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Interference-adjusted power learning curve model with forgetting

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ABSTRACT

Researchers in production and operations management have studied the effect of worker learning and forgetting on system performance for decades. It remains an active research topic. Those studies have assumed that production interruptions (or production breaks) cause forgetting, which deteriorates performance. Research on human working memory provides enough evidence that continuous forgetting, precisely cognitive interference, results from overloading the memory with information. Despite the evidence, few studies have incorporated it into learning curve models. This paper presents an enhanced version of the power learning curve that accounts for a variable degree of interference when moving from a production cycle to the next. It adopts the concept of memory trace decay to measure the residual (interference-adjusted), not the nominal (maximum) cumulative experience. We test the developed model against learning data from manual assembly and inspection tasks, with varying numbers of repetitions and breaks. We also test three alternative power-form learning and forgetting curve models from the literature. The results show that the interference-adjusted model fits the data very well. The proposed learning and forgetting model and its individualized cumulative metrics can help identify struggling workers early and release precocious learners earlier than expected. As such, the model gives insights for managers on the occurrence of interference to enable individual learning support.

1. Introduction

Learning is a phenomenon reflected by performance improvements of individuals gaining experience by (repetitively) carrying out activities or tasks. A learning curve estimates the speed of an individual (or group) improvement using a regression line that fits scatterplots of raw learning data. Learning curves have been a central research topic for psychologists for more than 100 years (Thurstone, 1919) and later an industrial production management tool (Wright, 1936). The Wright learning curve (WLC, 1936), a power-form model, continues to be widely used and well accepted by scientists and practitioners (Glock et al., 2019). Wright empirically showed that labor time/cost decreases by a constant rate each time cumulative production output doubles using data for aircraft production. However, WLC and many other traditional learning curves have a drawback that limits their usability; they do not account for production breaks, and consequently, regression in performance due to forgetting. Productivity losses due to forgetting are potentially dramatic, especially for short intermittent production runs (Baloff, 1970).

Ebbinghaus (1885) was probably the first to develop learning and forgetting (LaF) curves to study human memory. He found that the longer a person studies, the longer the learned information is retained. LaF topic has enjoyed increasing attention among researchers in industrial ergonomics (Anzanello and Fogliatto, 2011) and production and operations management (POM) (Glock et al., 2019) fields since its inception. Production could be interrupted for many reasons, e.g., manufacturing products in batches with breaks in-between (Adler and Nanda, 1974), strikes, and production-line downtime (Sikström and Jaber, 2002). Other reasons are worker cross-training to perform different tasks (Hewitt et al., 2015; Sayin and Karabati, 2007), rest time needed to recover during and between the working days, and vacation or other personal absences. Prolonged interruptions for weeks or months can lead to significant loss of knowledge or forgetting (Anderlohr, 1969; Globerson et al., 1998; Sanli and Carnahan, 2018). In the POM field, Hancock (1967) presumably was the first to investigate how production breaks affect workers' performance. His findings suggested that short interruptions, e.g., coffee breaks, do not affect the learning processes, whereas overnight and longer breaks do. Next, we present a succinct review of mathematical learning curves that incorporate forgetting effects.

The first attempts to mathematically model LaF in the POM field

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were by Hoffmann (1968) and Adler and Nanda (1974), who incorporated the effects of production breaks. Hoffmann (1968) modeled forgetting as a displacement on the original learning curve to measure the lost (not remembered) experience when the next batch resumes. He concluded that retained learning explains the nonlinearity observed in the start-up of manual runs for products similar to those performed in previous runs. Adler and Nanda (1974) noted that a break in production of a minimum of several days slows the production rate when production commences in the next cycle. Like Hoffmann (1968), they assumed a displacement in the cumulative number of units experienced (remembered) and treated it as an input parameter and, unlike Hoffmann (1968), did not explain how it was determined. The reason for this was probably to keep the mathematics of their lot-sizing model with LaF effects less complex, an approach that has also been adopted by other researchers (Glock et al., 2019; Jaber and Bonney, 1999). Carlson and Rowe (1976) modeled the forgetting curve as a mirror image of the learning curve. They treated a learning curve intercept as a dependent parameter on the number of units produced in the current cycle and those remembered from the previous ones. They also assumed a fixed forgetting rate, i.e., not affected by accumulated experience. Globerson et al. (1989) suggested several potential functions to measure the time to perform the first repetition after a production break for their experimental data separating two learning sessions. They found a bivariate learning curve to fit their data the best. The learning curve that Globerson et al. (1989) proposed is dependent on the performance of the last repetition in the first session and the length of the break that separates the two sessions. Elmaghraby (1990) presented a learning forgetting model similar (in form) to that of Carlson and Rowe (1976) and assumed that the curve's intercept and its exponent remain unchanged for every cycle. Unlike the models in Carlson and Rowe (1976) and Elmaghraby (1990), Jaber and Bonney (1996) developed a learn-forget curve model (LFCM) where the forgetting exponent and the intercept change from cycle to cycle (Fig. 1). In the LFCM, the forgetting exponent (rate) is dependent on the learning rate, the equivalent number of units remembered at the start of a cycle, the length of a break between two subsequent learning sessions, and the time for total forgetting to occur. They showed that their model produced results as good as those of Globerson et al. (1989), i.e., <0.5% error. In a follow-up paper, Jaber and Bonney (1997) compared their model with those of Carlson and Rowe (1976) and Elmaghraby (1990) and showed that the LFCM is consistent with the two hypotheses when the other models are not. Those hypotheses are: (1) the performance time on the forgetting curve reverts to its original value (i.e., the time to produce the first unit), and (2) the performance times on the LaF curves are the same at the point of

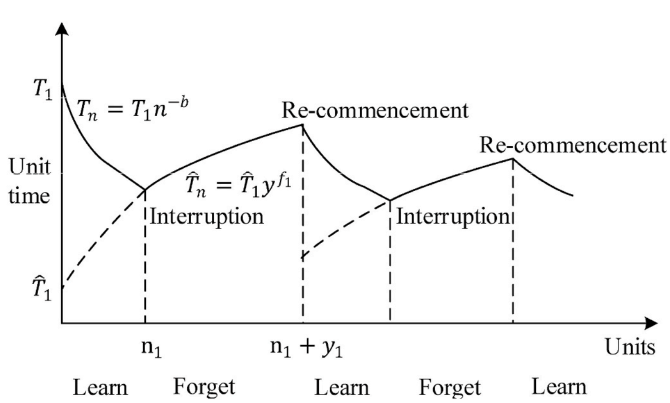

Fig. 1. Learn-forget curve model (LFCM) adapted from Jaber and Bonney (1996). n_1 = number of units at the first cycle, b = learning exponent, T_1 = time to produce first unit, $\hat{T}_1$ = forgetting curve intercept, f_1 = forgetting exponent (rate), and y_1 = number of units that would have been accumulated if interruption did not occur at the first cycle. For detailed explanations of LaF functions, T_n and $\hat{T}_n$, see Eqs. (1) and (2) in Section 2.

interruption (which is the intersection point between the two). Jaber et al. (2003) provided additional support in favor of the LFCM and showed that it embodies characteristics that have been identified in experimental and empirical studies. Those characteristics are (1) prior experience and the length of the break influence the forgetting intensity, (2) relearning and learning rates are the same, (3) LaF curves are of power forms and mirror images of one another, and (4) the learning rate (speed) impacts how fast or slow forgetting occurs. Jaber and Sikström (2004a) tested the LFCM against LaF empirical dataset taken from Nembhard and Osothsilp (2001) and showed that it fitted that data reasonably well. The LFCM has also been modified to include cognitive and motor elements (Jaber and Kher, 2002) and a job similarity index (Jaber et al., 2003). Jaber and Kher (2002) provided another modification of the LFCM (M-LFCM), which is of interest to this paper, by relaxing the assumption of a fixed time for total forgetting, supported by the experimental findings of Hewitt et al. (1992) for cognitive tasks. They suggested that the M-LFCM may provide better performance estimates than the LFCM, but this was not conclusive as neither the M-LFCM nor the LFCM was tested against empirical data. Some researchers modeled the LaF process as an integrated model that is time-based, e.g., the Power-Integration-Diffusion (PID) model (Sikström and Jaber, 2002). Since those models are not WLC-based, they are not relevant to this paper. Readers are referred to Jaber (2013) for a review of LaF models.

Nembhard and Osothsilp (2001) introduced the recency model (RCM) in the context of the WLC and showed that it fitted the data well – surpassing the other 13 tested models. Jaber and Sikström (2004a) extracted a dataset from Nembhard and Osothsilp (2001, Fig. 1) and showed otherwise by comparing the fits of the LFCM and the RCM models. However, this result was not conclusive due to limited testing data. One should compare the fits of the two models using learning data of a varying number of repetitions and breaks of different lengths before generalizing Jaber and Sikström's (2004a) findings. In this regard, Jaber and Sikström (2004b) numerically compared the three models, LFCM, RCM, and PID, and showed which type of learning (ranging from pure motor to pure cognitive) differentiates them. For example, the three models produced very close results for a moderate learning speed (number of motor task elements equals the cognitive ones). They concluded that the LFCM, RCM, and PID are best differentiated for LaF data characterized by high initial processing times, long production breaks, and tasks identified as being more motor than cognitive. Hoedt et al. (2020) recently tested how well the LFCM, M-LFCM, RCM, and PID can predict assembly performance in real-time. Their results showed that M-LFCM and PID performed the best.

It is clear from the above presentation that forgetting curve models available in the literature associate deterioration in performance following an interruption in production with the length of the break separating two subsequent learning sessions or production cycles. Those models are used in the POM field as decision-support tools for:

1. Optimizing the frequency and size of a production batch to satisfy demand and minimize production costs;
2. Determining the optimal training schedule given a fixed training budget (e.g., should training be in four shorter or two longer sessions per year?) (see; Sanli and Carnahan (2018), for multi-day training sessions);
3. Setting time standards;
4. Estimating labor costs during strikes (Globerson et al., 1998; Nembhard and Uzumeri, 2000).

