{J}\mathcal{M}_j$ according to*

Definition 1. Then, matrix $\mathbf{S}$ for the set of all possible combinations $\mathcal{S}$ can be expressed as

$$\mathbf{S} = \begin{bmatrix}\mathbf{A}\mathbf{S}^{j\le\eta} & \mathbf{B}\mathbf{S}^{j>\eta}\end{bmatrix}, \quad \text{where}$$

$$\mathbf{A} = \underbrace{\begin{bmatrix} \mathbf{1}_L & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{1}_L & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{1}_L \end{bmatrix}}_{H \text{ columns}} \text{ and } \quad \mathbf{B} = \left.\begin{bmatrix} \mathbf{I}_L \\ \mathbf{I}_L \\ \vdots \\ \mathbf{I}_L \end{bmatrix}\right\} H \text{ matrices}$$

such that $\mathbf{1}_L \in \mathbb{R}^{L\times 1}$ is a unit vector and $\mathbf{I}_L \in \mathbb{R}^{L\times L}$ an identity matrix.

Algorithm 1 A recursive algorithm for evaluating overall consistencies for level combinations.

Input: $\mathbf{M}, \mathbf{C}, j_1$ ▷ Numbers of levels, Consistency table, current recursion depth
Output: $\bar{\mathbf{c}}$ ▷ Overall consistencies in the current recursion (unscaled)

function OverallConsistencies($\mathbf{M}, \mathbf{C}, j_1$)
  Initialize $\bar{\mathbf{c}}$ with a $I \times 1$ vector of zeros.
  for $j_2 = j_1 + 1, \ldots, J$ **do**
    Initialize $\mathbf{c}'_{j_1 j_2}$ and $\mathbf{c}''_{j_1 j_2}$ with empty vectors.
    for $m_{j_1} = 1, \ldots, M_{j_1}$ **do**
      for $m_{j_2} = 1, \ldots, M_{j_2}$ **do**
        $\mathbf{c}''_{j_1 j_2} \leftarrow \begin{bmatrix}\mathbf{c}''_{j_1 j_2} & c_{j_1 j_2}(m_{j_1}, m_{j_2}) & \cdots & c_{j_1 j_2}(m_{j_1}, m_{j_2})\end{bmatrix}$ ▷ $c_{j_1 j_2}(m_{j_1}, m_{j_2})$ repeated $\prod_{u=j_2+1}^{J} M_u$ times
      $\mathbf{c}'_{j_1 j_2} \leftarrow \begin{bmatrix}\mathbf{c}'_{j_1 j_2} & \mathbf{c}''_{j_1 j_2} & \cdots & \mathbf{c}''_{j_1 j_2}\end{bmatrix}$ ▷ $\mathbf{c}''_{j_1 j_2}$ repeated $\prod_{u=j_1+1}^{j_2-1} M_u$ times
    $\bar{\mathbf{c}} \leftarrow \bar{\mathbf{c}} + \begin{bmatrix}\mathbf{c}'_{j_1 j_2} & \cdots & \mathbf{c}'_{j_1 j_2}\end{bmatrix}^{\mathrm{T}}$ ▷ $\mathbf{c}'_{j_1 j_2}$ repeated $\prod_{u=1}^{j_1-1} M_u$ times is $\mathbf{c}_{j_1 j_2}$
  if $j_1 = J - 1$ **then return** $\bar{\mathbf{c}}$
  else return $\bar{\mathbf{c}}$ + OverallConsistencies($\mathbf{M}, \mathbf{C}, j_1 + 1$)

Lemma 1 states that in the complete matrix $\mathbf{S}$, each row $\mathbf{s}_h^{j\le\eta}$ of $\mathbf{S}^{j\le\eta}$ (i.e, each possible combination of the levels of the first η factors) is repeated L times such that each repetition corresponds to a different row of $\mathbf{S}^{j>\eta}$ (i.e., a different combination of the levels

Table 3
Truncated consistency table of the electricity sales example with $\eta = 2$, $h = 1$.

	D 1	E 1	E 2	E 3
C 1	3	3	1	1
D 1		3	2	-2

of the remaining $J-\eta$ factors). In other words, matrix $\mathbf{S}$ consists of H blocks $\begin{bmatrix}\mathbf{1}_L \mathbf{s}_h^{j\le\eta} & \mathbf{S}^{j>\eta}\end{bmatrix}$ of size $L \times J$, each of which corresponds to one subproblem h.

To illustrate this result, consider partitioning the computation of overall consistencies of the levels of three uncertainty factors in Fig. 1 into subproblems by fixing the levels of the first $\eta = 2$ factors. There are $H = M_1 \cdot M_2 = 4 \cdot 3 = 12$ such subproblems, each comprising $L = M_3 = 3$ combinations, and the matrices $\mathbf{S}^{j\le\eta}$ and $\mathbf{S}^{j>\eta}$ corresponding to this partitioning are

$$\mathbf{S}^{j\le\eta} = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 4 \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \end{bmatrix}^{\mathrm{T}} \text{ and}$$

$$\mathbf{S}^{j>\eta} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^{\mathrm{T}}. \tag{8}$$

According to Lemma 1, $\mathbf{S}$ in Fig. 1 can be obtained by repeating each row of $\mathbf{S}^{j\le\eta}$ $L = 3$ times such that each repetition corresponds to a different row (or, in this case, element) of $\mathbf{S}^{j>\eta}$.

To compute the overall consistencies in a given subproblem h, we only need to consider for factors $j = 1, \ldots, \eta$ the rows and columns in consistency table $\mathbf{C}$ corresponding to the fixed levels of these factors. In particular, if the levels of both factors j_1 and j_2 are fixed (i.e., $j_1, j_2 \le \eta$), then the matrix of pairwise consistencies between these factors reduces to a single element $c_{j_1 j_2}(m_{hj_1}^{j\le\eta}, m_{hj_2}^{j\le\eta})$. On the other hand, if only the level of the first factor j_1 is fixed (i.e., $j_1 \le \eta$ and $j_2 > \eta$), then this matrix reduces to a row vector $\begin{bmatrix} c_{j_1 j_2}(m_{hj_1}^{j\le\eta}, 1) & \cdots & c_{j_1 j_2}(m_{hj_1}^{j\le\eta}, M_{j_2}) \end{bmatrix}$. More formally, the *truncated consistency table* for subproblem h is defined as follows.

