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Event-Triggered Control in Shared Networks: How the Computational Power of Sensors Affects Transmission Priorities

Tahmoores Farjam and Themistoklis Charalambous

***Abstract*— In this paper, we study the prioritized transmission schemes for event-triggered wireless networked control systems (WNCSs) with smart (i.e., with computational power) or conventional (i.e., without computational power) sensors. When considering conventional sensors, the estimated state available to the controller is based on the intermittently received raw measurements. We show that the priority measure is associated with the statistical properties of the observations conforming with the cost of information loss (CoIL). Next, we consider the case of smart sensors, and despite the fact that CoIL can also be deployed, we deduce that it is more beneficial to use the available measurements as suggested by the value of information (VoI). The derived VoI incorporates the channel conditions and is compatible with distributed implementation. The impact of adopting each priority measure on the performance is evaluated via simulations.**

***Index Terms*— Networked control systems, smart sensors, event-triggering, cost of information loss, value of information.**

I. Introduction

Modern control environments, such as industrial automation, consist of a multitude of spatially distributed components that are required to exchange information over a shared network. The tremendous increase of data traffic often renders the current transmission protocols incapable of accommodating the required traffic volume. Typically, the capacity constraints of the communication resources are such that the network can only accommodate transmissions from a limited number of components at any given time. However, meeting the performance requirements requires developing suitable methods to share the available communication resources efficiently with respect to a control objective.

For the case of multiple sensors sharing ideal communication channels, it has been proved that transmission based on a periodic schedule is optimal [1]. Consequently, the static optimal transmission sequence can be computed offline and distributed channel access can be provided by using time division multiple access (TDMA). Finding the optimal solution to time-based sensor scheduling problems in various scenarios has been an active area of research; see, for example, [2] and references therein. The optimal scheduling problem is subject to the curse of dimensionality, which renders finding the optimal schedule in large-scale networks infeasible within the limited time available for solving the problem.

To circumvent this, sub-optimal yet efficient contention-based dynamic scheduling methods can be utilized to allocate the resources based on time-varying transmission priorities. The celebrated try-once-discard (TOD) protocol is one of such methods which prioritizes transmission according to the deviation of the state from its nominal value [3]. Although originally proposed for deterministic systems and ideal channels, its application has been extended to stochastic systems with full state observation [4] as well as partial observations [5], [6] and non-ideal communication channels [7]. Instead of using measurement-based priorities and bit-wise contention resolution as the aforementioned works, a timer-based mechanism was proposed in [8] to prioritize transmissions based on a variance-based measure and its application was later extended to wireless networks with uncorrelated [9], [10] and correlated packet dropouts [11].

In a separate strand of research, adopting carrier-sense multiple access with collision avoidance (CSMA/CA) protocol and utilizing event-triggered methods, where transmission is triggered only upon the occurrence of certain events, has been extensively studied; see [12]. Such methods offer easier implementation and lower computational complexities compared to optimal periodic schedules. Furthermore, they can lead to lower communication frequency compared to time-triggered approaches, resulting in less congestion on the network and thus less packet dropouts. Hence, a common approach has been to design the triggering rules for each subsystem independently based on the assumption that transmission is always successful [13]–[16]. The practical implementation of such triggering rules in multi-subsystems networks with CSMA/CA was considered in [17]. It was shown that packet dropouts are inevitable due to the possibility of simultaneous transmissions, thus deteriorating the performance. Regarding wireless networks, even single-subsystem scenarios are prone to packet dropouts, which has only been addressed recently [18]–[20]. To the best of our knowledge, the effect of a deterministic channel access method on the performance of multiple event-triggered subsystems competing for transmission over a shared non-ideal (unreliable) network has not been addressed yet.

In this paper, we examine how the computational capabilities of sensors affect the priority measures that can be used for contention resolution between multiple event-triggered subsystems sharing a wireless network. This is inspired by [15], where the computation power of scalar sensors monitoring a single process is considered for designing triggering laws in wired networks. For conventional sensors, variance-based triggering (VBT) is proposed, which is closely related to the cost of information loss (CoIL) developed for multiple time-triggered vector processes in [8], [21]. Similarly, the

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measurement-based triggering (MBT) for smart sensors has been considered for time-triggered vector systems in [5], [6], [22] under the label value of information (VoI). The contributions of this paper are the following.

- We first consider the case of multiple subsystems equipped with conventional sensors that share a wireless network. Due to the limited/no computational power of the sensors, the computational power of the estimators can be used to negotiate channel access. The limited information of estimators is then utilized to derive the priority measure which is shown to be a function of CoIL.
- Next, the sensing and estimation architecture for smart sensors is considered. While in this case the priority measure can again be a function of CoIL, the additional information and computational power of sensors can be leveraged for improving performance. We achieve this by adopting the concept of VoI, which has been previously considered for resource allocation over perfect channels [5], [6]. We derive its closed form expression for unreliable channels which is shown to be suitable for distributed implementation.
- Finally, we address how the derived priority measures can be utilized for prioritizing channel access with TOD protocol and evaluate their performance.

