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# Optimization of a High-Speed Synchronous Reluctance Machine's Rotor Topology

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**Abstract**—Currently, due to the growing use of renewable energy technologies and hydrogen energy technologies, high-speed electric machines are of particular interest. High-speed electric machines have proven themselves well in compressor units and can improve the overall efficiency of the entire unit by eliminating the gearbox. However, the design and operation of these electric machines are accompanied by several difficulties, such as the resulting increased centrifugal forces, leading to increased mechanical stresses in the structure of the rotor. In this paper, a high-speed synchronous reluctance motor is considered as the object under study. The aim is to investigate the possibility of reducing mechanical stresses in the rotor steel by optimizing its topology while maintaining the output electromagnetic power.

**Keywords**—High-speed, synchronous reluctance machine, topology optimization, COMSOL.

## I. INTRODUCTION

Optimization of the rotor of synchronous reluctance motors has arisen earlier in the scientific and technical literature. For example, in [1], an attempt was made to optimize the rotor of an electric machine and the geometric parameters of the stator. The authors of [1] considered the distance between the stator slots, the size of the steel ribs in the rotor, and the size of the rotor air barriers as controllable parameters. The target function was the difference in direct and quadratic inductances. The optimization problem was solved using the finite element method, and the results obtained were confirmed by experimental studies.

In [2], the authors considered optimizing the rotor geometry of a synchronous reluctance machine using the hybrid particle swarm optimizer and grey wolf optimizer algorithm. Geometric parameters of air barriers and steel rotor ribs were used as controlled variables. The objective function in this study was the pulsation of the electromagnetic torque ( $\Delta T$ ) and its average value ( $T_{avg}$ ). The objective function was described as follows:

$$f = \Delta T^2 + \frac{1}{T_{avg}}$$

Papers [3,4] were devoted to similar issues and were solved in approximately the same way. It is worth noting that all the above works approached the solution of optimization problems precisely with the help of parametric optimization, that is, when there are some controlled parameters, changing the values of which, one seeks the optimal value of the required output. Also, due to the standard rotational speed, the authors did not consider the issues of rotor mechanics. However, in high-speed electrical machines, these issues

occupy a central place since the operation of the machine, due to mechanical stresses, becomes impossible.

During the electromagnetic analysis of the machine under consideration, it was revealed that the most promising rotor geometry for an electric machine with a power of 130 kW and a rotor speed of 31500 rpm maybe the geometry shown in Figure 1. These specifications have been fixed in a previous work on a permanent magnet machine [5].

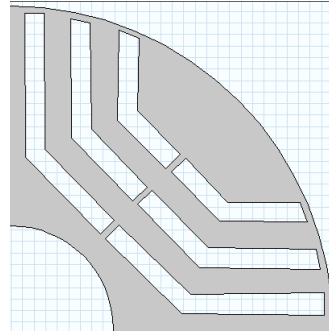


Fig. 1. Original rotor geometry

However, despite the advantages of electromagnetic characteristics, this geometry has issues associated with the mechanical stresses that can arise during operation. Figure 2 shows the distribution of mechanical stresses in the rotor.

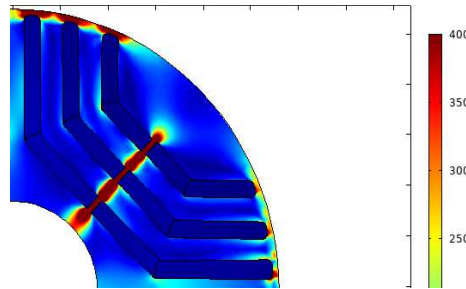


Fig. 2. Distribution of mechanical stresses in the rotor

As it can be seen from Figure 2, excessive mechanical stresses arise in the ribs of the steel between the flux barriers and the ribs of the steel between the flux barriers and the air gap. Mechanical stresses in these areas reach 1200 MPa, while the maximum allowable stress for AISI 4140 steel is only 410 MPa.

One of the solutions to this problem can be the implementation of non-magnetic steel into the flux barriers of the rotor. However, by this will require complex manufacturing procedure without guaranty of its efficacy.

The second solution is to optimize the arrangement of the steel and air sections of the rotor in such a way as to reduce the resulting mechanical stresses.