Definition 5. Let $\mathbf{S}^{j\le\eta}$ as in Lemma 1. The truncated consistency table $\mathbf{C}^h$ for subproblem $h \in \{1, \ldots, H\}$ is

$$\mathbf{C}^h = \begin{bmatrix}\mathbf{C}^h_{j_1 j_2}\end{bmatrix},$$

$$\mathbf{C}^h_{j_1 j_2} = \begin{cases} \begin{bmatrix} c_{j_1 j_2}(m_{hj_1}^{j\le\eta}, m_{hj_2}^{j\le\eta}) \end{bmatrix} & \text{if } j_1, j_2 \le \eta, \\ \begin{bmatrix} c_{j_1 j_2}(m_{hj_1}^{j\le\eta}, 1) & \cdots & c_{j_1 j_2}(m_{hj_1}^{j\le\eta}, M_{j_2}) \end{bmatrix} & \text{if } j_1 \le \eta, j_2 > \eta, \\ \mathbf{C}_{j_1 j_2} & \text{if } j_1, j_2 > \eta. \end{cases}$$

For example, consider subproblem $h = 1$ in the electricity sales company case with a focus on three uncertainty factors only (cf. the area bordered by a blue rectangle in Table 2). Here, the levels of the first $\eta = 2$ factors 'C. Competitive field' and 'D. Customer churn rate' are fixed at $\begin{bmatrix} m_{11}^{j\le\eta} & m_{12}^{j\le\eta} \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, i.e., 'Traditional: private & municipal' and 'Low, under 8% / year'. The truncated consistency table $\mathbf{C}^1$ for this subproblem is presented in Table 3.

The overall consistencies in each subproblem h can now be evaluated using Algorithm 1 with truncated consistency table $\mathbf{C}^h$ and vector $\mathbf{M}^- = \begin{bmatrix} 1 & \cdots & 1 & M_{\eta+1} & \cdots & M_J \end{bmatrix}$ of the numbers of uncertainty factor levels (where the first η elements are equal to one because the levels of these factors have been fixed). Let us denote by $\bar{\mathbf{c}}^h = \begin{bmatrix} \bar{c}_1^h & \cdots & \bar{c}_L^h \end{bmatrix}^{\mathrm{T}}$ the vector of overall consistencies in subproblem h. These consistencies can be used to determine the set of the N most consistent combinations in subproblem h using Definition 2. This can be done efficiently by using a partitioning algorithm to find the set $\mathcal{L}_h^*$ of indices of the N highest consistencies in $\bar{\mathbf{c}}^h$. In case several such sets exist, then set $\mathcal{L}_h^*$ is selected among these sets at random. Set $\mathcal{I}^*$ of the indices of the N most consistent level combinations in the complete problem can then be determined by finding the N largest elements in the union $\bigcup_{h=1}^H \{\bar{c}_\ell^h | \ell \in \mathcal{L}_h^*\}$, and by mapping the indices of these elements from the subproblem-specific sets $\mathcal{L}_h^*$ to the set of original combination indices. This result is formalized in Theorem 2 below.

Theorem 2. *Let $\bar{\mathbf{c}}^h = \begin{bmatrix} \bar{c}_1^h & \cdots & \bar{c}_L^h \end{bmatrix}^{\mathrm{T}}$ be the vector of overall consistencies in subproblem h and let $\mathcal{L}_h^*$ be the set of indices of the N highest consistencies in this vector. Let $\mathcal{T}_h = \{\mathbf{t} = (\bar{c}_\ell^h, i) \in \mathbb{R} \times \mathbb{N} \mid i = \ell + (h-1)L,\ \ell \in \mathcal{L}_h^*\}$, $\mathcal{T} = \bigcup_{h=1}^H \mathcal{T}_h$, and let t_1 and t_2 be the first and second element of tuple $\mathbf{t}$. The set $\mathcal{I}^*$ of row indices of $\mathbf{S}$ corresponding to the set $\mathcal{S}^*$ of the N most consistent level combinations is*

$$\mathcal{I}^* = \left\{ t_2 : \mathbf{t} \in \underset{\mathcal{T}^* \subset \mathcal{T}, |\mathcal{T}^*| = N}{\arg\max}\ t_1 \right\}.$$

Finally, the index set $\mathcal{I}^*$ can be used to find the row vectors $\mathbf{s}_i$, $i \in \mathcal{I}^*$ of matrix $\mathbf{S}$ corresponding to set $\mathcal{S}^*$. Theorem 3 shows how to retrieve the elements m_{ij} of these row vectors in closed form.

Theorem 3. *Let matrix $\mathbf{S}$ be as in Definition 1*. Then,

$$m_{ij} = 1 + \left\lfloor \frac{(i-1) - \sum_{t=j+1}^J (m_{it} - 1) \prod_{u=t+1}^J M_u}{\prod_{u=j+1}^J M_u} \mod M_j \right\rfloor, \quad j = J, \ldots, 1,$$

where the sum and product over an empty set are by convention 0 and 1, respectively.

For example, the elements of row $i = 863$ in matrix $\mathbf{S}$ in (1) are

$$m_{863,6} = 1 + \frac{862 - 0}{1} \mod 3 = 1 + 1 = 2,$$

$$m_{863,5} = 1 + \frac{862 - (2-1)}{3} \mod 3 = 1 + (287 \mod 3) = 3, \ldots,$$

$$m_{863,1} = 1 + \frac{862 - (4-1)4 \cdot 3^3 - (4-1)3^3 - (3-1)3^2 - (3-1)3 - (2-1)}{4^2 \cdot 3^3}$$
$$\mod 4 = 1 + (1 \mod 4) = 2.$$

This procedure for finding $\mathcal{S}^*$ efficiently is summarized in Algorithm 2, henceforth referred to as the Consistency Value Evaluation (CVE) algorithm.

Algorithm 2 The Consistency Value Evaluation (CVE) algorithm.

Input: $\mathbf{M}, \mathbf{C}, \eta, N$
Output: $\mathcal{S}^*$
1: **function** CVE($\mathbf{M}, \mathbf{C}, \eta, N$)
2: Create $\mathbf{S}^{j\le\eta}$ from $\begin{bmatrix} M_1 & \cdots & M_\eta \end{bmatrix}$ and let $\mathbf{M}^- = \begin{bmatrix} 1 & \cdots & 1 & M_{\eta+1} & \cdots & M_J \end{bmatrix}$
3: Initialize the set of highest consistencies and their indices $\mathcal{T}$
4: **for** $h = 1, \ldots, H = \prod_{j=1}^{\eta} M_j$ **do**
5: Create $\mathbf{C}^h$ as in Definition 5
6: $\bar{\mathbf{c}}^h \leftarrow$ OverallConsistencies($\mathbf{M}^-, \mathbf{C}^h, 1)/[J(J-1)/2]$ ▷ Get consistencies with Algorithm 1 and scale
7: Get the indices $\mathcal{L}_h^*$ of the N highest overall consistencies in $\bar{\mathbf{c}}^h$ with a set partitioning algorithm
8: Store them in $\mathcal{T}_h = \{\mathbf{t} = (\bar{c}_\ell^h, n) \mid n = \ell + (h-1)L, \ell \in \mathcal{L}_h^*\}$ and add $\mathcal{T}_h$ to $\mathcal{T}$
9: Construct $\mathcal{I}^*$ using Theorem 2 and sort its elements based on descending $\bar{c}_\ell^h$.
10: Construct $\mathcal{S}^*$ as in Definition 2 using Theorem 3
11: **return** $\mathcal{S}^*$