The remainder of the paper is organized as follows. Section II provides the necessary preliminaries and system model. In Section III and IV, we present the sensing and estimation architecture and the corresponding priority measures for the case of conventional and smart sensors, respectively. We evaluate the performance of the proposed schemes in Section V and finally draw conclusions in Section VI.

Notation: Vectors and matrices are denoted by lowercase and uppercase letters, respectively. A random vector x from a multivariate Gaussian distribution with mean μ and covariance X is denoted by $x \sim \mathcal{N}(\mu, X)$. The Euclidean norm of a vector x is denoted by $\|x\|$ and $\sigma_{\max}(X)$ denotes the spectral radius of matrix X. $\mathbb{S}_+^n$ is the set of n by n positive semi-definite matrices. The transpose of matrix X is denoted with X^T and its inverse with X^{-1}. $f^n(\cdot)$ is the n-fold composition of $f(\cdot)$ with the convention that $f^0(X) = X$, and $g \circ f(\cdot) \triangleq g(f(\cdot))$. The n by n identity matrix is represented by I_n.

II. Problem Formulation

We consider the scenario in which N dynamical subsystems use a shared communication network to accomplish their control tasks. Each subsystem $i \in \{1, \ldots, N\}$ consists of a plant ($\mathcal{P}_i$), dedicated sensor ($\mathcal{S}_i$), estimator ($\mathcal{E}_i$), and controller ($\mathcal{C}_i$); see Fig. 1. Packet transmission from $\mathcal{S}_i$ to $\mathcal{E}_i$ is supported by a time-slotted shared network. We consider the scenario in which the network consists of a wireless channel (which by definition it is non-ideal), where i.i.d. packet dropouts are possible even when channel access is collision-free. The extension of the proposed scheme to networks with multiple wireless channels is straightforward as it will be discussed in Remark 2.

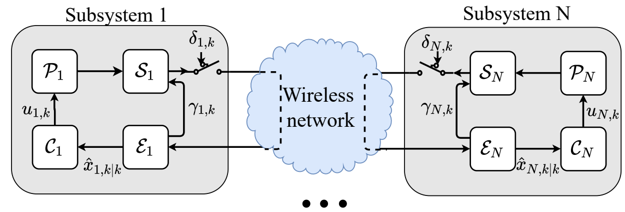

Fig. 1. Schematic diagram of N subsystems competing for access over a shared wireless network.

A. Plant and outputs

The states of each subsystem $i \in \{1, \ldots, N\}$ evolves according to the following linear time-invariant (LTI) process:

$$x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + w_{i,k}, \tag{1}$$

where $x_{i,k} \in \mathbb{R}^{n_i}$ and $u_{i,k} \in \mathbb{R}^{m_i}$ are the states and inputs at time step k, respectively, with A_i and B_i being the system and input matrices of appropriate dimensions. Furthermore, $w_{i,k} \in \mathbb{R}^{n_i}$ is the i.i.d. process disturbance with $w_{i,k} \sim \mathcal{N}(0, W_i)$ and the initial state is $x_{i,0} \sim \mathcal{N}(\bar{x}_{i,0}, X_i)$.

The output measured by the sensor is given by

$$y_{i,k} = C_i x_{i,k} + v_{i,k}, \tag{2}$$

where $C_i \in \mathbb{R}^{p_i \times n_i}$ is the output matrix and $v_{i,k} \in \mathbb{R}^{p_i}$ is the i.i.d. measurement noise described by $v_{i,k} \sim \mathcal{N}(0, V_i)$. We assume that $w_{i,k}$, $v_{i,k}$ and $x_{i,0}$ are mutually independent.

B. Controller and the quadratic cost

The aim of the controller is to minimize the standard quadratic cost over the infinite horizon which is given by

$$J_{0:\infty} = \lim_{K \to \infty} \frac{1}{K} \mathbb{E}\left\{ \sum_{k=0}^{K-1} \sum_{i=1}^{N} \left(x_{i,k}^T Q_i x_{i,k} + u_{i,k}^T R_i u_{i,k} \right) \right\},$$

where Q_i and R_i are weighting matrices of appropriate dimensions. The controller uses the certainty equivalence law and is given by [5]

$$u_{i,k} = L_{i,\infty} \hat{x}_{k|k}, \tag{3}$$

where $\hat{x}_{k|k}$ is the *a posteriori* state estimate provided by $\mathcal{E}_i$. Furthermore, $L_{i,\infty}$ is the constant feedback gain given by

$$L_{i,\infty} = -(B_i^T \Pi_{i,\infty} B_i + R_i)^{-1} B_i^T \Pi_{i,\infty} A_i, \tag{4}$$

and $\Pi_{i,\infty}$ is the solution of the following discrete-time algebraic Riccati equation (DARE)

$$\Pi_{i,\infty} = A_i^T \Pi_{i,\infty} A_i + Q_i - L_{i,\infty}^T (B_i^T \Pi_{i,\infty} B_i + R_i) L_{i,\infty}.$$