## II. BASICS OF TOPOLOGY OPTIMIZATION

### A. Optimization in problems of solid mechanics

Topology optimization is method that optimizes the shape of a structure freely without defined geometrical parameters. Such a methodology is suitable for problems where a generic geometry is not available. COMSOL Multiphysics solves topological optimization problems by describing a structure using an artificial volume factor  $\theta_c$ , which plays the role of a control variable in the optimization problem [7]. This variable takes on the values zero or one in solid mechanics, which corresponds to empty or solid domains, respectively. In order to accurately describe the statement of the optimization problem, it is necessary to limit the complexity of the design; otherwise, the optimization can be reduced to infinite small changes and features that cannot be solved by any finite element method due to the finite size of the mesh. The design detail can be limited by using a filter that creates a different field,  $\theta_f$ , which is guaranteed to contain no functions less than some given value  $R_{\min}$ , since it is calculated within the framework of the solution of the Helmholtz problem [8]:

$$\theta_f = R_{\min}^{-2} \nabla^2 \theta_f + \theta_c$$

Then the filtered variable is projected using the hyperbolic tangent [9]:

$$\theta = \frac{(\tanh(\beta(\theta_f - \theta_\beta)) + \tanh(\beta\theta_\beta))}{(\tanh(\beta(1 - \theta_\beta)) + \tanh(\beta\theta_\beta))}$$

Projection options can be selected automatically by COMSOL.

In general, to guarantee a binary solution, the Solid Isotropic Material with Penalization method (SIMP) is applied [10]:

$$\theta_p = \theta_{\min} + (1 - \theta_{\min}) \theta(\mathbf{x})^p$$

where  $\mathbf{x}$  - spatial coordinate.

Then the expression for Young's modulus  $E$ , which is a controlled variable in problems of solid mechanics, takes the following form:

$$E(\mathbf{x}) = E_0 \theta_p,$$

where  $E_0$  is the initial value of Young's modulus at a given point and  $p \geq 1$  is a penalty factor due to which intermediate densities provide less stiffness compared to their "cost" in weight.

### B. Optimization in problems of electromagnetism

Optimization problems can be solved not only for solid mechanics but also in electromagnetism, such as optimizing the rotor topology of a synchronous reluctance motor to maximize the electromagnetic torque. The formulation of the problems, in this case, has a similar structure to that described above, but the controlled variable is the relative magnetic permeability:

$$\mu_r = 1 + \theta(\mathbf{x}) \cdot (\mu_{r0} - 1),$$

where  $\mu_{r0}$  is the initial magnetic permeability of the material.

A peculiarity in solving these problems arises if the magnetization curve of electrical steel is initially set in the form  $B = f(H)$ . In this case, it is necessary to bring this curve to the form  $\mu_r = f(B)$ , which can be used in optimization problems.

## III. PROBLEMS OF OPTIMIZING THE HIGH-SPEED SYNCHRONOUS RELUCTANCE MOTOR'S TOPOLOGY

### A. Minimization of mechanical stresses on the rotor

Optimization of the rotor topology in the framework of solid mechanics was reduced to the need to reduce the mechanical stresses arising in the rotor. Therefore, the objective function can be minimization of the deformation energy density, which can be expressed:

$$W = \tilde{W}(\mathbf{B}, \mathbf{R}),$$

where  $\mathbf{B}$  is left Cauchy-Green strain tensor,  $\mathbf{R}$  the rotation tensor from polar decomposition  $\mathbf{F}$ , and  $\mathbf{F}$  the two-point strain gradient tensor.

For linear isotropic materials undergoing small deformations, the strain energy density function can be written as:

$$W = \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \sigma_{ij} \varepsilon_{ij} = \frac{1}{2} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + 2\sigma_{xy} \varepsilon_{xy} + 2\sigma_{yz} \varepsilon_{yz} + 2\sigma_{xz} \varepsilon_{xz})$$

where  $\sigma$  and  $\varepsilon$  stresses and strains along the corresponding axes.

On the other hand, minimizing the maximum value of mechanical stresses can act as an objective function. For this purpose, one can define a domain probe in COMSOL to monitor the peak stress values.

As a limitation, the distribution of mass along the length was chosen, that is, the integral of the density of the material over a given area. For the original geometry shown in Figure 1, the mass distribution was 47 kg/m.