In this algorithm, the for-loop can be multi-threaded to enable task parallelism (i.e., the use of multiple cores of contemporary computers simultaneously). Matrix $\mathbf{S}^{j\le\eta}$ in row 2 can be created by taking the Cartesian product over the sets

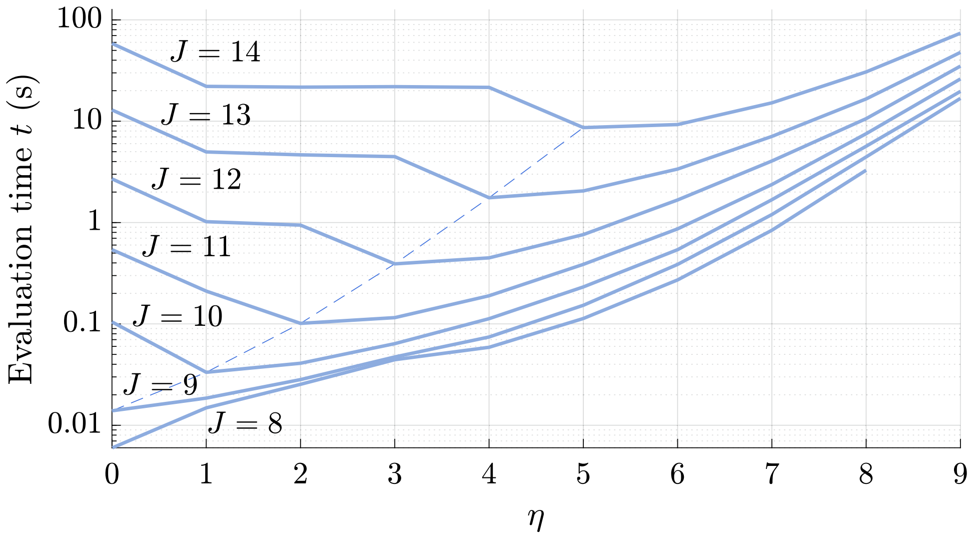

Fig. 2. Mean evaluation times from 10 test runs the CVE algorithm with varying J and η. The dashed line connects the optimal η values with the smallest computation time for each J.

of levels $\mathcal{M}_j, j = 1, \ldots, J$, for which routines are available in most mathematical programming languages[4]

We ran tests on the efficiency of the CVE algorithm for different values of J and η using Matlab R2019b on a standard laptop (six cores, 2.6GHz, 1536KB L2 cache, 32GB memory). During these tests, the numbers of levels were constant $M_j = 4$ for all factors $j = 1, \ldots, J$, the consistency tables $\mathbf{C}$ were randomly generated, and the number of consistent level combinations was $N = \min(0.1 \cdot I, 2000)$. The results of these efficiency tests are shown in Fig. 2 for $J = 8, \ldots, 14$ and $\eta = 0, \ldots, 9$. The optimal values of η corresponding to the smallest computation times for different values of J are connected by a dashed line. Based on Fig. 2, each additional factor increases the (smallest) computation time roughly by a factor of four. Nevertheless, the CVE algorithm is efficient enough in solving the set of the N most consistent level combinations even in large cases: for instance, with $J = 14$ uncertainty factors, this set could be solved in less than 10 seconds. In particular, partitioning the complete enumeration problem into smaller subproblems decreases the computation time significantly: with $J = 14$ uncertainty factors, for example, the computation time for solving the complete problem at once (i.e,. setting $\eta = 0$) would be approximately 60 seconds. It is worth noting that the optimal value $\eta = J - 9$ (for $J > 8$) corresponds to evaluating CVE with vectors that just fit the cache.

4. Visualization method for supporting an interactive scenario process

In participatory scenario development, the process participants need to understand the logic through which the final set of scenarios is obtained. Moreover, the participants may have preferences (other than maximizing the consistencies of the least consistent level combinations) that they would like to see reflected in the final scenario set. Such preferences might include, for instance, having a business-as-usual scenario or the most desirable scenario in the final set. Ideally, the final set of level combinations could be determined in a flexible way such that these kinds of preferences could be taken into account. This can be achieved with the help of a suitable visualization tool that enables the participants to manually select level combinations according to their preferences, and visually observe the consistency and diversity of these combinations. In Section 4.1, we discuss the use of Multiple Correspondence Analysis for visualizing multidimensional combinations in a two-dimensional plane, and in Section 4.2 we present an MCA-based software tool designed to facilitate the interactive identification of the final set of combinations.

4.1. Multiple correspondence analysis for visualizing consistent level combinations

Multiple Correspondence Analysis is a data analysis method used to detect underlying structures in a categorical data set by representing this set as points in a low-dimensional Euclidean space (Greenacre & Blasius, 2006). Here, we use MCA specifically to illustrate dissimiliarities between $(M - J)$-dimensional consistent level combinations $\mathbf{s}^*_n$, $n = 1, \ldots, N$ in a two-dimensional plane. In particular, let us represent these combinations by the rows $\mathbf{z}_n$ of the indicator matrix $\mathbf{Z}$ in Definition 3. Following the standard convention in MCA, the dissimilarity between two rows of $\mathbf{Z}$ is characterized by the chi-squared distance function.

Definition 6. The chi-squared distance between rows $\mathbf{z}_{n_1}$ and $\mathbf{z}_{n_2}$ of $\mathbf{Z}$ corresponding to consistent level combinations $\mathbf{s}^*_{n_1}$ and $\mathbf{s}^*_{n_2}$ is

$$d(\mathbf{s}^*_{n_1}, \mathbf{s}^*_{n_2}) = d(\mathbf{z}_{n_1}, \mathbf{z}_{n_2}) = \sqrt{\frac{1}{J}\sum_{m=1}^{M}\frac{1}{p_m}(z_{n_1 m} - z_{n_2 m})^2},$$

where $p_m = \sum_{n=1}^{N} z_{nm}/N$ is the proportion of consistent combinations that include factor level $m \in \{1, \ldots, M\}$.