By assuming that the pairs (A_i, B_i) and $(A_i, Q_i^{1/2})$ are controllable and observable, respectively, the given DARE has a unique positive semi-definite solution $\Pi_{i,\infty}$ [23]. By using the proposed controller, the single step quadratic cost at k can be written as [5], [21]

$$J_k = \sum_{i=1}^{N} J_{i,k}, \tag{5}$$

where

$$J_{i,k} = \operatorname{tr}(\Pi_{i,\infty} W_i) + \operatorname{tr}(\Gamma_{i,\infty} \mathbb{E}\{e_{i,k|k} e_{i,k|k}^T\}), \tag{6}$$

where $e_{i,k|k} \triangleq x_{i,k} - \hat{x}_{i,k|k}$ and $\Gamma_{i,\infty} = L_{i,\infty}^T(B_i^T \Pi_{i,\infty} B_i + R_i)L_{i,\infty}$.

C. Capacity constrained network

We consider the case where the triggering criterion and associated thresholds are prespecified as required by the available energy budget. Let $\theta_{i,k} = \{0,1\}$ denote whether subsystem i competes for transmission at k. In case the triggering threshold is crossed $\theta_{i,k}$=1, and $\theta_{i,k}$=0 otherwise. We assume that all the nodes on the network are synchronized and the duration of a transmission frame is less than the sampling time of subsystems and thus the effect of delay can be ignored. The channel access decision is given by

$$\delta_{i,k} = \begin{cases} 1, & \text{if } \theta_{i,k} = 1 \text{ and sensor } i \text{ transmits at } k, \\ 0, & \text{otherwise.} \end{cases}$$

To ensure that channel access is collision-free, we impose the following constraint

$$\sum_{i=1}^{N} \delta_{i,k} \leq 1, \quad \forall k \geq 0. \tag{7}$$

Due to the unreliable nature of the wireless medium, transmitted data packets might not be received successfully at the receiver, i.e., the corresponding estimator. We assume acknowledgement/negative-acknowledgement (ACK/NACK) feedback mechanism is in place which informs the transmitter about the status of the sent packet. This can be represented as another binary variable $\gamma_{i,k}$ which is defined as

$$\gamma_{i,k} = \begin{cases} 1, & \text{if } \delta_{i,k}\text{=1 and packet is successfully received,} \\ 0, & \text{otherwise.} \end{cases}$$

We consider the case of memoryless wireless channels where the packet dropouts are assumed to be i.i.d. random variables. The probability of successful transmission over each communication link is thus given by $q_i = \mathbb{P}\{\gamma_{i,k} = 1 | \delta_{i,k} = 1\}$.

D. Contention resolution

In this work, we intend to utilize TOD protocol for collision-free distributed channel access. In the celebrated work [3], dynamic identifiers, which depend on the performance criterion, were proposed for contention resolution over wired networks and its application was later extended to wireless networks in [7]. Here, we briefly review how this protocol operates.

Let $\mathrm{ID}_{i,k}$ denote the identifier of subsystem i, at time step k, which represents its priority, which consists of a time-varying dynamic segment ($\mathrm{ID}_{i,k}^d$) and a time-invariant static segment (ID_i^s). In the beginning of each frame, subsystems compete for channel access based on $\mathrm{ID}_{i,k}$ and the one with the highest priority, i.e., dominant $\mathrm{ID}_{i,k}$, claims the channel. Let $f(\cdot)$ be a continuous, nonnegative, and monotonically non-decreasing function; then, the dynamic identifier is determined by $\mathrm{ID}_{i,k}^d = [f(m_{i,k})]$, where $[\cdot]$ denotes the function round to the nearest integer and $m_{i,k}$ denotes the priority measure. Assuming that the dynamic segment consists of n contention bits, it is constrained between 0 and $\mathrm{ID}_{\text{max}}^d$, where $\mathrm{ID}_{\text{max}}^d = 2^n - 1$ denotes the identifier's upper bound. The dynamic identifier assignment could be defined as

$$\mathrm{ID}_{i,k}^d = \begin{cases} 0 & \text{if } \mathrm{ID}_{i,k}^d \leq 0, \\ [f(m_{i,k})] & \text{if } \mathrm{ID}_{i,k}^d < \mathrm{ID}_{\text{max}}^d, \\ \mathrm{ID}_{\text{max}}^d & \text{otherwise.} \end{cases} \tag{8}$$

Moreover, ID_i^s is a prespecified unique identifier assigned to each subsystem. Although it is possible for multiple subsystems to have the same dynamic identifier in (8), the unique ID_i^s ensures that channel access is provided in a collision-free manner satisfying (7). Fig. 2 illustrates how contention is resolved between two subsystems with the same dynamic identifier competing for channel access at k.