The POM literature suggests that forgetting, or knowledge depreciation, can occur continuously over time (Li and Rajagopalan, 1998) and "*throughout the learning process*" (Badiru, 1994, p.44) due to lack of training, reduced retention skills, a lapse in performance, extended breaks in practice, and natural forgetting (Badiru, 2012). Hogan et al. (2020, p.432) recently stated that "*the impact of forgetting may not wholly* *eclipse the impact of learning but will hamper the learning rate while performance continues to increase at a slower rate*." They modified the power-form WLC model for diminishing learning rate, $T_x = T_1 x^{-b/(1+x/c)}$, where c is a positive decay value and decreases the effect of the learning rate, b, with increasing the number of units produced. They tested the model against learning data from 69 production programs and a total of 169 unique end products from the US Department of Defense. Compared to the WLC, the model with a diminishing learning rate had fewer errors for 43%, more errors for 5%, and equal error for 52% of the observations. The results showed that the WLC underestimates the required resources, resulting in potential overruns of labor costs. Hogan et al. (2020) studied learning data in production lots by varying the number between 5 and 21 lots, as individual product data were not available. They calculated the lot midpoints and model parameters iteratively as they could not find a closed-form solution. Their model relaxed the assumption of a fixed learning rate, but they did not account for forgetting due to breaks between the lots. For this reason, it is not suitable for the comparative study in this paper.

For psychologists, interference explains long-term memory loss (or forgetting), observing a lower likelihood of memory retrieval of a given "*image*" as time passes (Anderson, 1983; Mensink and Raaijmakers, 1988). In this line of research, the seminal work of Melton (1963) argued for the continuity of short-term and long-term memory, which is consistent with modern theories of memory. Learning and memory processes involve three stages: encoding, retention, and retrieval (Anderson, 1983). Encoding refers to the formation of the memory trace (initial learning of information). Retention refers to storing an information piece in memory, and retrieval is one's ability to access the acquired information if necessary. Melton (1963) emphasized the role of the storage mechanism, especially consolidation, which refers to the assimilation of the memory traces and improves performance (McGaugh, 2000). Learning material, such as assembly instructions, comprises several information elements or units that a subject has to encode and remember. According to Melton (1963), the parameter of interest is the number of encoded meaningful groups of information, i.e., chunks. The intra-material interference, which is proportional to the number of 'chunks', is vital for short-term forgetting. Fig. 2(a) illustrates how the forgetting slope increases as the number of chunks increases from one to seven. Fig. 2(b) depicts how the forgetting slope decreases as the number of repetitions increases before retention. This suggests that repetition increases experience, thus, reducing the number of chunks allowing a subject to better recall the information (Melton, 1963). Gaining experience makes building up a salient picture of new information faster, as attention is more (intensively) focused on relevant information (Bruder and Hasse, 2019).

The probability of retrieving a memory trace is proportional to the associative strength of the cues (which control the learning stimulus) to that trace relative to that of all associations (the interfering and unrelated ones included) (Anderson, 1983; Mensink and Raaijmakers, 1988). Strong cues accompanied with fewer irrelevant ones reduce interference and improve retrieval of relevant knowledge and skill (Arthur et al., 1998).

The term *cognitive interference*, proposed by Sweller (2011), is based on cognitive load theory (CLT) which has to do with the distribution of cognitive resources during learning and problem-solving. Like previous studies, the CLT assumes that short-term or working memory is limited; it can handle only 7 ± 2 information units and actively process 2–4 units (Van Merriënboer and Sweller, 2010). In practice, dividing human attention among many information sources, overloads working memory and, subsequently, deteriorates performance. Cognitive interference could be:

1. Content-dependent, where too many information sources disturb performance (especially at the early learning stage) (Battig, 1972), or where other (un)related activities (both when repeating a task and during a break) do affect (Anderson, 1983; Raaijmakers, 2003);
2. Time-dependent, where memory traces from each repetition decay over time but consolidate (strengthen) in long-term memory, which improves performance (McGaugh, 2000);
3. Worker-dependent, for example, those with learning disabilities may have difficulty maintaining task-relevant information while facing interference (as they suffer from working memory deficits related to controlled attentional processes) (Swanson and Siegel, 2001).

One could easily observe from learning data for tasks performed in separate sessions, such as assembly learning data in Fig. 3, that, in addition to a break, deterioration in performance may occur between adjacent repetitions. Another thing that one can observe is that empirical learning data resemble the teeth of a saw, random variations in performance, which is conspicuous at its earliest learning stages. Two reasons may affect variations for novice learners at assembly work. First, if not fully instructed, they may change strategies (methods and sequences) each time they repeat the assembly (Lim and Hoffmann, 2014). Second, they cannot absorb a large amount of information at once, reflecting various difficulties in working (Peltokorpi and Niemi, 2019). This finding is consistent with the concept of cognitive interference, which plays a crucial role in the early stage of learning (Battig, 1972) and variance in performance (Melton, 1963). In addition to experimental psychologists, researchers in industrial ergonomics have acknowledged the importance of cognitive processes (Chan et al., 2017; Fan et al., 2018; Fish et al., 1997; Goonetilleke et al., 1995) and interference (Schwerha et al., 2007) in estimating the time of a psychomotor

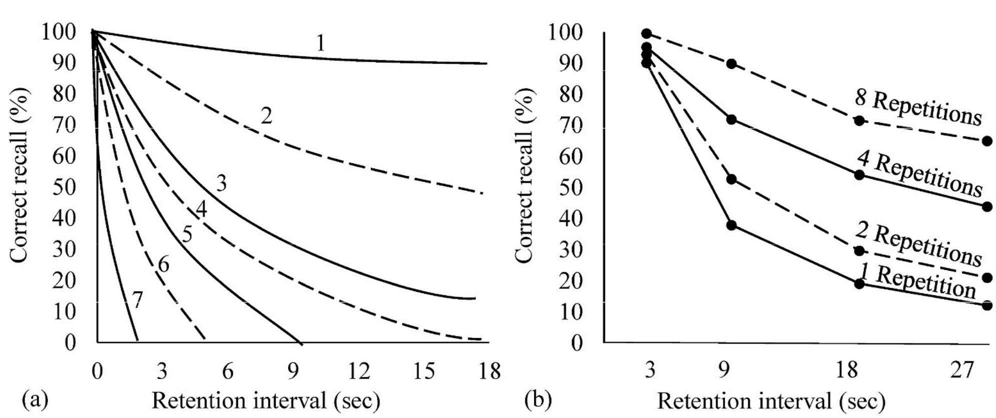

Fig. 2. Recall as a function of retention interval and (a) the number of chunks in the learning material; (b) the number of repetitions (1 s visual presentations of 3 chunks learning material) before retention interval (Melton, 1963).

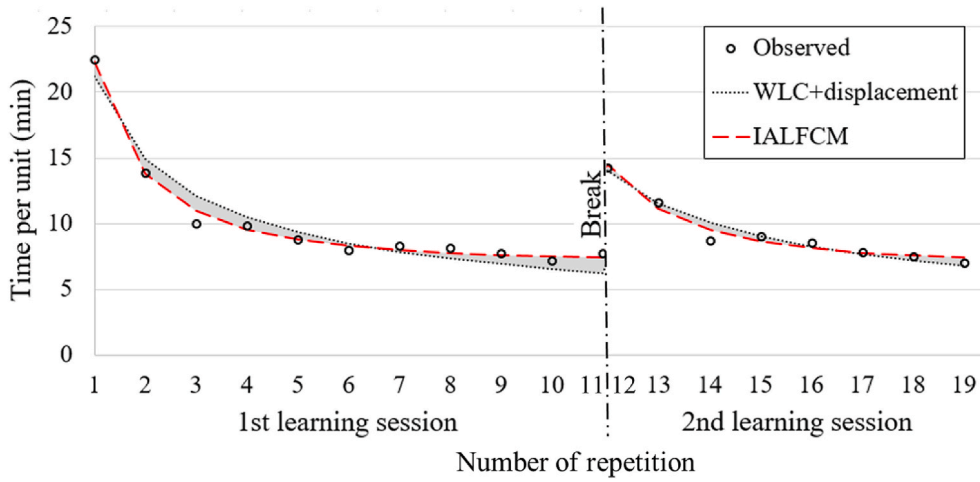

Fig. 3. Illustrative example of the behavior of IALFCM for two learning sessions separated by a break of 70 days. WLC moves to the original learning curve when the second learning session starts. The models have been fitted to assembly data from Bailey and McIntyre (1992).

task over repetitions. Lee and Duffy (2015) addressed interference at repetitive tasks and divided its causes into interruption, distraction, task switching, and task interleaving. The difference between the first two is that "*an ongoing task can continue in a situation with a distraction condition, but an ongoing task needs to be stopped in an interruption condition*" (Lee and Duffy, 2015, p.140). Despite the growing interest in interference in the learning process, only recent studies (Jaber et al., 2021; Peltokorpi and Jaber, 2022) parameterized interference into an industrial learning curve model. None of those studies considered the combined effect of interference and production break.

This paper contributes to the literature by developing an enhanced and a promising version of Wright's learning curve, WLC, that accounts for cognitive interference when learning (repeating a task) and forgetting due to breaks. To do this, we have modified a recently developed interference-adjusted learning curve model by Jaber et al. (2021) to include multiple sessions with production breaks in-between them. Fig. 3 illustrates the behavior of the IALFCM (Interference-Adjusted Learning-Forgetting Curve Model) for two learning sessions separated by a break of 70 days. In addition to forgetting as arising from production interruption, it shows that the experience gained during a learning session is not fully retained by the end of a learning session, i.e., when a break starts. The calculations to produce the fits (curves) in Fig. 3 are presented in the numerical example in Section 3.1. The reference model (WLC + displacement) is like that of Hoffmann (1968), who modeled forgetting as a displacement on the original learning curve to measure lost experience at the start of the next session.