This distance function is suitable because it respects the property of different categorical uncertainty factor levels being equidistant: i.e., the squared difference $(z_{n_1 m} - z_{n_2 m})^2 \in \{0, 1\}$ is equal to one if and only if one combination contains factor level m and the other one does not, regardless of what exactly the level of that factor in the other combination is. The scaling by $1/p_m$ provides another useful property: levels that are included in only a small proportion of consistent combinations have a larger weight in the metric. Thus, combinations containing rarer levels are particularly distinctive, which makes it easier for us to find the final set of level combinations that together attain all factor levels.

The goal of MCA in our context is to find the best projection of the set $\mathcal{Z} = \{\mathbf{z}_1, \ldots, \mathbf{z}_N\}$ of the rows of $\mathbf{Z}$ onto a two-dimensional

[4] E.g., method PRODUCT of ITERTOOLS Python package or NDGRID in Matlab.

plane. The goodness of a projection in MCA is assessed by the amount of preserved inertia of $\mathcal{Z}$, defined as the mass ($=1/N$) weighted sum of squared of distances of $\mathbf{z}_n$ from their center of gravity $\begin{bmatrix} p_1 & \cdots & p_M \end{bmatrix}$. To find this projection, the indicator matrix $\mathbf{Z}$ is first transformed such that the sum of the squared elements of the transformed matrix is the inertia of $\mathcal{Z}$. Then, a singular value decomposition for this transformed matrix is found:

$$\frac{1}{\sqrt{NJ}}\left(\mathbf{Z} - \mathbf{1}_N\mathbf{1}_M^{\mathrm{T}}\mathbf{P}\right)\mathbf{P}^{-1/2} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}},$$

where $\mathbf{P} = \mathrm{diag}(p_1, \ldots, p_M)$. Let us denote by $\tilde{\mathbf{U}}$ the matrix consisting of the first two columns of $\mathbf{U}$ corresponding to the two greatest singular values σ_r, and let $\tilde{\boldsymbol{\Sigma}} = \mathrm{diag}(\sigma_1, \sigma_2)$. The two-dimensional representation of $\mathbf{Z}$ that best preserves the inertia of the rows $\mathbf{z}_n$ (Seeve, 2018) is

$$\mathbf{Y} = \tilde{\mathbf{U}}\tilde{\boldsymbol{\Sigma}}.$$

The rows $\mathbf{y}_n, n = 1, \ldots, N$ of $\mathbf{Y}$ of are called the *principal coordinates*. The rth column of $\mathbf{Y}$ explains a share $PI(r) = \sigma_r^2 / \sum_{u=1}^{M} \sigma_u^2,\ r = 1, 2$ of the total inertia of the set $\mathcal{Z}$ (Greenacre & Blasius, 2006).

4.2. Software tool for interactive scenario identification

To effectively deploy the methods presented in this paper, an interactive software tool[5] called Scenario Builder™ was developed for a Finnish management consultancy Capful specializing in scenario work and strategy development. The tool automates various tasks related to Consistency analysis, such as setting up the morphological field and constructing a draft consistency table. The tool also implements the CVE algorithm of Section 3 for finding the set of consistent level combinations, and MCA for projecting this set onto a two-dimensional plane.

The most crucial feature of the tool is an interactive user interface (UI) for visualizing the principal coordinates of the two-dimensional projection by a scatter plot, where the sizes of the markers are proportional to the consistencies of the corresponding combinations. The scatter plot for the set of $N = 100$ most consistent level combinations in the electricity sales company example is shown in Fig. 3. The explained share of inertia of the two first principal coordinates in this scatter plot is $PI(1) + PI(2) = (\sigma_1^2 + \sigma_2^2)/\sum_{u=1}^{M} \sigma_u^2 = (0.809^2 + 0.494^2)/2.5 = 36.1\%$, which is significantly higher than the expected share of two randomly selected dimensions $2/(M - J) = 2/(21 - 6) = 13.3\%$. The consistencies of the combinations in $\mathcal{S}^*$ range between 1.2 and 1.73. The figure highlights level combinations in the optimal final set $\mathcal{S}^{\mathrm{f}}$, where the consistency of the least consistent combination $\mathbf{s}_4^{\mathrm{f}}$ is 1.33.

In the interactive UI, the markers in the scatter plot are associated with uncertainty factor level combinations such that a mouse click on a marker triggers an illustration of the corresponding factor levels in an adjacent morphological field. This linking helps mitigate the limited share of explained inertia in MCA, because the dissimilarity of two combinations can be directly observed once they have been selected in the scatter plot. For instance, in the scatter plot of Fig. 3, combinations $\mathbf{s}_1^{\mathrm{f}}$, $\mathbf{s}_2^{\mathrm{f}}$ and $\mathbf{s}_3^{\mathrm{f}}$ seem to be very far apart. Indeed, the highlighted levels in the adjacent morphological field suggest that apart from $\mathbf{s}_2^{\mathrm{f}}$ and $\mathbf{s}_3^{\mathrm{f}}$ both containing level $m_6 = 3$ (strong growth for the Finnish economy), these combinations do not share any other levels. On the other hand, combination $\mathbf{s}_4^{\mathrm{f}}$ seems to be closer to $\mathbf{s}_2^{\mathrm{f}}$ and $\mathbf{s}_3^{\mathrm{f}}$ than to $\mathbf{s}_1^{\mathrm{f}}$. This impression is verified by the morphological field, which shows that combination $\mathbf{s}_4^{\mathrm{f}}$ shares one level with both $\mathbf{s}_2^{\mathrm{f}}$ and $\mathbf{s}_3^{\mathrm{f}}$ but no levels with $\mathbf{s}_1^{\mathrm{f}}$.

The example on the electricity sales company case in Fig. 3 illustrates how an optimal or near optimal solution can be identified manually by utilizing the interactive features of the UI. Assume, for instance, that the user is looking to identify four level combinations to be used as a basis for scenario development by using these features. A natural first choice would be combination $\mathbf{s}_1^{\mathrm{f}}$, which is highly consistent (as demonstrated by its relatively large marker) and distinctive in that its marker is on the outer edge of the visualization. Selecting two additional combinations that would be maximally dissimilar compared to $\mathbf{s}_1^{\mathrm{f}}$ would probably lead the user to select $\mathbf{s}_2^{\mathrm{f}}$ and either combination $\mathbf{s}_3^{\mathrm{f}}$ or the one above it near the top-right corner of the plot.