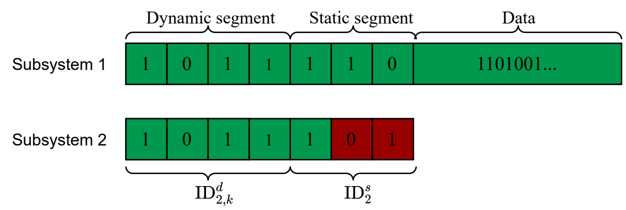

Fig. 2. Example of two subsystem competing for transmission at k. The first 7 bits are dedicated to contention resolution with $n = 4$ bits for the dynamic identifier in (8). Although $\mathrm{ID}_{1,k}^d = \mathrm{ID}_{2,k}^d$, assuming 1 is the dominant bit, channel access is granted to Subsystem 1 since it has the dominant static identifier.

E. Problem of interest

Our objective is distributed allocation of the available communication resources such that the following problem is solved.

Problem 1.

$$\min_{\delta_{i,k}} \quad \mathbb{E}\{J_k | \mathcal{I}^k\},$$
$$\textit{subject to} \quad (7),$$

where J_k is given in (5) and $\mathcal{I}^k$ denotes the information available at the decision makers.

III. Scenario 1: Conventional Sensors

In this section, we consider the scenario in which conventional sensors are utilized. First, we discuss how the state estimate is calculated at the estimator side based on the intermittent raw measurements received from the sensor. Then, we prove that the solution of Problem 1 can be obtained by utilizing CoIL when the estimators act as the decision makers.

A. Sensing and estimation

In this scenario, all sensors are assumed to lack any computational power and in the event sensor i receives a transmission request at k, i.e., $\delta_{i,k} = 1$, it sends the data packet containing its most recent raw measurement, i.e., $y_{i,k}$ in (2). Consequently, the information available at the corresponding estimator at k is given by $\mathcal{I}_{i,k} = \{\delta_{i,0}, \gamma_{i,0} y_{i,0}, \ldots, \delta_{i,k}, \gamma_{i,k} y_{i,k}\}$. Define the *a priori* and *a* *posteriori* state estimates and the corresponding error covariances as

$$\hat{x}_{i,k|k-1} \triangleq \mathbb{E}\{x_{i,k}|\mathcal{I}_{i,k-1}\}, \qquad \hat{x}_{i,k|k} \triangleq \mathbb{E}\{x_{i,k}|\mathcal{I}_{i,k}\},$$
$$P_{i,k|k-1} \triangleq \mathbb{E}\{(x_{i,k}-\hat{x}_{i,k|k-1})(x_{i,k}-\hat{x}_{i,k|k-1})^T|\mathcal{I}_{i,k-1}\},$$
$$P_{i,k|k} \triangleq \mathbb{E}\{(x_{i,k}-\hat{x}_{i,k|k})(x_{i,k}-\hat{x}_{i,k|k})^T|\mathcal{I}_{i,k}\},$$

respectively. For simplicity, we assume that the estimator ignores any additional information associated with $\delta_{i,k} = 0$. The seminal work on Kalman filtering with intermittent observations can then be applied in this setting [24], where $\hat{x}_{i,k|k}$ fed to the controller is obtained by

$$\hat{x}_{i,k|k-1} = A_i\hat{x}_{i,k-1|k-1} + B_i u_{i,k-1}, \tag{9a}$$
$$P_{i,k|k-1} = h_i(P_{i,k-1|k-1}), \tag{9b}$$
$$K_{i,k} = P_{i,k|k-1}C_i^T(C_iP_{i,k|k-1}C_i^T + V_i)^{-1}, \tag{9c}$$
$$\hat{x}_{i,k|k} = \hat{x}_{i,k|k-1} + \gamma_{i,k}K_{i,k}(y_{i,k} - C_i\hat{x}_{i,k|k-1}), \tag{9d}$$
$$P_{i,k|k} = \begin{cases} P_{i,k|k-1}, & \text{if } \gamma_{i,k} = 0, \\ g_i \circ h_i(P_{i,k-1|k-1}), & \text{otherwise,} \end{cases} \tag{9e}$$

where the functions $h,\, g : \mathbb{S}_+^n \to \mathbb{S}_+^n$ are defined as

$$h_i(X) \triangleq A_iXA_i^T + W_i, \tag{10}$$
$$g_i(X) \triangleq X - XC_i^T(C_iXC_i^T + V_i)^{-1}C_iX. \tag{11}$$

B. Priority measure and contention resolution

Due to their lack of computational power, sensors use basic triggering mechanisms such as *send-on-delta* (SoD). However, despite not having access to the actual measurements, estimators can use their computational power for executing more sophisticated triggering rules, which can greatly enhance performance [15], [16]. This is also known as *variance-based triggering* (VBT). Once the value of error covariance at the estimator exceeds a specified threshold, it sends a transmission request to the corresponding sensor. Proposition 1 shows how CoIL can be utilized in this context for prioritizing channel access. Note that estimators compete for claiming the channel at the start of frame k. Thus, the information available to each decision maker is $\mathcal{I}_{i,k-1}$ and $\mathcal{I}^k \triangleq \cup_{i\in\{1,\ldots,N\}}\mathcal{I}_{i,k-1}$ in Problem 1.

