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Investigation of radial top-hat electric field distributions for corneal reflectometry using modified Fourier optics method

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***Abstract*—An efficient Fourier optics method for synthesizing a required incident beam to compute the internal and scattered electromagnetic fields from known spherical surface electric field distribution is presented.**

I. INTRODUCTION

CONVENTIONAL methods for evaluating electromagnetic beams created from known spherical electric-field surface distributions may include full-wave simulations, geometric optics (GO), or physical optics (PO). These methods are extensively studied and accurate given the model fidelity and a suitable wavelength range. However, they cannot assess the internal and scattered electric fields on the multi-layered spherical objects without considerable computational effort and complexity.

Fourier optics is a powerful method to model an incident electric field that addresses the above-mentioned challenges [1]. For spherical targets, the field can be easily expanded via vector spherical harmonics (VHS) presentation to accommodate the geometry. The VHS presentation is mapped with the T-matrix method to evaluate internal and scattered fields from multi-layered dielectric spheres [2]. Nevertheless, this method is nominally limited to sources defined on a plane.

In this study, the Fourier optics method is expanded to model beam propagation from arbitrary electric field distributions positioned on spherical surfaces. The method's main idea is that Riemann's surface integral combines differential surface elements, which can be approximated as locally planar elements. The Fourier optics method is applied to each of these elements, and the total electric field is the sum of the fields created from them by the superposition principles [3].

The method is readily applicable to the field of THz sensing of corneal tissue hydration and geometry due to the cornea's layered spherical shell nature [4]. The derived formula for the incident electric field of the beam from a spherical surface distribution with differential elements is:

$$\boldsymbol{E}_{inc}(\boldsymbol{r}) = \frac{1}{4\pi^2}\iint_{\Omega} E_0(\theta,\varphi) sin\theta \boldsymbol{E}_t(k\boldsymbol{r};\theta,\varphi) d\theta d\varphi \,, \qquad (1)$$

where $E_0(\theta,\varphi)$ is the complex surface amplitude, $k = 2\pi/\lambda$ is the wavenumber, and r, θ, φ are the spherical coordinates and $\boldsymbol{r} = re_r$ and e_r is a radial unit-vector. Moreover, the electromagnetic beam from a differential source element with realistic polarization in (1) is presented with VSH as:

$$\boldsymbol{E}_t(k\boldsymbol{r};\theta,\varphi) = p^2 \sum_m \sum_n D_{mn}[a^t_{emn}M^1_{emn} + a^t_{omn}M^1_{omn} + b^t_{emn}\boldsymbol{N}^1_{emn} + b^t_{omn}\boldsymbol{N}^1_{omn}\,], \qquad (2)$$

where p is a computational factor given in [2], D_{mn} is the normalization factor, $a^t_{emn}, a^t_{omn}, b^t_{emn}$ and b^t_{omn} are the VSH coefficients, $M^1_{emn}, M^1_{omn}, N^1_{emn}$ and N^1_{omn} are the VSH of the first kind [5].

II. RESULTS

The electric field with radial Gaussian and top-hat distributions was created on a spherical surface and simulated with MATLAB. The incident beam which makes such field distributions are obtained with the developed modified Fourier optics method. The optical axis of the incident beam is along the x-axis, and the beam is evaluated at the transverse zy - plane at 40 mm from the origin of the sphere (see Figure 1). The results are shown in Figures 3 – 4.

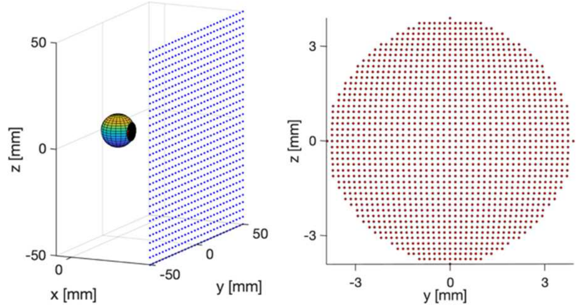

***Fig. 1** The simulation arrangement, where red dots represent the source element locations and blue dots represent the evaluating plane.*

The results of top-hat simulations are compared to identical PO simulations, and they are with a precise match down to -41 dB difference in amplitude and difference less than 0.32 degrees (average difference is 0.27 degrees) in phase at 275 GHz for the spherical surface with radius of curvature (RoC) of 7.8 mm that coincident with the beam curvature.

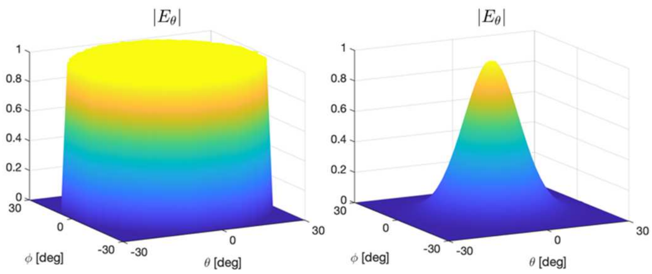

***Fig. 2** The projected electric field distributions located at the spherical surface.*

A comparison simulation was made with a Gaussian distribution at the sphere´s surface. This beam is not a well-known Gaussian beam from a planar distribution, as the wave-front of the incident beam is synthesized from the equal phase spherical Gaussian distribution to match the cornea´s surface.