Assume that the user chooses $\mathbf{s}_3^{\mathrm{f}}$. To complete the set, the user would then need to find a combinations comprising all levels that are not included in $\mathbf{s}_1^{\mathrm{f}}$, $\mathbf{s}_2^{\mathrm{f}}$, and $\mathbf{s}_3^{\mathrm{f}}$, i.e., a combination $\mathbf{s}^* = \begin{bmatrix} 4 & 4 & 4 & m_4 & m_5 & 2 \end{bmatrix}$ for some $m_4, m_5 \in \{1, 2, 3\}$. Finding such a combination by simply clicking through the remaining combinations one by one would be relatively arduous, which is why another type of visualization offered by the UI is helpful: in particular, the UI enables highlighting a set of combinations containing some specific levels from a subset of uncertainty factors.

Following this idea, Fig. 4 illustrates the part of the scatter plot of Fig. 3 containing five highlighted level combinations that have level 4 in the first three factors. These levels are also highlighted in the adjacent morphological field. The highlighted combination closest to the top-left corner of the scatter plot (i.e., $\mathbf{s}_4^{\mathrm{f}}$) includes level $m_6 = 2$ which is not included in any of the three previously selected combinations. Consequently, the user chooses $\mathbf{s}_4^{\mathrm{f}}$ as the fourth combination, thereby ending up with the same final set that would have been obtained through optimization.

5. Scenario-based strategic planning for Finnish security of supply in 2030

The National Emergency Supply Organization[6] (NESO) is a network of governmental agencies and bodies responsible for maintaining and developing the security of supply in Finland. In particular, the objective of NESO is to ensure the continuity of production and infrastructure vital to society under all circumstances in such a way that the living conditions of the population and the critical functions of society are secured also in the event of disruptions and emergencies, including a state of defense (Ministry of Economic Affairs & Employment, 2018).

In the face of growing global and local uncertainty, foresight has become an important part of NESO's core activities. Since 2012, NESO has utilized scenarios in its foresight efforts. During the summer and fall of 2017, NESO collaborated with Capful in a project to develop scenarios for 2030. The objective of this project was to generate foresight information to support both the Finnish government's decision on the objectives of the security of supply as well as NESO's own strategic and operative decision-making. The scenarios were also to be used as a basis for the continuous monitoring of NESO's operational environment.

The scenario project consisted primarily of meetings of the project group composed of 10 representatives from NESO and Capful. Additionally, many representatives from partner organizations of NESO as well as external subject matter experts contributed to this work through participating in interviews and workshops. The project began with acquiring information about relevant trends and developments from existing reports (e.g., Global Risk Report 2017 and the Finnish Government's Defence Report 2017) and 20 expert interviews. These developments were then assessed by the project group with respect to their impact and uncertainty. Those ten developments that were seen to be the most impactful and

[5] https://www.capful.fi/en/services/corporate-clients/scenario-builder/, link accessed No. 6th 2020.

[6] National Emergency Supply Organization web page, https://www.nesa.fi/organisation/ (accessed No. 6th 2020).

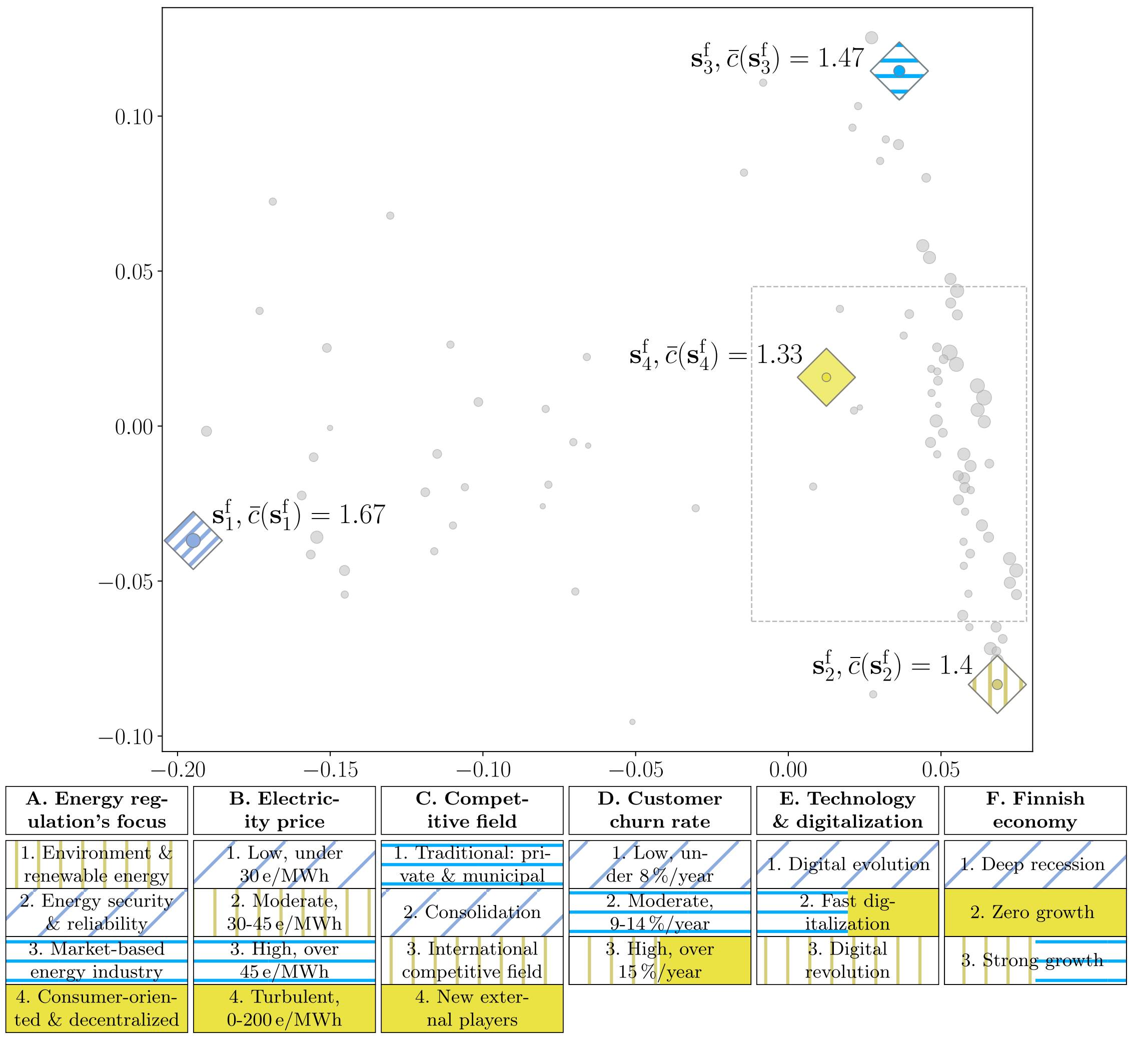

Fig. 3. Principal coordinates of the $N = 100$ most consistent electricity sales combinations $\mathcal{S}^*$ and visual grouping of a solution $\mathcal{S}^f$ to (7).