We compare the performance of our model with three other learning-forgetting models, based on the WLC, by fitting the models against learning data from various assembly settings (Bailey, 1989; Bailey and McIntyre, 1992, 1997; Nembhard and Osothsilp, 2001; Arkite NV, https://arkite.com/, personal communication, March 2020). Our results show that adjustment for interference improves the fit to learning data and captures the variation in performance well, which is a characteristic of assembly and other labor-intensive tasks. This paper has four more sections. Section 2 reviews the relevant learning-forgetting models. Section 3 introduces the interference-adjusted learning curve model with forgetting. Section 4 is for fitting the selected models to empirical data and for discussing the results and insights. Section 5 summarizes and concludes the study.

2. Learning curve models with forgetting

The POM literature documents that WLC (Wright, 1936) fits empirical data quite well; another advantage is that its mathematical form is easy to understand and use (Anzanello and Fogliatto, 2011; Glock et al., 2019; Jaber, 2013). It is, therefore, used in this study to describe the learning phenomenon. This section, therefore, reviews learning curve models with forgetting that appear in the industrial ergonomics and POM literature. The WLC is of the form:

$$T_n = T_1 n^{-b} \tag{1}$$

where

T_n time to produce unit n;
n unit number or cumulative output;
T_1 time to produce the first unit, a fitting parameter;
b learning exponent, $0 \le b < 1$, a fitting parameter, measuring the rate at which T_n decreases as cumulative output doubles; i.e., $2^{-b} = T_{2n}/T_n$.

Carlson and Rowe (1976) suggested that forgetting is a function of the break length and the performance time just before a break (i.e., a mirror function of WLC):

$$\widehat{T}_n = \widehat{T}_1 y^f \tag{2}$$

where

$\widehat{T}_n$ time for nth unit of lost experience of the forgetting curve;
y the number of units that would accumulate if interruption did not occur;
$\widehat{T}_1$ equivalent time of the first unit (intercept) of the forgetting curve, a fitting parameter;
f forgetting slope, $0 \le f < 1$, a fitting parameter.

The parameter $\widehat{T}_1$ varies with each break, but f does not. Globerson et al. (1989) showed that the forgetting function in Eq. (2), among seven, fitted the best to a dataset from data entry tasks with an interruption. The break between the two learning sessions varied from one up to 82

Table 1
Summary of the similarities and differences of the LaF models under study.

Model	Eq. (#)	Cause	CFV	FCE	FIP	Form	Comments
LFCM	3	BL(d)	u_i	f_i	T_1, b, n_i, d, d_m	Pwr	$f_{i-1} \neq f_i \neq f_{i+1}\ \forall i = 1,2,\ldots$; f_i depends on FIP; $u_{i-1} \leq u_i \leq u_{i+1}$. Total forgetting when $d \geq d_m$; d_m is fixed
M-LFCM	3	BL(d)	u_i	f_i	T_1, b, n_i, d, $d_{i,m}$	Pwr	Same for LFCM expect that $d_{i-1,m} \geq d_{i,m} \geq d_{i+1,m}$
RCM	4	BL(d) + PT	R_{n_i}	a	t_0, t_i, t_{n_i}, n_i	Pwr	$R_{n_{i-1}} \leq R_{n_i} \leq R_{n_{i+1}} \forall i = 1,2,\ldots$
IALFCM	8h	BL(d) + INTF	U_i	a_i	a_i, t, n_i	Exp	INTF causes exponentially decaying memory traces that are consolidated. $U_{i-1} \leq U_i \leq U_{i+1}\ \forall i = 1,2,\ldots$.

BL: Break Length; Pwr: Power; Exp: Exponential; PT: Passage of time from a repetition to interruption; INTF: Interference.

days. Contrary to the assumptions of Carlson and Rowe (1976), Elmaghraby (1990) assumed $\widehat{T}_1$ and f are both fixed, i.e., that the forgetting speed is independent of the break length.

2.1. Learn-forget curve model (LFCM)

Jaber and Bonney (1996) extended the WLC model in Eq. (1) by incorporating forgetting effects, which resulted in the Learn-Forget Curve Model (LFCM). The LFCM assumes that the forgetting exponent f depends on the learning slope, the equivalent cumulative units produced by the point of interruption, and the time it takes for total forgetting to occur, allowing $\widehat{T}_1$ and f to vary from one learning session to another. The LFCM is of the following form:

$$T_{n_i} = T_1(u_i + n_i)^{-b} \tag{3}$$

where

- u_i residual knowledge, measured in units, from previous cycles at the beginning of cycle i;
- n_i units produced in cycle i up to the point of interruption;
- u_{i+1} the number of units produced at the beginning of cycle $i+1$, $u_{i+1} = (u_i + n_i)^{(1+f_i/b)} y_i^{(-f_i/b)}$;
- f_i forgetting slope for cycle i, $f_i = \frac{b(1-b)\log(u_i+n_i)}{\log(1+d_m/t(u_i+n_i))}$;
- d_m break time to which total forgetting occurs, Jaber and Bonney (1996) assumed to be given a priori, and Jaber and Sikström (2004b) as a fitting parameter, as we do;
- $t(u_i + n_i)$ time to produce $u_i + n_i$ units, where the latter is the production accumulated by the end of cycle i, $t(u_i + n_i) = \frac{T_1}{1-b}(u_i + n_i)^{1-b}$;
- y_i number of units that would have been accumulated if production was not interrupted for d_i units of time, $y_i = \left\{\frac{1-b}{T_1}\left[t(u_i + n_i) + d_i\right]\right\}^{1/(1-b)}$.

2.2. Modified learn-forget curve model (M-LFCM)

The assumption of a fixed time for total forgetting is not consistent with the experimental finding of Hewitt et al. (1992) that the performance at the beginning of the next cycle is dependent on the interruption time and the performance at the point of interruption. Based on this, Jaber and Kher (2004) modified the LFCM in Eq. (3), referred to as M-LFCM, by assuming the time to total forgetting varies from cycle to cycle, i.e., $d_{1,m} \neq d_{2,m} \neq d_{3,m} \ldots \neq d_{i,m}$. Correspondingly, $f_i = \frac{b(1-b)\log(u_i+n_i)}{\log(1+d_{i,m}/t(u_i+n_i))}$. In the next section, we fit both LFCM and M-LFCM to data.

2.3. Recency model (RCM)

Nembhard and Osothsilp (2001) modified the WLC by incorporating a factor indicating the 'recency' effect of experiential learning, which they borrowed from Nembhard and Uzumeri (2000). The recency theory advocates that a subject remembers the most recent trials (repetitions) rather than those performed earlier in the process. Thus, unlike LFCM and M-LFCM, the recency model (RCM) assumes each repetition is not equally effective. RCM is of the following form:

$$T_{n_i} = T_1\left(n_i R_{n_i}^a\right)^{-b}, \text{ where } R_{n_i} = \frac{\sum_{i=1}^{n_i}(t_i - t_0)}{n_i(t_{n_i} - t_0)} \tag{4}$$

In Eq. (4), the recency effect is determined by multiplying the cumulative work n by the discounting factor R_n^a, where a is the forgetting exponent, a fitting parameter, describing "*… the degree to which the individual forgets the task*" (Nembhard and Osothsilp, 2001, p.286). R_{n_i} is the recency measure and is the ratio of the average elapsed time to the elapsed time of the most recent unit produced. The elapsed time (including breaks) for unit n is $t_n - t_0$, where the time to complete unit n is t_n, and that at which the first ($n_i = 1$) starts is t_0. R_{n_i} is bounded by 0 and 1, with 0 indicating the experience of the distant past and 1 immediately preceding the current unit. For a constant production rate, R_{n_i} tends toward a value of 0.5. The forgetting exponent $a > 0$, where a closer to zero means less forgetting and away from it means otherwise. The RCM is fitted to the data in the next section.

2.4. Modified Wright's learning curve (MWLC) model and its approximate version (AMWLC)

Jaber et al. (2021) modified the power learning curve model by accounting for a variable degree of cognitive interference while a subject is learning when moving from one repetition to the next. The idea is based on cognitive load theory (Sweller, 1994, 2011), which assumes that human working memory is limited. The model captures the cognitive interference by accounting for memory traces of repetitions to measure the residual (interference-adjusted), not the nominal (maximum) cumulative experience, and has two versions of memory trace decay functions, power and exponential. Jaber et al. (2021) showed that the exponential decay form fits experimental learning data (without breaks, i.e., one learning session) much better than the power one. Therefore, they selected the exponential representation for their actual tests. The modification of the WLC, MWLC henceforth, takes the following form:

$$T_n = T_1 n_e^{-b} = T_1\left(\sum_{j=1}^{n} e^{-\alpha(t-t_j)}\right)^{-b} \tag{5}$$

where n_e is the equivalent number of units remembered (the residual cumulative experience). Jaber et al. (2021) provided an extensive explanation of how they arrived at the above relationship. As applying the model in Eq. (5) is cumbersome, they found the following simple and approximate expression:

$$M = K\sum_{j=1}^{n} e^{-\alpha(nt-(j-1)t)} = K\frac{1-e^{-\alpha tn}}{e^{\alpha t}-1} = K\frac{1-e^{-\gamma n}}{e^{\gamma}-1} \tag{6}$$

where $\gamma = \alpha t$ is a fitting parameter. Again, when there is no interference, $\alpha = 0$, applying l'Hopital's rule, $n_e = n$; $0 < n_e < n$ otherwise. Note that $n_e = M/K$, where M and K are the numbers of information items accumulated and recalled, respectively. The approximate model, henceforth referred to as the AMWLC, takes the following form:

$$T_n = T_1 n_e^{-b} = T_1\left(\frac{1-e^{-\gamma n}}{e^{\gamma}-1}\right)^{-b} \tag{7}$$

where, for simplification, $K = 1$. The AMWLC (Eq. (7)) was found to fit assembly learning data far better than the WLC (Eq. (1)), the MWLC (Eq. (5)), and the Plateau (bounded) learning curve model of (Baloff, 1970, 1971). Note that Jaber et al. (2021) compared the models against many experimental and empirical learning data. The WLC did not perform well. In the next section, we borrow their model in Eq. (7) and extend it to the case of multiple learning sessions or production breaks.