Proposition 1. *Let $\mathcal{F}_k \triangleq \{i : i \in \{1,\ldots,N\}, \theta_{i,k} = 1\}$ denote the set of subsystems which cross the triggering threshold. Define CoIL for subsystem i at k as*

$$\mathrm{CoIL}_{i,k} \triangleq \mathrm{tr}\left(\Gamma_{i,\infty}\left[P_{i,k|k-1} - g_i \circ h_i(P_{i,k-1|k-1})\right]\right). \tag{12}$$

Then Problem 1 is solved by letting $\delta_{i^*,k} = 1$, where

$$i^* = \underset{i\in\mathcal{F}_k}{\arg\max} \quad \mathrm{CoIL}_{i,k}q_i. \tag{13}$$

Proof. By following the approach of [10], [21, Lemma 2], Problem 1 can be equivalently written as

$$i^* = \underset{i\in\mathcal{F}_k}{\arg\max} \quad \mathbb{E}\{J_{i,k}|\mathcal{I}_{i,k-1},\delta_{i,k}=0\} - \mathbb{E}\{J_{i,k}|\mathcal{I}_{i,k-1},\delta_{i,k}=1\}. \tag{14}$$

From (6) and the law of total expectation we obtain

$$\begin{aligned}\mathbb{E}\{J_{i,k}|\mathcal{I}_{i,k-1},\delta_{i,k}=0\} &= \mathrm{tr}\,(\Pi_{i,\infty}W_i) \\ &\quad + \mathrm{tr}(\Gamma_{i,\infty}\mathbb{E}\{e_{i,k|k}e_{i,k|k}^T|\mathcal{I}_{i,k-1},\delta_{i,k}=0\}) \\ &= \mathrm{tr}\,(\Pi_{i,\infty}W_i) + \mathrm{tr}(\Gamma_{i,\infty}P_{i,k|k-1}),\end{aligned} \tag{15}$$

where the second equality follows from (9e). Similarly,

$$\begin{aligned}&\mathbb{E}\{J_{i,k}|\mathcal{I}_{i,k-1},\delta_{i,k}=1\} = \mathrm{tr}\,(\Pi_{i,\infty}W_i) \\ &+(1-q_i)\,\mathrm{tr}(\Gamma_{i,\infty}P_{i,k|k-1}) + q_i\,\mathrm{tr}(\Gamma_{i,\infty}g_i\circ h_i(P_{i,k-1|k-1})).\end{aligned} \tag{16}$$

Finally, substituting (15) and (16) in (14) yields (13). □

Since computation of $\mathrm{CoIL}_{i,k}q_i$ requires no information exchange between subsystems, from Proposition 1 it follows that Problem 1 can be solved in a distributed fashion. The solution can be obtained by implementing TOD and letting $m_{i,k} = \mathrm{CoIL}_{i,k}q_i$ in (8). Note that this scheme does not necessarily provide the optimal solution to Problem 1, since the limited number of contention bits n means that multiple subsystems could have the dominant dynamic identifier at some k. Nevertheless, the mapping function $f(\cdot)$ in (8) can be fine-tuned based on the network configuration to increase the probability of the subsystem with the largest value for $\mathrm{CoIL}_{i,k}q_i$ having the dominant dynamic identifier. During the contention period, all sensors listen to the shared medium. Once the contention period ends, the sensor corresponding to the dominant estimator can infer permission to transmit from the unique static identifier and transmit its data packet without collisions.

Remark 1. *Neither the triggering mechanism nor the contention resolution requires the actual measurements from the sensor. Implementing VBT enables the estimator to determine the exact number of sampling times that the triggering condition will not be met after it successfully receives a packet. During this period, the sensor can enter sleeping mode since taking measurements and listening on the medium are unnecessary. Therefore, energy consumption can be further reduced by sending the sleeping duration to the sensor in addition to ACK/NACK through the feedback channel.*

Remark 2. *This scheme can be easily extended to networks with multiple channels $j \in \{1,\ldots,M\}$. In such scenarios, an additional constraint is added, such that each subsystem occupies a single channel only at any specific time slot k, i.e., $\sum_{j\in\{1,\ldots,M\}}\delta_{i,j,k} \le 1$, $\forall i,k$. This condition can be satisfied by having subsystems to back off from all other channels when they are granted access to one channel. Similar to what has been done with timers in [9]–[11], following the approach in Proposition 1, the solution can be obtained by setting $m_{i,j,k} = \mathrm{CoIL}_{i,k}q_{i,j}$.*

IV. Scenario 2: Smart sensors

In this section, we consider the case of smart sensors which contain an embedded microprocessor. This allows them to do local computations and preprocess the raw measurements before transmission. We first discuss how the computational resources can be utilized to enhance the estimation performance. Next, we use the concept of VoI to derive the corresponding priority measure which is shown to be compatible with TOD.