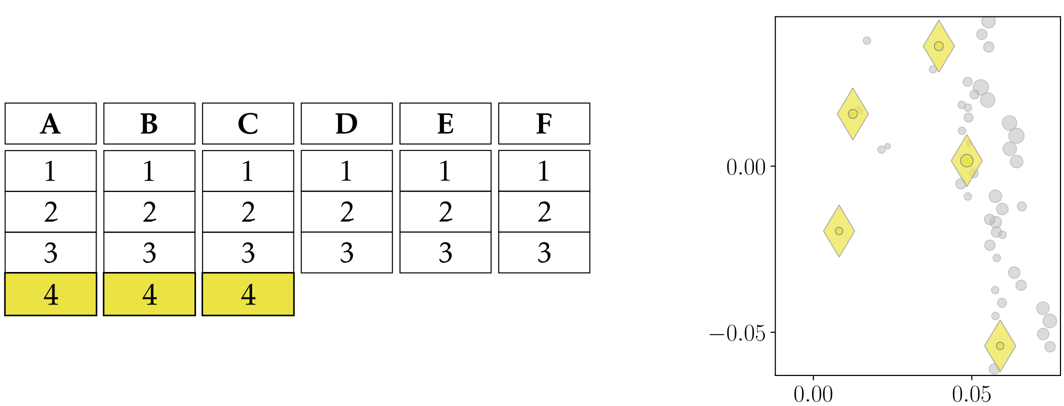

Fig. 4. Illustration of consistent level combinations with level 4 in the first three factors. The part of principal coordinate scatter plot on the right of this figure is illustrated with a gray rectangle in Fig. 3.

uncertain were selected as the uncertainty factors based on which the scenarios would be built. Four plausible levels for each of these factors were defined by the project group, together with narratives about how the developments corresponding to these levels would play out between 2018 and 2030. The titles of the uncertainty factors and their levels are shown in Table 4. The pairwise consistency values between each pair of factor levels were assessed by Capful representatives. These consistency values were then discussed, verified, and partly revised with NESO stakeholders in project meetings. Table 5 presents the final consistency table.

From the consistency analysis, the CVE algorithm in Section 3 was used to filter out the set $\mathcal{S}^*$ of $N = 10{,}000$ most consistent level combinations from the $4^{10} \approx 1{,}000{,}000$ possible combinations. Using the optimization formulation (7), it would have been possible to find a final set of four level combinations from $\mathcal{S}^*$ that together would cover all the factor-specific levels.

Table 4
Uncertainty factors and their levels in the NESO case.

A. Globalization & int. relations	B. Geopolitical focus points	C. Security situation	D. Finland's role in int. politics	E. Resources & pop. movements
1. Slower & safer globalisation	1. Traditional industrial nations	1. Territorial conflicts	1. Impartial & neutral Finland	1. Pronounced regional resource scarcity
2. Fast marked-driven globalisation	2. Focus shifts to market-driven East	2. Conflicts related to (hybrid)influence	2. Finland allied at the European level	2. Declined availability of resources
3. Blocification & the birth of new alliances	3. States w/ top-down leadership increase role	3. Escalation of terrorism	3. Trans-Eur. alliances & NATO membership	3. Improved resource availability & efficiency
4. Protectionism & deglobalisation	4. Power from nations to supranat. networks	4. Conflicts de-escalate	4. Bilateral agreements	4. Less extreme climate phenomena

F. Global data openness & security	G. The global economy	H. Development of Europe	I. Smart systems & machines	J. Economic structure & work
1. Open & public data	1. Positive & equally distributed growth	1. Slow diminishing of the EU's role	1. Digital evolution - AI aiding people	1. Traditional
2. Cyber risks, protection & isolation	2. Positive but polarised growth	2. Harmonisation & strong interdependence	2. Digital leap - AI alongside people	2. Everything-as-a-service
3. Data controlled by the few	3. Slower economic growth	3. Blocs within Europe	3. Digital revolution - AI makes decisions	3. New wave of globalisation
4. Post-truth era	4. Economic crises & turbulence	4. Decline of the EU	4. Digital stagnation	4. Age of robotisation

Table 5
Consistency values between each pair of factor-specific levels.

		B				C				D				E				F				G				H				I				J			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
A	1	3	2	-3	-1	-3	-2	1	3	1	2	1	-1	1	-3	3	1	2	-2	-2	-1	3	-1	1	-2	-1	3	-1	-3	2	2	1	-1	2	2	1	0
	2	1	3	-2	3	-1	1	-1	1	0	1	0	-2	1	-1	1	0	1	-1	2	0	1	3	-1	2	1	1	2	1	1	2	2	-1	-1	2	3	2
	3	1	1	2	-2	3	3	1	-1	-1	2	2	1	2	2	-1	-1	0	2	0	1	-1	1	2	3	1	-1	3	1	1	0	0	1	2	1	-2	0
	4	1	-1	2	-3	3	3	3	-2	2	-1	-1	3	2	3	-3	-1	-2	3	1	3	-2	-1	3	2	3	-2	1	3	1	0	0	2	2	1	-3	0
B	1					-1	2	1	2	-2	3	2	-3	2	-1	2	1	1	-1	-1	1	2	2	2	1	-2	3	2	-2	2	2	1	0	2	3	-1	1
	2					-1	2	0	2	2	1	-1	0	1	2	1	1	1	0	1	1	2	2	-1	1	2	-1	2	2	1	2	2	-1	-1	2	2	1
	3					2	3	1	-2	-2	2	2	1	1	2	-2	-1	-2	2	2	3	-2	1	2	2	1	1	0	-1	1	0	-1	1	1	0	-2	-1
	4					-2	2	1	0	1	0	-1	-1	0	1	-1	0	2	0	2	1	1	2	-1	2	-1	3	1	-1	-1	2	2	-2	-2	2	3	1
C	1									0	2	3	1	3	2	-2	-2	-2	2	1	2	-3	1	1	2	1	2	2	1	1	0	0	2	0	0	-1	0
	2									0	3	2	1	1	1	-2	-2	-2	3	2	2	-2	1	1	1	1	2	2	1	1	0	0	2	0	0	-1	2
	3									-2	2	1	0	2	2	-2	-2	0	2	0	1	-2	2	1	1	1	1	2	2	1	0	0	2	0	0	-1	0
	4									1	2	0	-1	-2	-2	3	3	3	1	-1	-2	3	-1	0	-2	1	3	-1	-3	1	1	0	-3	1	1	2	0
D	1													1	1	0	0	0	0	0	0	1	0	0	1	2	-3	-1	3	0	0	0	0	0	0	2	0
	2													2	2	2	1	2	0	1	0	2	1	1	0	-2	3	1	-3	0	0	0	0	0	0	2	0
	3													1	1	0	0	0	0	0	0	0	0	0	0	1	1	2	1	0	0	0	0	0	0	0	0
	4													1	2	-2	-2	-1	2	0	0	-2	1	0	1	1	-2	-1	2	0	0	0	1	0	0	2	0
E	1																	0	1	1	1	-3	2	2	2	1	1	3	1	1	0	0	2	1	1	2	0
	2																	-1	2	2	2	-3	3	3	2	1	1	2	1	1	0	-1	2	2	1	1	0
	3																	2	-1	-1	-2	3	-1	-2	-2	-1	1	-1	-1	1	3	3	-3	0	2	1	1
	4																	1	-1	0	0	3	1	-2	-2	-1	1	-1	-1	2	3	3	-2	0	1	0	1
F	1																					2	-1	-1	-1	-2	3	-2	-3	1	3	2	-3	1	3	1	1
	2																					-1	2	3	2	2	-2	2	2	1	0	-1	2	2	-1	3	1
	3																					-2	2	1	1	2	-2	3	1	1	1	2	1	1	1	2	1
	4																					-1	1	1	1	1	-1	1	1	1	0	0	2	1	0	0	0
G	1																									1	2	-2	-2	1	2	-2	-2	1	2	1	1
	2																									1	-2	3	2	0	1	3	-1	1	2	2	3
	3																									2	1	1	2	1	-1	1	1	2	-1	-1	0
	4																									2	-1	3	1	1	0	2	2	1	-1	2	2
H	1																													0	0	0	1	1	0	2	0
	2																													0	0	0	-1	2	0	1	0
	3																													0	0	0	0	0	-2	0	0
	4																													0	0	0	2	1	0	2	0
I	1																																	3	1	1	-1
	2																																	-1	2	2	2
	3																																	-3	2	2	3
	4																																	3	-3	-1	-3