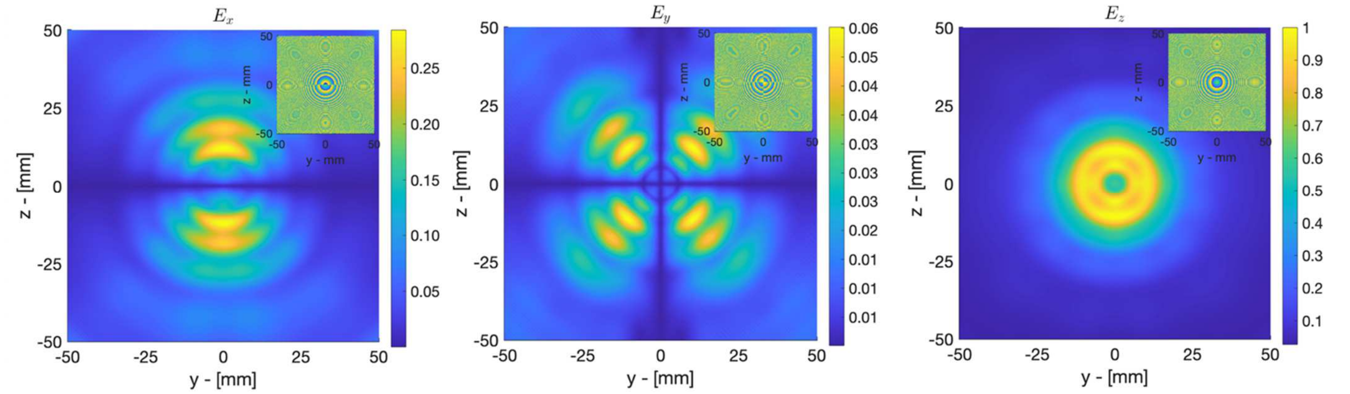

Fig. 3 *The electric field components from the top-hat distribution at the zy - plane at 40 mm from the origin of the sphere, normalized by the main polarization* $\boldsymbol{E_z}$.

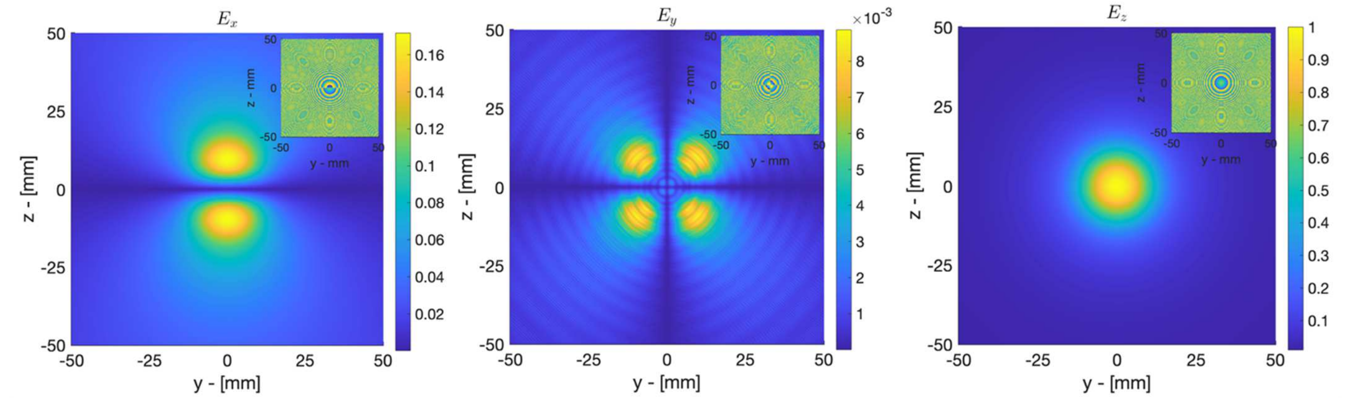

Fig. 4 *The electric field components from the Gaussian distribution at the zy - plane at 40 mm from the origin of the sphere, normalized by the main polarization* $\boldsymbol{E_z}$.

Figures 3 and 4 illustrate the required electric field components at a plane aperture for creating the incident beam with a spherical top-hat or Gaussian electric field distribution at the cornea´s surface with the matching wavefront.

III. Summary

The presented modified Fourier optics method was compared to numerous arrangements with known analytical methods or PO simulation results. Each comparison demonstrates good agreement combined with the novel ability to evaluate the internal and scattered fields from the multi-layered sphere. With this proposed method, the complex electric field distribution at the spherical surface can be discretized and distributed arbitrarily. If the phase is uniform and fixed at each discretization point, the incident beam's wavefront matches the spherical surface across the illuminated area. This feature of wavefront matching enables analysis with standard Fresnel equations and stratified medium theory.

The minimum wavelength of the electric field to simulate an incident beam from the spherical surface is approximately $RoC \geq 2\lambda$, due to the nature of the Fourier-optics. When RoC is below the limit, the simulation error increases faster with the total phase and at the periphery of the surface. The vector spherical harmonic presentation, on the other hand, creates a computational upper limit for the frequency. Thus, more VHS modes are needed to compute the presented results at higher frequencies for a given RoC. For this reason, the presented method is optimal for simulating the electromagnetic behavior of the cornea with THz frequencies due to the size parameter range defined by the band of interest and the corneal RoC.

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