Table 1 differentiates the four LaF models, LFCM, M-LFCM, RCM, and IALFCM. The headings of Table 1 are 'Model' (acronym of the LaF model), 'Eq. (#)' (Learning curve equation), 'Cause' (what factor(s) causes (cause) forgetting), 'CFV' (Controllable Forgetting Variable), 'FCE' (Forgetting Curve Exponent), 'FIP' (Forgetting Influential Parameters), 'Form' (the form of the forgetting curve; i.e., power or exponential), and 'Comment' (specific features of the LaF model). The notations listed in Table 1 have been defined above, except for the proposed IALFCM in the next section (see Appendix for a list of abbreviations and notations). The basic learning function for each model is the same power-form WLC (Eq. (1)), which has two fitting parameters, T_1 and b.

3. The proposed Interference–Adjusted Learning-Forgetting Curve Model (IALFCM)

There is evidence from the literature that cognitive interference, or the rate of a memory trace decay, could vary when moving from one cycle to another (Raaijmakers and Shiffrin, 1981). In practice, memory traces from previous cycles do not continue decaying at the same speed (exponent) as a new decay exponent also comprises the interference effects caused by a break. The above is also in line with the assumption of M-LFCM that the time for total forgetting to occur varies from cycle to cycle. Fig. 4 is a schematic of a memory trace that depletes over time.

Assume now that between cycles 1 and 2, there is a break of length d_1. Then, from cycle 1, the following is the number of units remembered at the beginning of the second cycle (no experience prior to the first repetition at the first cycle, $U_o = 0$):

$$U_1 = \sum_{i=1}^{n} e^{-\alpha_1(nt-(i-1)t+d_1)} = \frac{e^{-\alpha_1 d_1} - e^{-\alpha_1(n_1 t+d_1)}}{e^{\alpha_1 t}-1} = \frac{e^{-\alpha_1 d_1}\left(1-e^{-\alpha_1 n_1 t}\right)}{e^{\alpha_1 t}-1} \tag{8a}$$

where α_i is a decay parameter for cycle $i = 1\ldots i$, and t is a fixed time step in calculation procedure, are the fitting parameters, and $\alpha_i t = \gamma_i$ is a decay exponent. Note that on the right side of Eq. (8a), we use n_1 to refer to the number of repetitions performed in the first cycle. Note again that when there is no forgetting, $U_1 = n_1$, a total transfer of learning occurs between cycles 1 and 2. The equivalent number of units in the second cycle is computed as:

$$n_{e_2} = U_1 e^{-\alpha_2 n_2 t} + \frac{1-e^{-\alpha_2 t n_2}}{e^{\alpha_2 t}-1} \tag{8b}$$

Similar to Eq. (8a), the equivalent number of units remembered from cycles 1 and 2 at the beginning of the third cycle is computed as:

$$U_2 = U_1 e^{-\alpha_2(n_2 t+d_2)} + \frac{e^{-\alpha_2 d_2} - e^{-\alpha_2(n_2 t+d_2)}}{e^{\alpha_2 t}-1} = e^{-\alpha_2 d_2}\left[U_1 e^{-\alpha_2 n_2 t} + \frac{1-e^{-\alpha_2 n_2 t}}{e^{\alpha_2 t}-1}\right] \tag{8c}$$

where d_2 is the breaktime between cycles 2 and 3. Again, when there is no forgetting, $\alpha_2 = 0$, $U_2 = n_1 + n_2$ or $U_2 = 2n$ when $n = n_1 = n_2$; i.e., full transfer of learning. The equivalent number of units in the third cycle would be written as:

$$n_{e_3} = U_2 e^{-\alpha_3 n_3 t} + \frac{1-e^{-\alpha_3 t n_3}}{e^{\alpha_3 t}-1} \tag{8d}$$

Similar to Eq. (8c), the equivalent number of units remembered from the third and all earlier cycles at the beginning of the fourth cycle is computed as:

$$U_3 = U_2 e^{-\alpha_3(n_3 t+d_3)} + \frac{e^{-\alpha_3 d_3} - e^{-\alpha_3(n_3 t+d_3)}}{e^{\alpha_3 t}-1} = e^{-\alpha_3 d_3}\left[U_2 e^{-\alpha_3 n_3 t} + \frac{1-e^{-\alpha_3 n_3 t}}{e^{\alpha_3 t}-1}\right] \tag{8e}$$

where d_3 is the break time between cycles 3 and 4. So the general forms are:

$$U_i = e^{-\alpha_i d_i}\left[U_{i-1} e^{-\alpha_i n_i t} + \frac{1-e^{-\alpha_i n_i t}}{e^{\alpha_i t}-1}\right] \tag{8f}$$

$$n_{e_i} = U_{i-1} e^{-\alpha_i n_i t} + \frac{1-e^{-\alpha_i t n_i}}{e^{\alpha_i t}-1} \tag{8g}$$

The general form of the Interference-Adjusted Learning-Forgetting Curve Model, henceforth IALFCM, is:

$$T_{n_i} = T_1 {n_{e_i}}^{-b} \tag{8h}$$

where T_{n_i} and T_1 are estimated times to produce n th unit in cycle i and the first unit and b is the learning exponent, like Wright's learning curve

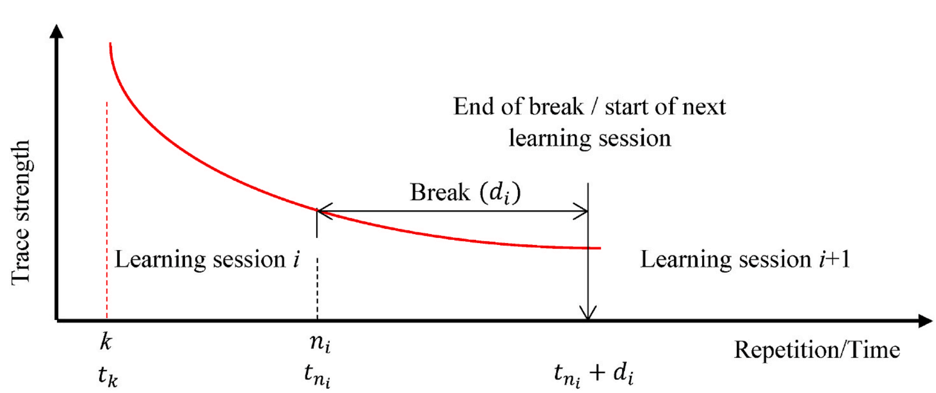

Fig. 4. Schematic showing decay of memory trace for repetition k < n over time.

Table 2
Results from fitting the IALFCM and the WLC + displacement model to a sample dataset from Bailey and McIntyre (1992).

j	*i*	*Observed*		*IALFCM*		*WLC + displacement*	
		n_i	$\widehat{y}_j$	n_{e_i}	y_j	n_{e_i}	y_j
1	1	1	22.48	0.72	22.35	1	21.17
2		2	13.9	1.23	13.82	2	14.88
3		3	10.02	1.59	10.95	3	12.11
4		4	9.83	1.86	9.56	4	10.46
5		5	8.8	2.04	8.78	5	9.34
6		6	7.97	2.18	8.29	6	8.51
7		7	8.3	2.27	7.98	7	7.87
8		8	8.13	2.34	7.77	8	7.35
9		9	7.77	2.39	7.63	9	6.92
10		10	7.17	2.43	7.53	10	6.56
11		11	7.78	2.45	7.46	11	6.25
12	2	1	14.3	1.16	14.5	2.18	14.02
13		2	11.65	1.57	11.12	3.18	11.57
14		3	8.72	1.86	9.55	4.18	10.07
15		4	9.03	2.07	8.68	5.18	9.03
16		5	8.58	2.22	8.14	6.18	8.26
17		6	7.83	2.33	7.8	7.18	7.65
18		7	7.55	2.41	7.57	8.18	7.16
19		8	7.05	2.47	7.41	9.18	6.75

(WLC, Wright, 1936), and n_{e_i} is the residual (after interference/forgetting) experience in cycle *i*.

3.1. Numerical example of IALFCM

In the example of Table 2 below, $\widehat{y}_j$ refers to the observed value of repetition *j*, where $j = 1,2, \ldots,19$. Between the cycles 1 and 2, there is a break of length $d_1 = 70$ days, which corresponds to 100800 min. The optimal values of the parameters of the IALFCM model that produce the results in Table 2 for which the *MSE* ($\sum_{j=1}^{19}(\widehat{y}_j - y_j)^2/19 = 0.158$) is minimum are $y_1 = 16.58$, $\alpha_1 = 0.0000139$, $\alpha_2 = 0.0000134$, $t = 24035.581$, and $b = 0.891$. The optimal parameter values for the reference model (WLC + displacement) are $y_1 = 21.17$ and $b = 0.509$, with $MSE = 0.764$. The IALFCM produced lower *MSE* for both the first ($MSE = 0.141$ vs 1.111) and the second ($MSE = 0.182$ vs 0.286) learning session. The resultant decay exponents for cycles are $\gamma_1 = \alpha_1 t = 0.335$ and $\gamma_2 = \alpha_2 t = 0.323$, i.e., IALFCM captures interference and with a 4% lower exponent in the second cycle than in the first one. By the end of the first and second sessions, a worker retained $n_{e_{11}} = 2.45$ and $n_{e_{19}} = 2.47$ units of experience, and not 11 and 9.18 units as for the WLC + displacement model. The observed and estimated values of Table 2 are illustrated in Fig. 3 in Section 1.