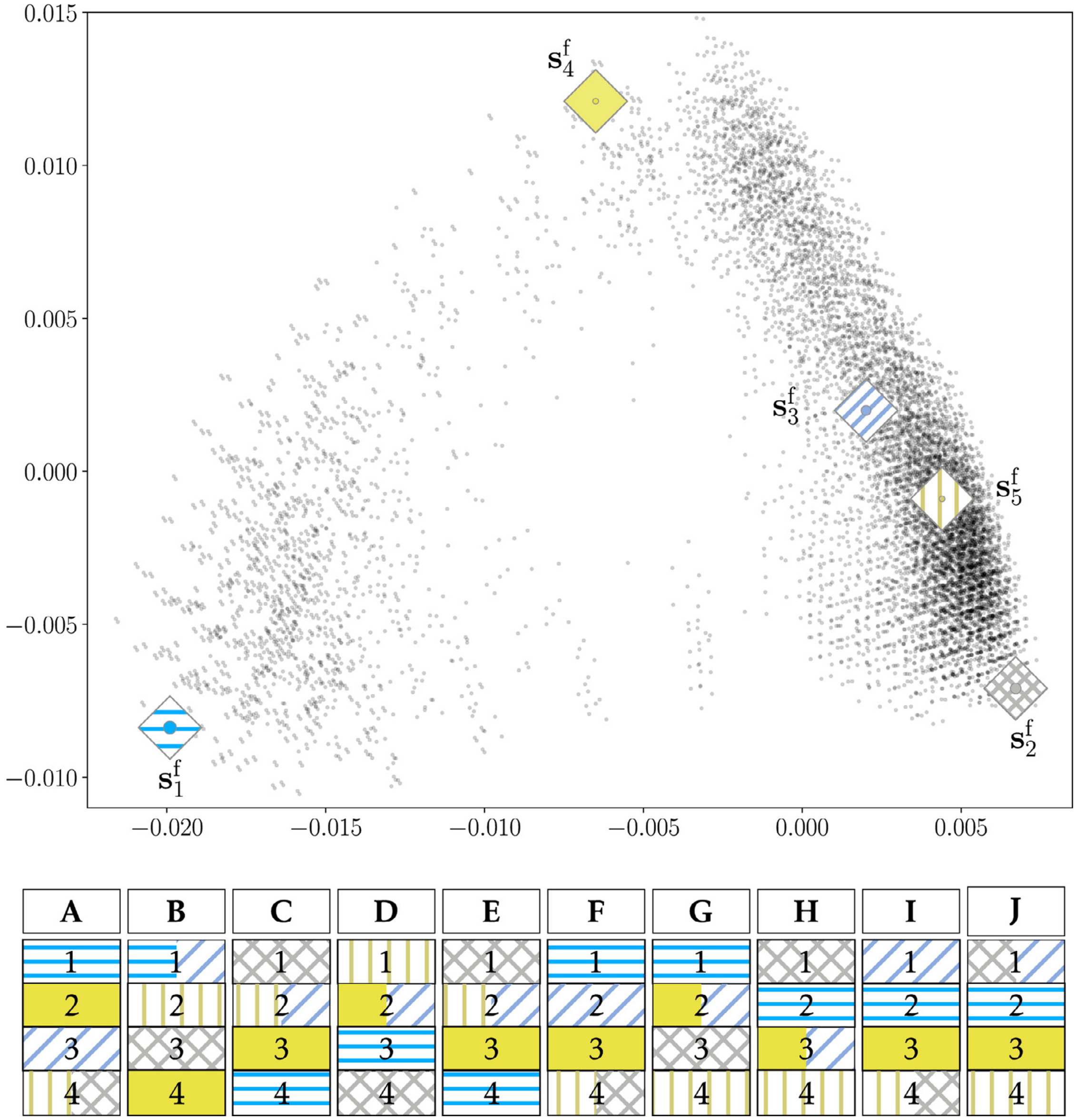

Fig. 5. Visualization of the 10,000 most consistent combinations of levels and the final five combinations of the NESO case.

Nevertheless, the client wanted to have five scenarios, some of which should correspond to combinations a few factor-specific levels that were not necessarily in any of the most consistent combinations of all factor levels. Hence, the interactive features of the visualization tool discussed in Section 4 were utilized to find the final set of combinations. The principal coordinates of the $N = 10{,}000$ most consistent combinations and the final set of five level combinations[7] are illustrated in the scatter plot of Fig. 5. The first (horizontal axis) and second (vertical axis) principal dimensions explain 21.88% and 8.86% of the inertia of the $M - J = 40 - 10 = 30$-dimensional scenario cloud, respectively. The distances between the first four selected level combinations are well depicted by the scatter plot, while the dissimilarity between the third and fifth combination is mostly contained in the unexplained variance.