Figure 3 shows the result of optimization of the rotor topology while minimizing the energy of the plastic deformation with a limitation of the final mass distribution of 40 kg/m.

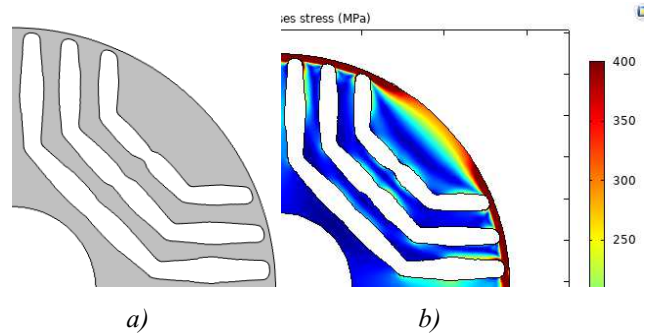


Fig. 3. The result of optimization while minimizing the energy of plastic deformation and mass distribution of 40 kg / m (a) and the distribution of mechanical stresses (b)

Figure 3 shows that mechanical stresses in the steel ribs between the flux barriers were solved, while the problem of increased loads on the periphery remained.

Figure 4 shows the result of optimization of the rotor topology when minimizing the peak value of mechanical stresses with a limitation of the final mass distribution of 40 kg / m.

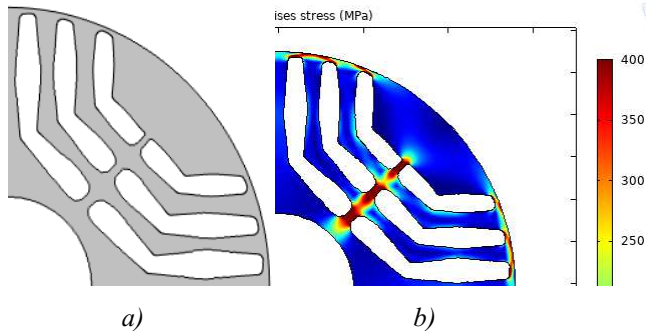


Fig. 4. Optimization result at peak stress values and mass distribution of 40 kg / m (a) and mechanical stress distribution (b)

From Figure 4, it can be seen that the problem of mechanical stresses at the periphery was practically solved, and the loads on the ribs between the flux barriers were reduced. However, the mechanical stresses in this area did not fulfill the permissible limit of 410 MPa.

Figure 5 shows the result of optimizing the rotor topology with the problem of minimizing the energy of plastic deformation and with the limitation of the output mass distribution of 30 kg / m.

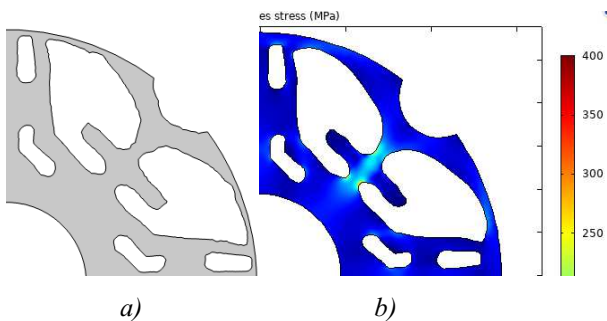


Fig. 5. Optimization result with minimization of plastic deformation energy and weight distribution of 30 kg / m (a) and distribution of mechanical stresses (b)

As follows from Figure 5, the problem of increased mechanical stresses has been entirely resolved. The maximum stresses arising from such a rotor configuration do not exceed 250 MPa.

Figure 6 shows the result of optimization of the rotor topology when minimizing the peak values of mechanical stresses and limiting the output mass distribution to 30 kg / m.



Fig. 6. Result of optimization at peak stress values and weight distribution of 30 kg / m

The geometry in Figure 6 is entirely unrealistic since some parts of the material turned out to be suspended in space and not connected at all.

Thus, it can be argued that the best way to reduce mechanical stress in the rotor of a synchronous high-speed machine is a topology in Figure 5.