4. Computational analysis

The ability of the IALFCM and the three alternative LaF models, the LFCM, the M-LFCM, and the RCM, to produce good fits is tested on empirical data from manual assembly and inspection tasks. The datasets were selected for three reasons. First, they were available for us. Second, they are highly relevant to industry (unlike, e.g., data entry task (Globerson et al., 1989)), more cognitive than motor tasks, thus subject to significant LaF effects (Abubakar and Wang, 2019). Third, the datasets represent different numbers of learning sessions and breaks with varying lengths. The *MSE* (Mean Square of Errors) is used to assess the quality of fits of a learning curve to data, which is in line with the literature (Badiru, 1994; Nembhard and Sun, 2019; Sikström and Jaber, 2012; Stratman et al., 2004; Towill, 1977). The mathematical form is given as:

$$\text{Minimize } MSE = \frac{1}{N}\sum_{i=1}^{n}(\overline{T}_i - T_i)^2 \tag{9}$$

Subject to:

$T_1 > 0$

$0 \le b < 1$

$0 \le \alpha_i < 1$

$t > 0$

For comparison of the models, we use the following baseline:

$$T_n = T_1(u_i + n_i)^{-b} \tag{10}$$

where T_1, u_i and b are the fitting parameters, and $u_1 = 0$, $0 \le u_i \le \sum_{j=2}^{j-1} u_j$. The baseline assumes a fixed learning exponent and varying prior experience for each learning cycle. As a performance indicator of the models, we use

$$\Delta_j = MSE_j - MSE_{baseline} \tag{11}$$

where j = LFCM, M-LFCM, RCM, IALFCM.

We used the Evolutionary Engine for Excel Solver to optimize the parameter values for each model. For each dataset and model, the results from the fits are presented, compared, and analyzed.

4.1. Testing the models against Bailey (1989) and Bailey's and McIntyre's (1992, 1997) data

The three product assembly datasets (Bailey, 1989; Bailey and McIntyre, 1992, 1997) were collected from experiments performed by students in a laboratory setting. The experiments consisted of two or three learning sessions separated by breaks of varying lengths. The selection of subjects' learning data from Bailey's experiments is based on the versatility of learning data, in terms of the number of breaks and their lengths, to ensure a proper comparison of how the LaF models fit various datasets.

Bailey (1989) found that the forgetting exponent, *f*, is dependent on the amount learned before the break and the break length, *d*, but that it is independent of the initial performance time, T_1, or the learning exponent, *b*. He also found that the learning exponent of the second cycle depends on that of the first. He initially considered 8 h of working in the first learning session. To reduce fatigue, ten individuals completed the session for two consecutive days, 4 h per day. We have considered overnight breaks (1-day) in the fits since they caused forgetting; i.e., $T_{1,i+1} > T_{n,i}$. Table 3 presents the descriptive statistics for the ten individuals (IDs #1 − 10) in terms of the total number of repetitions, the number of repetitions in cycle *i*, and the length of a break, in days, separating cycles *i* and $i + 1$.

Bailey and McIntyre (1992) compared different relearning curves (i.e., for the second cycle). Their results suggested that the functional form that is best for the learning curve is also best for the relearning curve and that the choice among alternative models becomes more crucial to increasing the level of forgetting. We have fitted the models against learning data from four individuals with the longest breaks (IDs #11 − 14, Table 3). Bailey and McIntyre (1997) studied a relearning curve starting anew (i.e., $n = 1$ for the first post-break repetition and determining new parameter values). This approach provided more accurate predictions than backing up an existing learning curve, especially when a small number of repetitions are available to fit a relearning curve. We have tested the fits of the models against data from twenty-five subjects. These subjects (IDs #15 − 39, Table 3) form three subgroups, each with their dedicated break length of about 13, 34, and 42 days. Table 3

Table 3
Descriptive statistics of individual learning data from Bailey's experiments. ID# = Individual id, Rep. Total = Total number of repetitions, $Repetitions_i[Breakdays_i]$ = The number of consecutive repetitions and break days for each learning cycle, i, Δ_j = performance (MSE_j) of the models, j, in relation to the baseline, with the best model highlighted in grey. An underlined grey number in square brackets indicates 1-day break does not cause forgetting.

Dataset	ID#	Rep. total	$Repetitions_{i=1}[Breakdays_{i=1}]$ $Repetitions_{i=2}[Breakdays_{i=2}]$ $Repetitions_{i=3}$	$\Delta_j = MSE_j - MSE_{baseline}$			
				$j = LFCM$	$M - LFCM$	RCM	$IALFCM$
Bailey (1989)	1	27	7[1]10[14]10	0.078	0.000	0.783	−0.906
	2	23	6[1]8[14]9	0.196	0.000	−0.766	−0.841
	3	21	7[1]8[58]6	0.276	0.000	1.404	−1.356
	4	15	7[1]2[58]6	0.002	0.000	2.412	−0.839
	5	12	2[1]5[79]5	2.280	0.000	4.622	−4.283
	6	23	6[1]9[100]8	0.033	0.000	3.860	−3.437
	7	17	4[1]7[100]6	0.169	0.000	6.080	−0.055
	8	13	3[1]5[114]5	0.419	0.000	29.381	−9.268
	9	17	5[1]6[114]6	0.065	0.000	5.403	−2.771
	10	21	6[1]8[114]7	0.038	0.000	1.216	−1.468
			Average IDs #1−10:	**0.356**	**0.000**	**5.440**	**−2.522**
Bailey and McIntyre (1992)	11	13	8[119]5	0.000	-	−0.840	−0.658
	12	10	5[133]5	0.000	-	0.139	4.489
	13	13	7[133]6	0.000	-	−0.519	−0.438
	14	10	5[175]5	0.000	-	1.458	2.109
Bailey and McIntyre (1997) (1)	15	27	14[13]13	0.000	-	0.116	−0.290
	16	43	22[13]21	0.000	-	0.251	−0.574
	17	37	19[13]18	0.000	-	−0.047	−0.017
	18	19	9[13]10	0.000	-	−0.270	0.000
	19	31	15[13]16	0.000	-	0.180	−0.240
	20	32	16[14]16	0.000	-	0.394	−0.265
	21	29	13[14]16	0.000	-	0.032	−0.384
	22	28	13[14]15	0.000	-	0.010	−0.463
	23	25	11[14]14	0.000	-	0.162	−0.375
	24	26	13[14]13	0.000	-	1.369	−2.392
Bailey and McIntyre (1997) (2)	25	33	16[34]17	0.000	-	0.055	−0.147
	26	23	10[34]13	0.000	-	1.622	−1.143
	27	21	9[34]12	0.000	-	0.552	−0.844
	28	26	12[34]14	0.000	-	−0.265	0.000
	29	20	9[34]11	0.000	-	3.614	−5.954
	30	25	11[35]14	0.000	-	0.170	1.791
	31	23	10[35]13	0.000	-	0.803	−1.119
	32	27	11[35]16	0.000	-	−0.099	−0.116
	33	17	7[35]10	0.000	-	0.550	−2.203
	34	18	8[35]10	0.000	-	1.902	−3.612
Bailey and McIntyre (1997) (3)	35	19	9[41]10	0.000	-	−0.306	0.028
	36	30	15[41]15	0.000	-	0.091	−0.528
	37	28	13[42]15	0.000	-	0.549	−1.282
	38	26	12[42]14	0.000	-	0.608	−2.052
	39	28	13[43]15	0.000	-	0.319	−0.304
			Average IDs #11−39:	**0.000**	**-**	**0.435**	**−0.586**

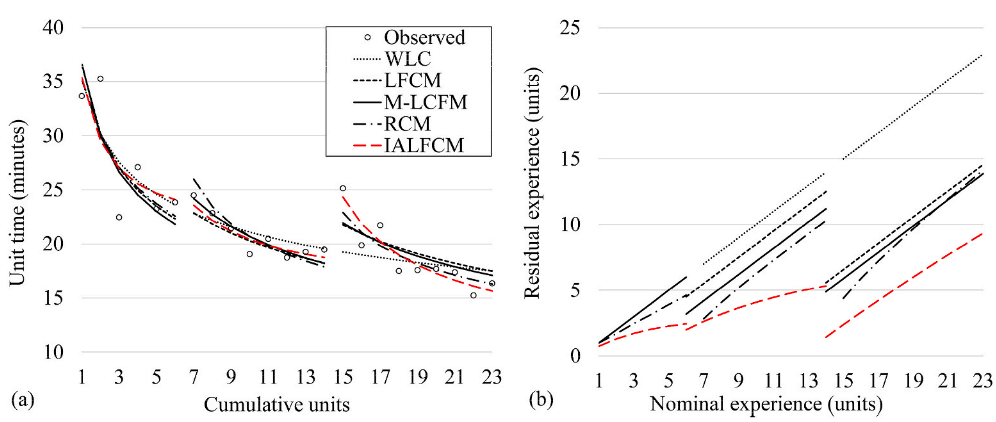

Fig. 5. (a) An example of observed learning data (ID#2 in Table 3) and fitted trend lines of the learning-forgetting models; (b) Residual (after forgetting/interference) experience as a function of nominal experience.

compares the errors of model j to the baseline model in Eq. (10). The lower the result of the subtraction, $MSE_j - MSE_{baseline}$, the better the performance. Note that since the data of Bailey and McIntyre (1992, 1997) only consider one break, the M-LFCM reduces to the LFCM.