A. Sensing and estimation

It is well-known that preprocessing the raw measurements before transmission can enhance the estimation quality at the receiver side; see [16]. To this end, the sensor computes the MMSE state estimate, denoted by $\hat{x}^s_{i,k|k}$, which is then sent to the estimator. The value of $\hat{x}^s_{i,k|k}$ is determined by running the standard Kalman filter, i.e.,

$$\hat{x}^s_{i,k|k-1} = A_i\hat{x}^s_{i,k-1|k-1} + B_iu_{i,k-1}, \tag{17a}$$
$$P^s_{i,k|k-1} = h_i(P^s_{i,k-1|k-1}) \simeq h_i(\overline{P}_i), \tag{17b}$$
$$K^s_{i,k} = P^s_{i,k|k-1}C_i^T(C_iP^s_{i,k|k-1}C_i^T + V_i)^{-1} \simeq K^s_i, \tag{17c}$$
$$\hat{x}^s_{i,k|k} = \hat{x}^s_{i,k|k-1} + K_{i,k}(y_{i,k} - C_i\hat{x}^s_{i,k|k-1}), \tag{17d}$$
$$P^s_{i,k|k} = g_i \circ h_i(P^s_{i,k-1|k-1}) \simeq \overline{P}_i. \tag{17e}$$

By assuming that the pairs (A_i, C_i) and $(A_i, W_i^{1/2})$ are observable and controllable, respectively, the *a posteriori* error covariance in (17e) converges exponentially fast to the unique positive semi-definite solution of $g_i \circ h_i(X) = X$ [23]. Let $\overline{P}_i$ denote this solution and assume that the filter has already entered steady state and thus (17e) can be written as $P^s_{i,k|k} = \overline{P}_i$. As a result, the filter's gain in (17c) is also time-invariant. Thus, we drop the time subscript in the remaining of this paper and denote the filter's gain by K^s_i.

For this scenario, the information available to the estimator is given by $\mathcal{I}_{i,k} = \{\delta_{i,0}, \gamma_{i,0}\hat{x}^s_{i,0|0}, \ldots, \delta_{i,k}, \gamma_{i,k}\hat{x}^s_{i,k|k}\}$ and the state estimate and error covariance are obtained by

$$\hat{x}_{i,k|k} = (A_i + B_iL_{i,\infty})^{t_{i,k}}\hat{x}^s_{i,k-t_{i,k}|k-t_{i,k}},$$
$$P_{i,k|k} = h_i^{t_{i,k}}(\overline{P}_i),$$

where $t_{i,k}$ denotes the time elapsed since the last successful packet arrival, i.e.,

$$t_{i,k} \triangleq \min\{\kappa \geq 0 : \gamma_{i,k-\kappa} = 1\}.$$

In essence, the estimator uses $\hat{x}^s_{i,k|k}$ if the packet arrives at k; or simply runs a prediction step otherwise.

B. Priority measure and contention resolution

Using the real-time data for deterministic or stochastic triggering of the events is more beneficial since it conveys additional information about the state of the system compared to VBT. However, this results in more energy consumption, since the sensors are required to take measurements and process them constantly. Here, we show how the additional information and computational resources at the sensors can be leveraged to solve Problem 1 in a distributed manner. The information available at each sensor i is $\mathcal{I}^s_{i,k} = \{y_{i,0}, \ldots, y_{i,k}, \delta_{i,0}, \ldots, \delta_{i,k-1}, \gamma_{i,0}, \ldots, \gamma_{i,k-1}\}$. Therefore, the aggregate information of the decisions makers, i.e., smart sensors, is given by $\mathcal{I}^k = \cup_{i\in\{1,\ldots,N\}}\mathcal{I}^s_{i,k}$. Note that $t_{i,k-1}$ can be inferred from ACK/NACK which allows the sensor to keep track of the applied inputs at the receiver side.

Proposition 2. *Let $\check{e}_{i,k|k} \triangleq \hat{x}^s_{i,k|k} - (A_i + B_iL_{i,\infty})\hat{x}_{i,k-1|k-1}$ be the discrepancy between sensor's a posteriori state estimate and estimator's estimate in case it receives no data packet at k. Define VoI for subsystem i at k as*

$$\mathrm{VoI}_{i,k} \triangleq \mathrm{tr}\left(\Gamma_{i,\infty}\check{e}_{i,k}\check{e}^T_{i,k}\right). \tag{18}$$

Then, Problem 1 is solved by letting $\delta_{i^*,k} = 1$, where

$$i^* = \underset{i\in\mathcal{F}_k}{\arg\max} \quad \mathrm{VoI}_{i,k}q_i. \tag{19}$$

Proof. Using the new information available to the decision makers, (14) can be written as

$$\begin{aligned} i^* = \underset{i\in\mathcal{F}_k}{\arg\max} \quad & \mathbb{E}\{J_{i,k}|\mathcal{I}^s_{i,k}, \delta_{i,k} = 0\} \\ & - \mathbb{E}\{J_{i,k}|\mathcal{I}^s_{i,k}, \delta_{i,k} = 1\}. \end{aligned} \tag{20}$$