#### B. Maximizing the electromagnetic torque

As noted earlier, COMSOL also can solve problems of optimizing the topology of electromagnetic quantities. However, in this case, maximization of the output electromagnetic torque was used as the objective function, while the limitations were only in the external and internal boundaries of the material, that is, during optimization, the rotor cannot occupy the air gap area, as well as the shaft area. At the same time, in contrast to previous studies, the entire rotor was initially considered as a whole, filled with steel without air barriers.

Figure 7 shows the result of this optimization and comparison with the original geometry (Figure 1).

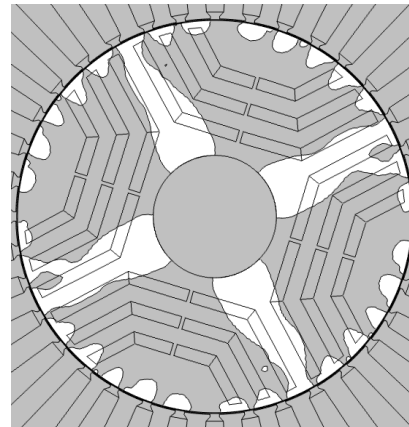


Fig. 7. Result of rotor optimization in order to maximize the electromagnetic torque

The topology in Figure 7 looks like a "petal" one, which is rather unrealistic from a technological point of view.

#### IV. ELECTROMAGNETIC COMPARISON OF TOPOLOGICALLY OPTIMIZED TOPOLOGY

##### A. Comparison of the electromagnetic characteristics of the original geometry and topologically optimized in the problem of solid mechanics

The main characteristic of a synchronous motor is the load characteristic, which is the dependence of the electromagnetic torque or electromagnetic power on the angle of the rotor in relation to the stator magnetic field.

For the initial geometry shown in Figure 1, the load characteristic of the electromagnetic torque dependence on the load angle is shown in Figure 8.

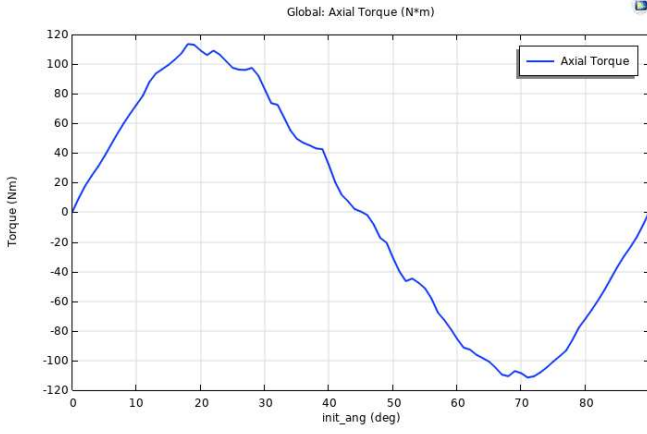


Fig. 8. Load characteristic of a synchronous motor with the original rotor geometry

For the optimized topology shown in Figure 5, the load characteristic of the electromagnetic torque's dependence on the load angle is shown in Figure 9.

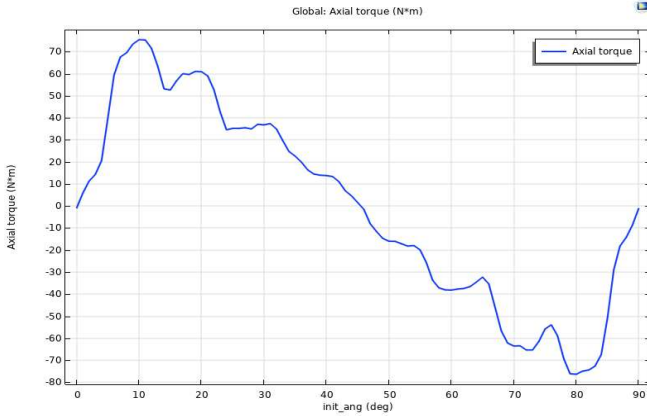


Fig. 9. Load characteristic of a synchronous motor with a rotor geometry optimized for the problem of solid-state mechanics

Figures 8 and 9 show that 1.5 times decrease in the output electromagnetic torque is observed when optimizing the rotor geometry. However, it should be noted that the electromagnetic torque shown in Figure 8 was initially unattainable due to excessive mechanical stresses in the rotor steel. That is, the operation of such an electric machine would be impossible. The load characteristic shown in Figure 9, in this case, is an entirely adequate and reasonable replacement since, based on mechanical stresses, the operation of such an electric motor would not be accompanied by material destruction or other mechanical problems.