Table 3 shows that either the M-LFCM (two breaks) or the LFCM (one break) produces a perfect fit in all cases, totaling 39, i.e., the same as the baseline in Eq. (10). When there are two breaks (IDs #1 − 10), the RCM performs the worst ($\Delta_{RCM,avg} = 5.440$, $\Delta_{RCM,best} = -0.766$, and $\Delta_{RCM,worst} = 29.381$), the LFCM the second-worst ($\Delta_{LFCM,avg} = 0.356$, $\Delta_{LFCM,best} = 0.002$, and $\Delta_{LFCM,worst} = 2.280$), whereas the IALFCM performs the best ($\Delta_{IALFCM,avg} = -2.522$, $\Delta_{IALFCM,best} = -9.268$, and $\Delta_{IALFCM,worst} = -0.055$). In the case of one break (IDs #11− 39), IALFCM performs the best ($\Delta_{IALFCM,avg} = -0.586$, $\Delta_{IALFCM,best} = -5.954$, and $\Delta_{IALFCM,worst} = 4.489$), and the RCM the worst ($\Delta_{RCM,avg} = 0.435$, $\Delta_{RCM,best} = -0.840$, and $\Delta_{RCM,worst} = 3.614$), and outperform the baseline in 23 and 7 cases, respectively. In general, IALFCM performs the best (1st), the M-LFCM comes second, LFCM comes third, and the RCM comes last, suggesting that a model with the most parameters tends to have the best performance. Fig. 5 illustrates the behavior of the models over three learning sessions (and two breaks) by using the observed learning data from ID#2. Fig. 5a presents the fitted trend lines of the models. The WLC, a reference model, does not account for forgetting and overestimates the performance after the break. The LFCM and the M-LFCM account for the forgetting effects resulting from interruptions (i.e., production breaks). The latter model is, unlike the LFCM, flexible as its input parameter, which is the time for total forgetting to occur, varies with each break. This feature makes it adjust more effectively to relearning curves. The RCM and the IALFCM, on the other hand, assume forgetting occurs during a break and learning sessions (IALFCM through interference), allowing for the residual experience from each repetition could be less than the nominal, measured in cumulative units. Fig. 5b illustrates this with residual experience increasing at a decreasing rate through repetitions (and not linearly as with the WLC, LFCM, and M-LFCM). Learning data in Fig. 5a shows plateauing, a characteristic that is typical for assembly tasks. The IALFCM and the RCM capture the plateauing phenomenon the best, as they both adjust the curvature of the trend line to the data points. The IALFCM has an advantage over the RCM and the other models as it accounts for varying degrees of interference (curvature of the trend line) over learning-forgetting cycles. It also assumes, unlike the RCM, interference (forgetting) effect applies to the first units in each session, which gives additional flexibility in estimating unit times. Fig. 5 is the only case of two breaks when RCM outperforms LFCM and M-LFCM (baseline). Table 3 shows that, on average, forgetting during a break (LFCM) fits Bailey's data better than continuous forgetting through production runs and breaks (RCM).

4.2. Testing the models against car safety-seat assembly data

The other data set is from Arkite NV (https://arkite.com/, personal communication, March 2020) research team. It was collected from a car safety-seat assembly plant and sheltered workplace and contained the start and end times of jobs for a total of 14 workers. We have tested a minimum break length of one day (overnight) for which forgetting could occur and split the data into learning cycles accordingly. This approach is like the one in Bailey. Fitting the WLC model in Eq. (1) to data points comprising each learning cycle showed considerable scatter around the trend line and revealed values that were exceptionally far above the trend line. This finding was probably due to careless registration. On average, 7% of the data points are outliers (a 1% maximum false discovery rate on Prism 8 software) and, therefore, were removed. Table 4 shows the descriptive statistics of (cleaned) learning data. Fitting the WLC to the mentioned data showed that, for 12 out of 14 workers, learning or forgetting does not occur (i.e., $b_i = 0$ or $T_{1,i+1} \le T_{n,i}$), at least for one cycle. Such periods are described as underlined numbers (black for repetitions and grey in square brackets for break days, respectively). It is worth noting that no learning often means no forgetting during the subsequent break and vice versa. Table 4 also presents the results from fitting the models to the data.

From Table 4, we analyze the cases with more than two breaks (IDs #3 − 14). LFCM ($\Delta_{LFCM,avg} = 800.139$, $\Delta_{LFCM,best} = 35.672$, and $\Delta_{LFCM,worst} = 3435.478$) and RCM ($\Delta_{RCM,avg} = 865.095$, $\Delta_{RCM,best} = 117.997$, and $\Delta_{RCM,worst} = 3144.527$) always perform worse than the baseline model. M-LFCM ($\Delta_{M-LFCM,avg} = 41.707$,

Table 4
Descriptive statistics of car safety seat assembly data.

ID#	I	$\sum_{i=1}^{I} Repetitions_i\ [\sum_{i=1}^{I-1} Breakdays_i] = Repetitions_{i=1}[Breakdays_{i=1}] \ldots Repetitions_{i+1}[Breakdays_{i+1}] \ldots [Breakdays_{I-1}]Repetitions_I$	$\Delta_j = MSE_j - MSE_{baseline}$			
			$j = LFCM$	$M-LFCM$	RCM	$IALFCM$
1	2	15[1] = 9[1]6	0.000	-	575.625	−358.894
2	2	34[28] = 29[28]$\underline{5}$	0.000	-	893.477	761.703
		Average IDs #1−10:	**0.000**	**-**	**734.551**	**201.405**
3	4	116[75] = 11[5]41[3]49[67]15	35.672	0.000	197.572	228.885
4	7	111[15] = 22[1]$\underline{19}$[1]$\underline{9}$$\underline{[7]}$19[1]15[4]11[1]16	1587.322	38.602	1820.638	−1438.829
5	8	167[15] = 10[3]12[3]17[1]33[1]36[5]17$\underline{[1]}$$\underline{22}$$\underline{[1]}$20	208.854	6.892	209.435	143.170
6	9	164[15] = 13[2]17[1]$\underline{16}$[3]33[1]15$\underline{[1]}$19[1]18[1]18[5]15	408.045	69.805	520.956	158.019
7	9	187[35] = 10[2]16[1]13$\underline{[4]}$18[1]21[1]46[1]26[24]16$\underline{[1]}$21	96.691	0.000	472.597	336.413
8	10	255[50] = 8[1]12[2]35$\underline{[1]}$38[3]38[1]38[37]21[1]20[3]22[1]23	70.482	0.000	380.448	405.172
9	10	199[29] = 13[2]20[1]18[4]20[2]23[1]26[3]19[1]$\underline{18}$$\underline{[10]}$$\underline{20}$[5]22	405.866	0.000	549.021	315.385
10	11	149[23] = 14[2]12[1]16$\underline{[6]}$$\underline{9}$[1]9$\underline{[1]}$12[6]$\underline{12}$[1]11[3]$\underline{17}$$\underline{[1]}$$\underline{21}$[1] $\underline{16}$	2442.958	197.655	2139.396	99.048
11	11	256[21] = 11$\underline{[3]}$20[4]22[2]26[1]23[3]$\underline{25}$$\underline{[1]}$27[2]24$\underline{[1]}$27[3]26 [1]25	60.500	4.728	117.997	−25.588
12	12	196[21] = 11[2]13[1]14$\underline{[3]}$18[1]18$\underline{[2]}$19$\underline{[4]}$11[1]18$\underline{[2]}$17$\underline{[1]}$$\underline{17}$[3]18$\underline{[1]}$$\underline{22}$	340.058	4.783	312.032	442.631
13	13	62[29] = 4[1]5[2]4$\underline{[7]}$$\underline{3}$$\underline{[1]}$6[6]$\underline{2}$[1]5[3]$\underline{4}$$\underline{[1]}$$\underline{7}$[2]3[1]5$\underline{[3]}$$\underline{10}$[1]$\underline{4}$	3435.478	0.000	3144.527	6054.841
14	18	475[132] = 14[1]16[1]14$\underline{[4]}$$\underline{15}$[1]$\underline{13}$$\underline{[2]}$$\underline{15}$$\underline{[4]}$17[1]14[1]$\underline{17}$[5]16$\underline{[35]}54\underline{[10]}$$\underline{26}$[4]$\underline{53}$[1]47[6]45[2]42[5]25[49]$\underline{32}$	509.747	178.017	516.526	−188.353
		Average IDs #3−14:	**800.139**	**41.707**	**865.095**	**544.233**

I = number of learning cycles. Note: underlined numbers indicate repetitions/breaks do not cause learning/forgetting.

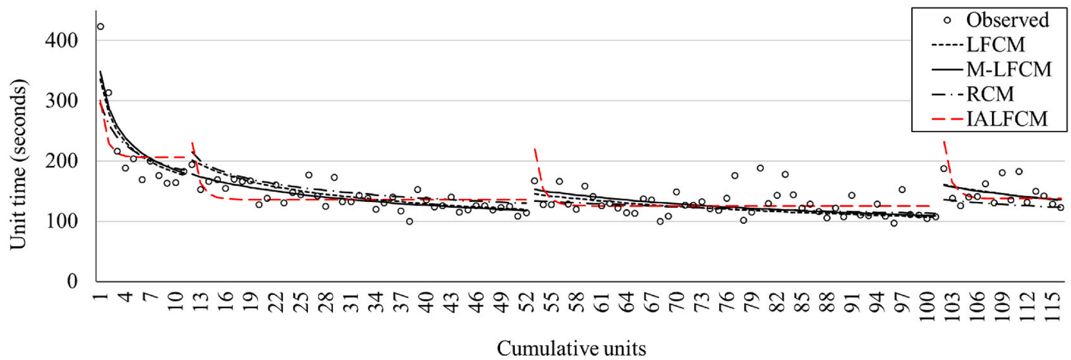

Fig. 6. Learning-forgetting curves fitted to the observed data (ID#3 in Table 4).

$\Delta_{M-LFCM,best} = 0.000$, and $\Delta_{M-LFCM,worst} = 197.655$) performs equally in 5 and worse in 7 cases than the baseline model. IALFCM ($\Delta_{IALFCM,avg} = 544.233$, $\Delta_{IALFCM,best} = -1438.829$, and $\Delta_{IALFCM,worst} = 6054.841$) performs better than the baseline model in 3 cases and worse in 9 cases, respectively. These results show that the M-LFCM fits, on average, the best (1st), the IALFCM comes second, the LFCM comes third, and the RCM comes last. The results show that the M-LFCM fits most of the data well, perhaps better for a smaller number of sessions ($I = 4 - 10$). The performance of the IALFCM varies depending on individual learning data. In some instances (IDs #3, 8, 12, and 13), it is worse, and in others (IDs #4, 10, 11, and 14) far better than the other models. Fig. 6 is an example where the IALFCM (trend line in red) does not fit the data that well. Note that the IALFCM has five fit parameters in this case, of which two are the same for the other models, which are T_1 and b. The third parameter, t, is a fixed time step in the calculation procedure, and the fourth and fifth parameters, α_i ($i = 1,2$), are cycle-specific. The speed of the decay rate (interference factor), $\gamma_i = \alpha_i t$, affects the curvature or plateauing behavior in when comparing to other learning curve models. The IALFCM, however, underestimated first-unit times and plateauing levels of most sessions in Fig. 6. The learning exponent was at the upper limit ($b = 1$), and memory traces entirely decayed during the breaks.