The final set of five level combinations was refined, concretized and focused in workshops and meetings, resulting in five plausible but mutually dissimilar scenarios for the future. The narratives developed for these scenarios included, e.g., (i) three-phase timelines of logical events occurring in years 2018–2021, 2022–2026, and 2026–2030, amplified with effective visualizations, and (ii) influence diagrams crystallizing the interdependencies between the driving forces and focal events in each scenario. Summarized narratives of the five final scenarios are presented in Appendix B in the supplementary material. More extensive depictions of the scenarios can be found in the public project report (National Emergency Supply Agency, 2018). Finally, the scenarios were described from the perspectives of industries critical to the security of supply in sector-specific workshops. These descriptions worked as a basis for the preparation of sector-specific contingency plans and measures that would be necessary regardless of which scenario would realize.

The methods supported the scenario process by helping to build a set of plausible but diverse scenarios in a project with tight time constraints. Moreover, the interactive software tool facilitated the examination of the configurations of the final scenarios

[7] In the project, level combinations were treated more flexibly than in this paper; e.g., a scenario could correspond to multiple levels from a specific factor with varying significance. The five level combinations presented in this figure are as similar as possible to those in National Emergency Supply Agency (2018).

and comparisons between them. During informal discussions in the workshops, the process participants gave positive feedback about the methods, particularly because these methods enabled the incorporation of all relevant uncertainty factors and levels without the need to choose the two most relevant factors at the outset. They also appreciated the use of pairwise consistency assessments as a basis of evaluating scenario plausibility, since they felt that “pairwise comparisons are the most effective means of human reasoning”.

6. Discussion and conclusions

Scenario-based methods have a long tradition in supporting strategy development for organizations operating in complex and unpredictable environments. Especially when building strategy for the long term, scenarios must be constructed based on mostly qualitative expert judgment elicited in a workshop setting instead of hard quantitative data. Earlier methods for such scenario processes range from intuitive and creative to analytic and quantitative. On the one hand, intuitive methods can support the engagement of experts from various domains and encourage thinking of the unprecedented. On the other hand, analytic methods provide structure and methodological rigor for evaluating the plausibility and diversity of scenarios.

In this paper, we have developed a visually supported scenario identification method that benefits from the strengths of both intuitive and analytic approaches. In this method, scenarios are developed based on combinations of uncertainty factor levels describing plausible developments for the key drivers of future change. The plausibilities of these level combinations are assessed based on qualitative expert judgments about pairwise consistencies between factor levels, translated into quantitative consistency values. The algorithms we have developed make it possible to efficiently filter out the most consistent combinations from exponentially many candidates. To support the selection of the final set of combinations to be used as a basis for scenario development, we have formulated an optimization problem the solution to which gives the smallest number of maximally consistent combinations that together cover all factor levels. Finally, we have developed an interactive software tool that utilizes Multiple Correspondence Analysis to visualize the diversity of the consistent level combinations in a two-dimensional plane. In this paper, we have presented a real-life application of the methods and software tool in building scenarios for the Finnish National Emergency Supply Organization. The methods have since been used to support the scenario development processes of many other companies and organizations, including Business Finland (a Finnish governmental organization for innovation funding and trade, travel, and investment promotion) and Royal Dutch Shell.

The proposed method offers several benefits for improving the quality and structure of scenario development processes. First, the efficient algorithms enable to incorporate numerous uncertainty factors and levels that are deemed relevant and plausible by a variety of experts with diverse backgrounds. Second, the optimization formulation presented in this paper provides an efficient and methodologically rigorous approach to identifying a diverse set of plausible level combinations to be used as a basis for scenario development. Due to this efficiency, time and effort of the scenario project participants can be focused on the parts of the process where the role of their creative input is most crucial, that is, the identification of the uncertainty factors and their levels, and fleshing out the final combinations into scenarios. Third, the interactive software tool enables a clear communication of the results of the process to participants with little background in mathematical modeling. Importantly, by linking the coordinates of the combinations in the two-dimensional visualization to their tabular illustrations, the tool can be used to inspect the entire space of plausible level combinations by hand, which may help the participants to challenge their mental models and develop new perspectives on the issue at hand. This feature also facilitates a manual selection of the final set of combinations according to the participants' specific preferences, which may be different from those suggested by our optimization formulation.

Our method also has some limitations. First, the number of uncertainty factors included in the analysis cannot be arbitrarily large due to time constraints related to carrying out the pairwise consistency assessments. If, for instance, all uncertainty factors have four levels, the maximum number of such factors in practical applications would range between 10 and 15 resulting in 720 and 1680 pairwise consistency assessments, respectively. Consequently, if the number of interesting driving forces identified by the process participants is very large, these driving forces need to be prioritized into a manageable set of uncertainty factors and factor-specific levels based on, e.g., their impact and uncertainty (van der Heijden, 2005). Another limitation of our method is that it only considers pairwise consistencies between uncertainty factor levels and thereby ignores potential higher-order dependencies from causal chains of events. In principle, consistency assessments could be elicited also for, e.g., combinations of levels of three uncertainty factors instead of two. Nevertheless, this would significantly increase the burden of elicitation: In the NESO case, for instance, the inclusion of such second-order dependencies would result in $\binom{10}{3} \cdot 4^3 = 7680$ elicitation tasks compared to the 720 pairwise consistency assessments. A practical solution to this problem could be to choose certain subsets of factors on which information about higher-order dependencies would be elicited. For example, the inclusion of second-order dependencies for three uncertainty factors with four levels each would increase the elicitation effort only moderately: the number of consistency assessments between these factors would grow from $3(3-1)/2 \cdot 4^2 = 48$ to $4^3 = 64$.

Finally, our method does not offer guidance for assessing the relative likelihoods of the developed scenarios. Yet, if scenario analysis is to support strategic choices, then some judgment about the relative likelihoods of these scenarios need to be made (Bunn & Salo, 1993). This could be achieved by (i) eliciting information about the relative likelihoods of the levels for each uncertainty factor and (ii) treating the consistency values as probability statements about dependencies between uncertainty factor levels. However, the theoretical foundations for a such a probabilistic treatment of consistency values remains a topic for future research.

As a final note, the methods developed in this paper are generic in that they could be applied in other contexts as well. Specifically, these methods are applicable to all problems where the quality of a solution is defined by the pairwise compatibilities of variable values, and where the ultimate goal is to seek a set of few solutions that together attain all possible values in a number of distinct variables. Examples of such problems include the development of a new business model or service concept as a combination of choices made with respect to factors such as value proposition, customer segment and revenue logic (Im & Cho, 2013; Lee, Song, & Park, 2009).

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Supplementary material

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