Let $e^s_{i,k|k} \triangleq x_{i,k} - \hat{x}^s_{i,k|k}$ and thus the first term on the right hand side of (20) can be written as

$$\begin{aligned} &\mathbb{E}\{J_{i,k}|\mathcal{I}^s_{i,k}, \delta_{i,k} = 0\} - \mathrm{tr}\left(\Pi_{i,\infty}W_i\right) \\ &= \mathrm{tr}(\Gamma_{i,\infty}\mathbb{E}\{e_{i,k|k}e^T_{i,k|k}|\mathcal{I}^s_{i,k}, \delta_{i,k} = 0\}) \\ &= \mathrm{tr}(\Gamma_{i,\infty}\mathbb{E}\{(e^s_{i,k|k} + \check{e}_{i,k|k})(e^s_{i,k|k} + \check{e}_{i,k|k})^T|\mathcal{I}^s_{i,k}, \delta_{i,k} = 0\} \\ &= \mathrm{tr}(\Gamma_{i,\infty}\mathbb{E}\{e^s_{i,k|k}{e^s_{i,k|k}}^T|\mathcal{I}^s_{i,k}, \delta_{i,k}{=}0\}) + \mathrm{tr}(\Gamma_{i,\infty}\check{e}_{i,k|k}\check{e}^T_{i,k|k}) \\ &= \mathrm{tr}(\Gamma_{i,\infty}\overline{P}_i) + \mathrm{tr}(\Gamma_{i,\infty}\check{e}_{i,k|k}\check{e}^T_{i,k|k}), \end{aligned} \tag{21}$$

where the first equality is obtained by rearranging the terms in (6) and the law of total expectation and the second equality follows from $e_{i,k|k} = e^s_{i,k|k} + \check{e}_{i,k|k}$. The facts that $\mathbb{E}\{e^s_{i,k|k}|\mathcal{I}^s_{i,k}\} = 0$ and $\check{e}_{i,k|k}$ is deterministically given by

$$\check{e}_{i,k|k} = \sum_{\tau=k-t_{i,k-1}+1}^{k} A_i^{k-\tau}K^s_i(y_{i,\tau} - C_i\hat{x}^s_{\tau|\tau-1}), \tag{22}$$

where K^s_i is the Kalman gain in (17c), yield the third equality. Using a similar approach for the second term on the right hand side of (20) yields

$$\begin{aligned} &\mathbb{E}\{J_{i,k}|\mathcal{I}^s_{i,k}, \delta_{i,k} = 1\} \\ &= \mathrm{tr}\left(\Pi_{i,\infty}W_i\right) + \mathrm{tr}(\Gamma_{i,\infty}\overline{P}_i)q_i \\ &\quad + (1 - q_i)\left(\mathrm{tr}(\Gamma_{i,\infty}\overline{P}_i) + \mathrm{tr}(\Gamma_{i,\infty}\check{e}_{i,k|k}\check{e}^T_{i,k|k})\right), \end{aligned} \tag{23}$$

Rearranging the terms in (21) and substituting that and (23) in (20) completes the proof. □

Sensors are able to compute $\check{e}_{i,k|k}$ locally due to the decoupled dynamics. Therefore, $\mathrm{VoI}_{i,k}$ can also be calculated without requiring additional information from other subsystems. As a result, TOD can be implemented for solving Problem 1 by letting $m_{i,k}{=}\mathrm{VoI}_{i,k}q_i$ in (8). As aforementioned, the solution obtained by this setup is not necessarily optimal. However, $f(\cdot)$ in (8) can be fine-tuned for near-optimal performance as shown in Section V.

Conjecture 1. *Using the TOD scheme with* $m_{i,k} = \mathrm{VoI}_{i,k} q_i$ *stabilizes the system in the Lyapunov mean-square sense if*

$$\lim_{t\to\infty} \mu_i(t)^{1/t} < \frac{1}{\sigma^2_{\max}(A_i)}, \quad \forall i \in \{1, \ldots, N\}, \tag{24}$$

where $\mu_i(t) \triangleq \mathbb{P}\{t_{i,k} = t\}$.

Sketch of the proof/intuition. It is proved in [10, Theorem 1] that condition (24) guarantees boundedness of $\mathbb{E}\{P_{i,k|k}\}$ which is sufficient for Lyapunov mean-square stability of the system. This condition was verified in a time-triggered system with smart sensors and a variance-based priority measure for contention resolution given by

$$\overline{\mathrm{CoIL}}_{i,k} = q_i \operatorname{tr}\left(\Gamma_{i,\infty}\left[h_i^{t_{i,k-1}+1}(\overline{P}_i) - \overline{P}_i\right]\right). \tag{25}$$

Since $\overline{\mathrm{CoIL}}_{i,k}$ is independent of the real-time measurements, its value only depends on the number of consecutive packet dropouts which results in deterministic channel access decisions. This allows for modeling the packet arrival sequence as an ergodic Markov chain whose unique stationary distribution can be utilized to determine $\mu_i(t)$ and verify (24). The same method can be applied to a system with smart sensors using VBT and TOD with $m_{i,k}{=}\overline{\mathrm{CoIL}}_{i,k}$, since the channel access decisions become deterministic. Although not analytically proved here, intuitively, and as confirmed by the simulations, the use of real-time data for decision making, i.e., using $m_{i,k}{=}\mathrm{VoI}_{i,k}q_i$, improves performance. Hence, $\mathbb{E}\{P_{i,k|k}\}$ in this scenario is upper bounded by the one using $m_{i,k} = \overline{\mathrm{CoIL}}_{i,k}$ which concludes the proof. □