Fig. 7 shows an example where the fit of the IALFCM outperforms the other models. The learning data points show an unusual rise on the third learning session/day, indicating excessive cognitive interference for this subject. It is worth noting here that the subjects experienced cognitive disabilities despite keeping the total cognitive load low. For some, it was high, which may cause additional interference. IALFCM captures such scatter in learning data the best by adjusting the interference factor. The M-LFCM captures, better than the other models, a typical early learning profile (i.e., the first two sessions) when performance shows a wide variation (scatter), but the trend is rapidly improving (e.g., IDs #7 – 9, 13). In these cases, IALFCM seeks to capture the scatter by adjusting its curvature and plateauing forms.

Table 4 shows that the LFCM and the RCM perform roughly equally. They fit very poorly when data show a large scatter (IDs #4, 10, and 13). The LFCM provides better fits when learning occurs through repetition and forgetting during breaks (IDs #3, 7, and 8), i.e., usual learning-forgetting profile, and RCM vice versa (IDs #10, 12, and 13). The recency measure of the RCM sensitivity to variations in break lengths is high. In this regard, the RCM fit to data separated by significantly long breaks is poor.

4.3. Testing the models against car radio inspection data

The last dataset from Nembhard and Osothsilp (2001, Fig. 1) represents an individual worker. The data are from a final inspection station of a car radio manufacturer for six months. In total, 71 permanent workers performed 130 inspection steps for each of the six radio models. In Fig. 8, each data point is a median of the production time of 20 units, presented as a function of cumulative production. The study considered that the forgetting effect begins after a break of 50 h, which captures over weekend breaks. There were seven breaks having lengths of 3.7, 6.5, 11.6, 3.8, 4.7, 3.7, and 11.2 days listed in chronological order. Nembhard and Osothsilp (2001) fitted the WLC model to each learning cycle while varying the learning slope showing that the initial production time increases following a break but decreases with cumulative experience. Table 5 summarizes the results from fitting the models to the data. Fig. 8 shows the learning-forgetting curves generated from observed data, whereas Fig. 9 shows the residual experience (after forgetting/interference) as a function of nominal experience measured in cumulative units.

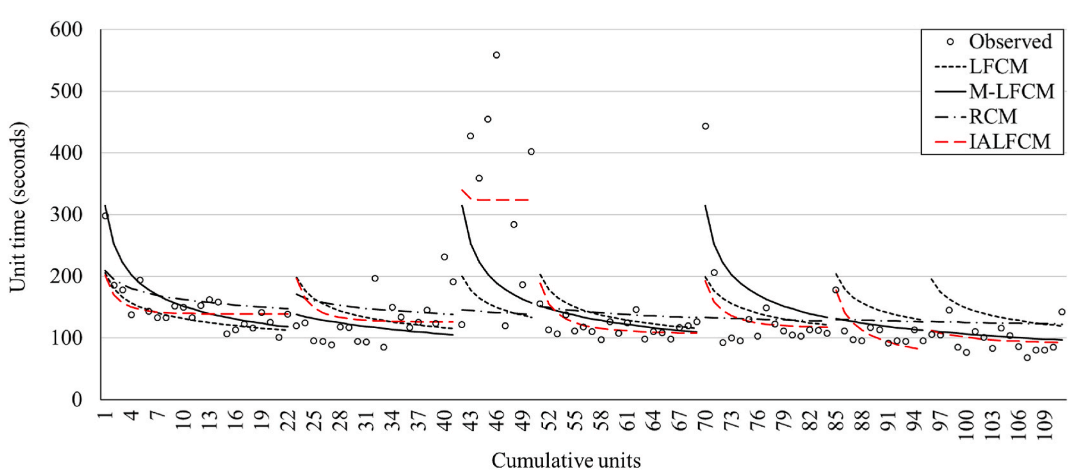

Fig. 7. Learning-forgetting curves fitted to the observed data (ID#4 in Table 4).

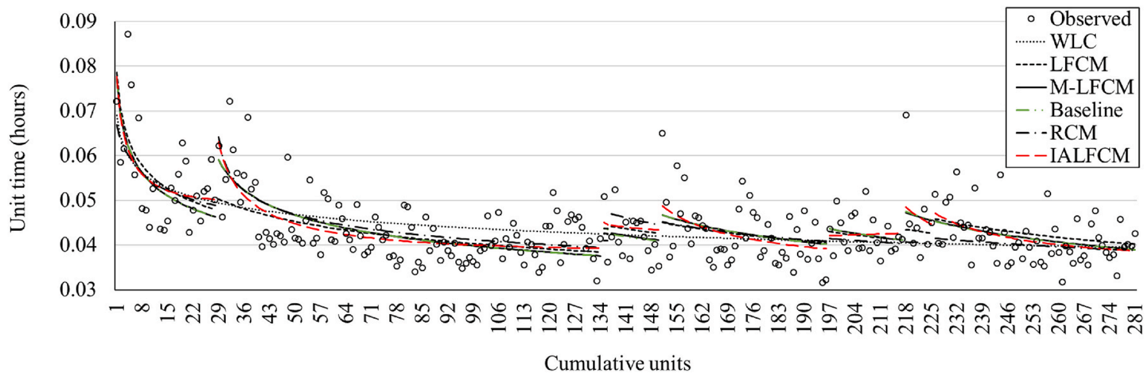

Fig. 8. Learning curves generated from observed learning data from Nembhard and Osothsilp (2001, Fig. 1).

Table 5
Results from fitting the models to data from Nembhard and Osothsilp (2001).

$\Delta_j = MSE_j - MSE_{baseline}$			
j = *LFCM*	*M – LFCM*	*RCM*	*IALFCM*
2.297E-06	1.128E-08	2.847E-06	– **1.439E-06**
Parameters			
$T_1 = 0.079$ $b = 0.148$ d_m = 1.31E+16	$T_1 = 0.077$ $b = 0.153$ $d_{1,m} = 3188.841$ $d_{2,m}$ = 1.063E+9 $d_{3,m}$ = 1.8E+308 $d_{4,m}$ = 3.039E+7 $d_{5,m}$ = 1.27E+12 $d_{6,m}$ = 8.412E+6 $d_{7,m}$ = 1.8E+308	$T_1 = 0.067$ $b = 0.113$ $a = 0.958$	$T_1 = 0.076$ $b = 0.186$ $t = 5.133$ $\alpha_1 = 0.018$ $\alpha_2 = 0.005$ $\alpha_3 = 0.000$ $\alpha_4 = 0.008$ $\alpha_5 = 0.003$ $\alpha_6 = 0.009$ $\alpha_7 = 0.001$ $\alpha_8 = 0.004$

The results in Table 5 show that the IALFCM fits car radio inspection data the best, followed by M-LFCM, LFCM, and RCM. The three best models approximately have the same estimated time to perform the first unit, T_1. IALFCM has the fastest learning rate, LR = 2^(-b) = 87.9% and RCM the slowest, LR = 92.5%. By adjusting the interference factor, or memory traces decay exponent, IALFCM can adjust the curvature to fit the observed data. Fig. 9 shows the residual experience over repetitions, the variable that differentiates behavior between the models, where the WLC is the reference model. It has two fundamental drawbacks: (1) it does not account for the effect of forgetting during breaks, and (2) it assumes repetitions are equally effective. The LFCM and M-LFCM, to some extent, correct the first drawback, i.e., an event of production interruption results in experience loss. The RCM and IALFCM correct both drawbacks, i.e., the continuous loss of experience occurring through production and interruption periods. The IALFCM estimates the lowest residual experience over the entire data, RCM second lowest for the first half, and M-LFCM for the second half of production. The LFCM estimates the largest residual experience, except the RCM for the last

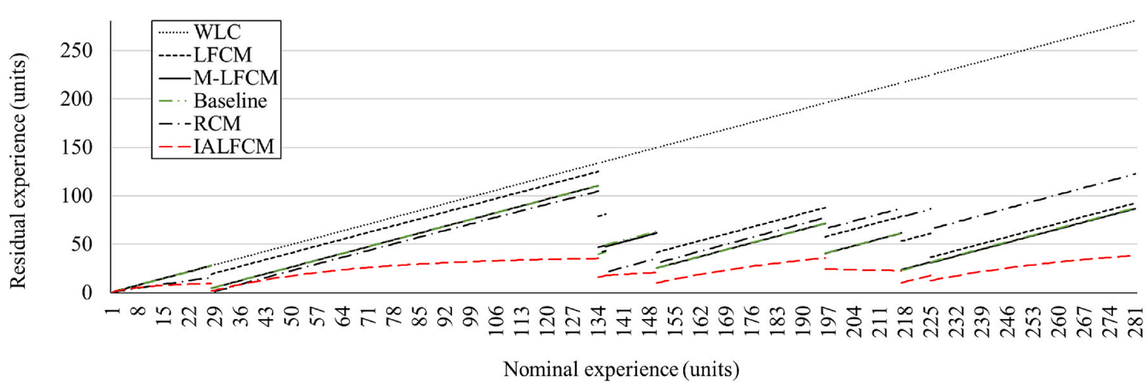

Fig. 9. Residual (after forgetting/interference) experience as a function of nominal experience.