The second most important characteristic is the pulsation of the output electromagnetic torque. For the initial geometry (Figure 1), the torque graph for the period is shown in Figure 10, and for the optimized geometry in Figure 11. In this case, the torque ripple can be determined by the following expression:

$$\Delta T = \frac{\max T - \min T}{T_{avg}} \times 100\%,$$

where  $\max T$  - maximum torque value for the period,  $\min T$  - minimum torque value for the period,  $T_{avg}$  - average value of the torque for the period.

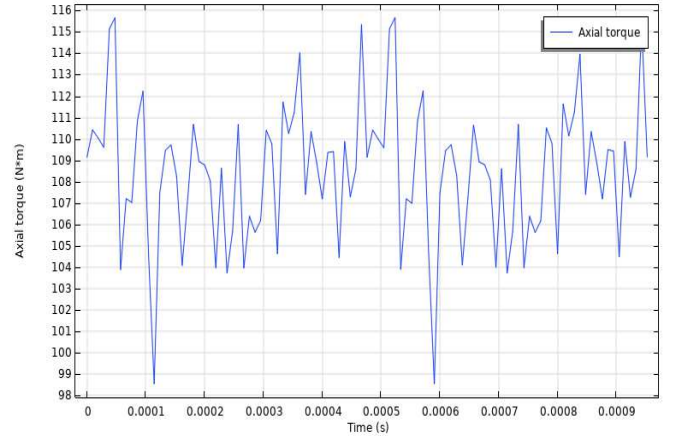


Fig. 10. Electromagnetic torque for the period with the initial geometry of the rotor

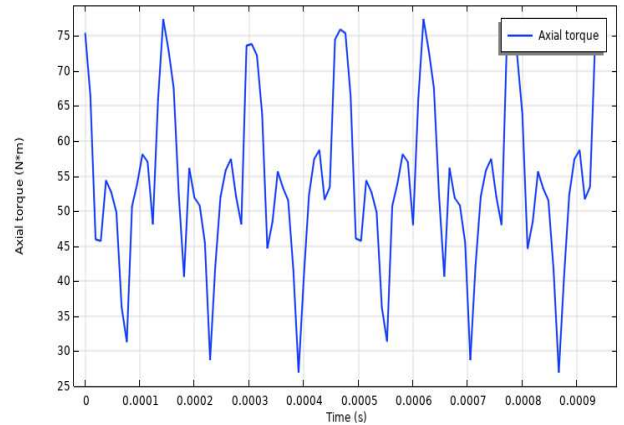


Fig. 11. Electromagnetic torque for a period with optimized rotor geometry

From Figures 10 and 11, it is calculated that the ripple of the torque for the first case is 15%, while in the second case the ripple reaches 90%.

### B. Improvements to Optimized Topology

As noted earlier, despite the fact that topologically optimized geometry gives good results in terms of mechanical parameters, it loses in terms of power and pulsation of the electromagnetic torque. It is also worth noting that the initial declared power is 130 kW, which at a rotation speed of 31,500 rpm corresponds to an electromagnetic torque of 39 Nm. Figure 9 shows that this value is quite achievable and gives a margin of 1.8 times in relation to the maximum torque. Thus, it is necessary to solve the problem of ripple.

By making small changes to the optimized topology, based on considerations of the distribution of the magnetic field in the rotor, it is possible to obtain the rotor shown in Figure 12. It can be seen that a slight change in the air barriers of the rotor does not affect the mechanical stresses in any way.

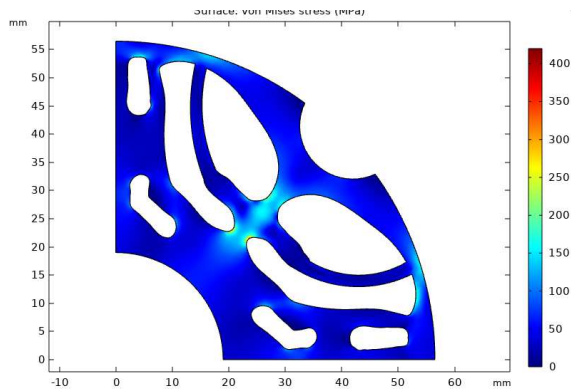


Fig. 12. Mechanical stresses after improvement of the optimized geometry

At the same time, as can be seen from Figures 13 and 14, such a change does not lead to a change in the load characteristic of the electric motor obtained by us, but at the same time, the resulting pulsation of the torque is 80%, and the average value of the electromagnetic torque also increases.

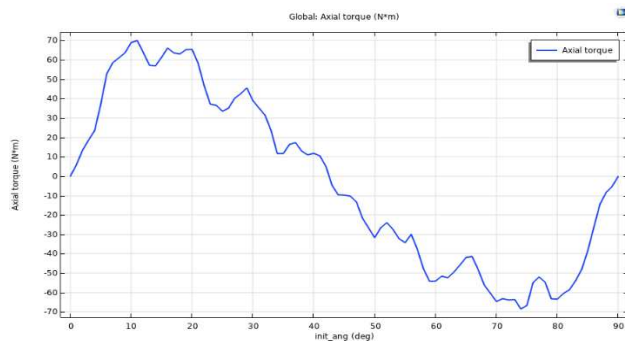


Fig. 13. Load characteristic after refinement of the optimized geometry

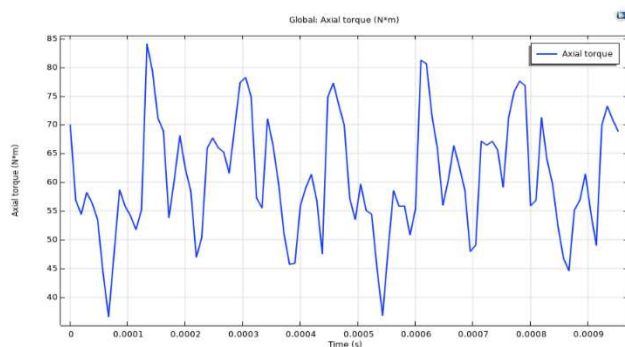


Fig. 14. Electromagnetic torque ripple after optimization of optimized geometry

The electromagnetic torque ripple can be further reduced by introducing permanent magnets into the most minor air barriers shown in Figure 12. Figure 15 shows a graph of the electromagnetic torque after the implementation of permanent magnets.

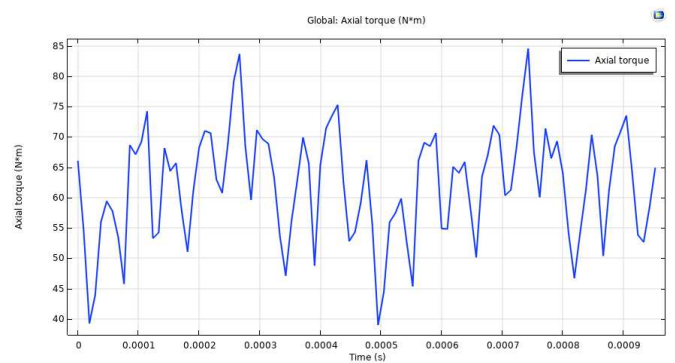


Fig. 15. The electromagnetic torque ripple after the introduction of permanent magnets

Due to the small size of the magnets in comparison with the remaining air barriers, they do not change the load characteristic of the synchronous reluctance motor, but at the same time, they slightly stabilize the acting electromagnetic torque. The ripple of the output torque, in this case, was 72%.

## CONCLUSION

Within the framework of this work, it is shown that a completely adequate model of the topology of the rotor of a high-speed electric machine can be created using topological optimization methods in the COMSOL Multiphysics environment. In this case, the tasks of minimizing mechanical stresses caused by centrifugal forces are completely solved. The work of the optimizer is investigated under various objective functions and constraints. In the course of optimization, it was found that due to the less use of steel, the electromagnetic power and the torque developed on the shaft decrease, as well as its pulsation increases. To reduce ripple, you can make minor adjustments to the optimized geometry and place small permanent magnets in the area of the air barriers. In this case, these methods do not affect the load characteristic but only reduce the output pulsations. Further reduction of the pulsations of the electromagnetic torque can be realized both by means of topology adjustments and by building an appropriate motor control system, which will be an integral part of the electromechanical converter.

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