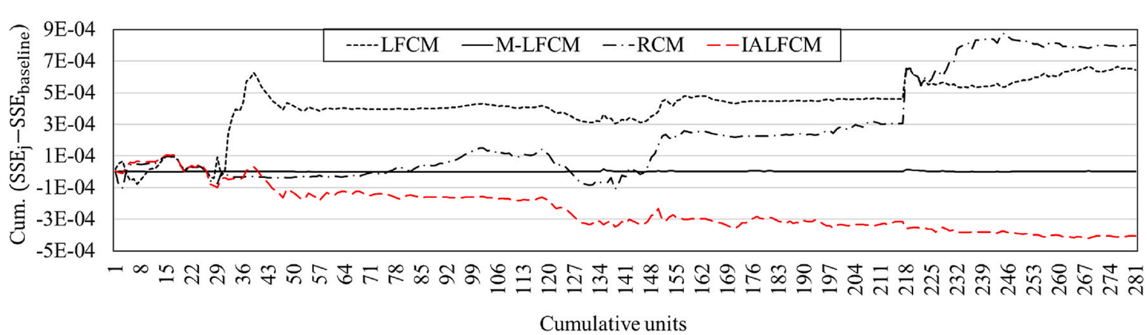

Fig. 10. Cumulative $SSE_j - SSE_{baseline}$.

three production runs. The lower the residual, i.e., the greater the correction of experience, the better the model performs. Fig. 10 illustrates the cumulative performance of the models, how much they deviate from the baseline ($SSE_j - SSE_{baseline}$). The performance of RCM deteriorates towards the end of production, where the LFCM slightly outperforms it by 1.4% lower SSE. This result is due to the recency measure resulting in later repetitions being less effective, while the effect of forgetting is negligible ($a = 0.958$). The RCM also does not consider the relearning curve starting anew (i.e., $n = 1$ for the first post-break repetition and determining new parameter values). This result makes the RCM inaccurate, especially when a small number of repetitions are available to fit a relearning curve (e.g., the second last in Fig. 8). The fits of the IALFCM showed that interference plays a significant role in the first cycle, where performance ceases quite fast, and interruption results in significant forgetting (Fig. 8). The second significant interference occurs after about 200 units, where IALFCM does not estimate improvement through repetitions at all.

4.4. Managerial insights

The results show that adjustment for cognitive interference enhances, on average, the predictive ability of the power learning curve more than other models with forgetting. Acknowledging that cognitive interference can potentially cause significant loss of knowledge over time would facilitate the decision-making process of industrial managers. In this regard, our model is the best, especially for a small number of production cycles (two or three) with relatively short interruptions. Such early production cycles are more likely to cause novice workers to experience interference due to processing a large amount of information while simultaneously focusing on other details that overload their working memory. Such learners typically are poor performers at first (post-break) repetitions, where interference explains why reaching plateauing is faster in subsequent repetitions.

Using the interference-adjusted model could support managers with more accurate budgeting of labor costs, especially for novice workers or when workers frequently learn new tasks. For example, assume a $15 hourly wage and 1700 h annually working per worker. Then, use the data of ten individuals from Bailey (1989), IDs #1 − 10, Table 3, and calculate the cumulative sum of the deviations estimated from the actual assembly hours $\sum_{i=1}^{n} \widehat{y}_i - y_{i_j}$ for each model $j =$ LFCM, M-LFCM, RCM, IALFCM. We then linearly extrapolate the cumulative deviation to the annual level for each LaF model and sum them up for ten workers. The IALFCM overestimates the actual labor cost by $74, M-LFCM by $489, LFCM by $504, and RCM by $904, suggesting that IALFCM estimates the budget much more realistic than other models do. Using the IALFCM for the first few learning cycles would help identify those struggling learners and release precocious ones earlier than expected. Managers could quickly notice if employees need more support in the learning process. Considering the size of information and how one presents it to learners will help ensure smooth improvements towards a target standard-time. Our model could help managers detect periods of excessive interference for workers with cognitive disabilities when scheduling work or assigning tasks to workers. This feature provides insights for their instructors, when such an undesirable interference is most likely to occur, how to avoid it, and promote their working ability.

Despite its better accuracy and novelty in parameterizing the interference effect, the IALFCM has limitations regarding its implementation. One of those is that the model becomes computationally expensive. For example, the number of fitting parameters increases proportionally to the number of production cycles. Another limitation is that as memory traces continue decaying over the break following the production cycle, long ones following plateaued performance may result in total decay (forgetting) and decrease the model accuracy as production resumes.

5. Conclusions

There is consensus among psychologists that interference of human cognition always results in forgetting. Cognitive interference depends on the content presented at each time point to a human whose working memory is limited. In practice, when a learner cannot absorb all the information, a working memory overload is experienced. This observation suggests that some knowledge is continually lost over time, making it critical when performing psychomotor tasks. Inspired by this phenomenon, this paper challenges some prominent forgetting curve models in production and operations management that usually depict the relationship between break length and intensity of deterioration of task performance. More precisely, this paper presented an enhanced version of the power learning curve that accounts for cognitive interference when learning (repeating a task) and forgetting due to breaks, referred to here as the Interference–Adjusted Learning-Forgetting Curve Model (IALFCM). In addition to forgetting as arising from production break, it shows that the experience gained during a learning session is not fully retained by the end of a learning session, i.e., when an interruption starts. To do this, a recently modified version of the Wright learning curve that accounts for interference (Jaber et al., 2021) has been extended to consider multiple learning sessions separated by production breaks. More precisely, the interference-adjusted number of cumulative units has been used as a proxy to measure residual experience. The model of this paper assumes that each repetition leaves a memory trace that decays exponentially with time. The IALFCM and three alternative models (Learn-Forget Curve Model, LFCM, its modified version, M-LFCM, and the Recency Model, RCM) have been tested against several empirical data sets from manual assembly and inspection tasks. The results showed that the IALFCM fitted the data very well. It produced much fewer errors than the other models. This finding was noticeable when learning data has one or two breaks or when the learning dataset points show plateauing and an unusual rise above the trend, reflecting various degrees of interference in learning processes. The interference parameter enables more curvature or plateauing, compared to other learning curve models. However, interference in learning, followed by a long break, would result in total decay (forgetting) of memory traces, thus decreasing model accuracy when production resumes. A limitation of the IALFCM is that it becomes computationally expensive when the number of production cycles increases, which increases the number of fitting parameters.

The model developed here is the first attempt to capture the relative level of interference for production runs separated by breaks. As such, it has many applications in production decision-making. For example, the model could detect an abnormally large interference, thus reflecting, e.g., memory overloading, worker fatigue, or cognitive disabilities in the learning process, requiring managers to provide additional support for better production planning. This observation would also help managers save on the training budget as managers could determine the frequency and lengths of training sessions. With its improved accuracy, the model also enables more realistic budgeting of labor costs.

The model of this paper and the findings reported would serve as seeds for future research. For example, one could set up an experiment to test how overloading memory with different information affects performance in a psychomotor task. Further, developing a real-time tracking system of operator performance with adaptive instructions (information amount) could provide a worker with the most efficient learning experience.

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Author statement

Jaakko Peltokorpi: Methodology, Software, Validation, Formal analysis, Investigation, Visualization, Writing – original draft, Project administration, Funding acquisition.

Mohamad Y. Jaber: Conceptualization, Methodology, Supervision, Funding acquisition, Writing – review & editing, Supervising.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Table A.1
List of abbreviations and notations in the article

Abbreviation	Explanation	Notation	Explanation
LaF	Learning and forgetting	n (n_i)	Unit number or cumulative production (in cycle i up to the point of interruption)
WLC	Wright's learning curve	T_n (T_{n_i})	Time to produce unit n (in cycle i)
POM	Production and operations management	T_1	Time to produce the first unit
PID	Power-Integration-Diffusion	b	Learning exponent
LFCM	Learn-forget curve model	$\widehat{T}_n$	Time for nth unit of lost experience of the forgetting curve
M-LFCM	Modified learn-forget curve model	$\widehat{T}_1$ ($\widehat{T}_{1,i}$)	Forgetting curve intercept (for cycle i)
RCM	Recency model	f_i	Forgetting exponent (slope) for cycle i
IALFCM	Interference-Adjusted Learning-Forgetting Curve Model	d_m ($d_{m,i}$)	Break time to which total forgetting occurs (at cycle i)
MWLC	Modified Wright's learning curve	u_i or U_i	Residual knowledge, measured in units, from previous cycles at the beginning of cycle i
AMWLC	Approximate modified Wright's learning curve	$t(u_i + n_i)$	Time to produce $u_i + n_i$ units, where the latter is the production accumulated by the end of cycle i
BL	Break length	d (d_i)	Break length (between cycles i and $i + 1$)
INTF	Interference	y_i	Number of units that would have been accumulated if production was not interrupted for d_i units of time.
Pwr	Power	R_n^a ($R_{n_i}^a$)	Discounting factor for unit n (in cycle i)
Exp	Exponential	R_n (R_{n_i})	Recency measure for unit n (in cycle i)
PT	Passage of time	$t_n - t_0$ ($t_{n_i} - t_0$)	Elapsed time for unit n (in cycle i)
CFV	Controllable Forgetting Variable	t_n (t_{n_i})	Time to complete unit n (in cycle i)
FCE	Forgetting Curve Exponent	a	Forgetting exponent
FIP	Forgetting Influential Parameters	M	Number of information items accumulated
SSE	Sum square of errors	K	Number of information items recalled
MSE	Mean square of errors	n_e (n_{e_i})	Residual (after interference/forgetting) experience (in cycle i), measured in equivalent number of units remembered
ID#	Individual identification	α (α_i)	Decay parameter (in cycle i)
LR	Learning rate	t	Fixed time step in calculation procedure
		γ (γ_i)	Decay exponent (in cycle i)
		$\widehat{y}_j$	Observed value of repetition j
		y_j	Estimated value of repetition j
		I	Total number of learning sessions/cycles
		Δ_j	Performance indicator of the model j
		c	Decay exponent

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