V. Numerical Results

A. The effect of priority measure on channel access decisions

To demonstrate the effect of the adopted priority measure on the channel access decisions, we consider an illustrative scenario where a wireless channel is shared between two subsystems with system matrices

$$A_1 = \begin{bmatrix} 1.1 & 0 \\ 0 & 0.9 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix},$$

and $B_i{=}C_i{=}Q_i{=}I_2$, $V_i{=}R_i{=}0.01I_2$, and $W_i{=}0.1I_2$ for $i \in \{1, 2\}$. Moreover, the probability of successful transmission is assumed to be $q_1{=}0.85$ and $q_2{=}0.5$. We consider the case where an infinitesimal triggering threshold is implemented for both subsystems. In other words, both subsystems constantly compete for transmitting their data packet. This allows us to isolate the impact of the sensors' capabilities on the choice of priority measure $m_{i,k}$ in (8) and thus on the contention resolution outcome. The contention is resolved by using 29-bit identifiers as it is the standard in CAN2.0B, where $n{=}20$ most significant bits represent $\mathrm{ID}^d_{i,k}$ while the remaining 9 bits allow the network to accommodate $2^9 - 1 = 511$ with unique static identifiers.

Fig 3 depicts the scenario of conventional sensors described in Section III and how utilizing $m_{i,k} = \mathrm{CoIL}_{i,k} q_i$ affects the evolution of CoIL. For the setup considered here, we choose a simple function $f(m_{i,k}) = \alpha m_{i,k}$ in (8) with $\alpha = 1000$ which results in unique dynamic identifiers

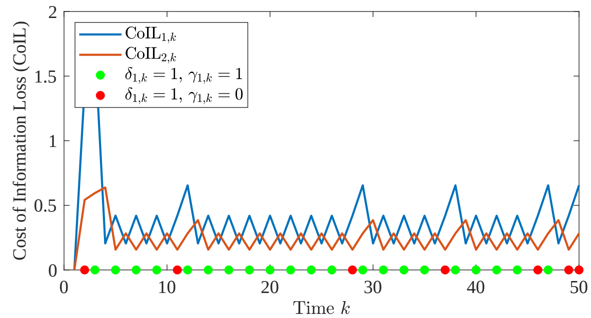

Fig. 3. Impact of using $m_{i,k} = \mathrm{CoIL}_{i,k} q_i$ in (8) when two subsystems with conventional sensors constantly compete for transmission over a single wir

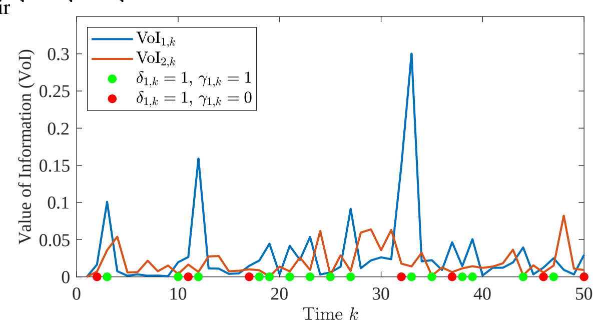

Fig. 4. Two subsystems with smart sensors constantly compete for transmission over a single wireless channel based on $m_{i,k} = \mathrm{VoI}_{i,k} q_i$.

and thus optimal channel access. As expected, Subsystem 1 wins the contention (shown as green and red dots) more frequently due to its unstable dynamics and faster growth rate of its CoIL. Furthermore, as CoIL is a function of the statistics of the random variables rather than their realization, it follows a predictable pattern. In contrast, VoI depends on the realization of the random variables ($y_{i,k}$ in (22)) as shown for the case of smart sensors in Fig. 4. This explains the more frequent transmissions by Subsystems 2 despite its stable dynamics and the irregular pattern of VoI.

B. Computation capabilities and control performance

Herein, we demonstrate how the choice of sensors, which indicates the possible choices for the triggering mechanism and the priority measure for contention resolution, impacts control performance. To this end, average number of communication attempts, i.e., crossing the triggering threshold, versus the average quadratic cost is depicted in Fig. 5. The results are averaged over 1000 simulations obtained for the system described in Section V-A on horizon $K{=}1000$.

Although all setups can be implemented with smart sensors, only two of them are compatible with conventional sensors, namely CoIL (Section III) and SoD. In SoD, transmission is triggered when the difference between the measured output and the last last successfully received one exceeds a threshold, i.e., $\left\| y_{i,k} - y_{i,k-t_{i,k}} \right\| \geq \Delta$ and contention is based on $m_{i,k}{=}\left\| y_{i,k} - y_{i,k-t_{i,k}} \right\|$. The additionally considered setups, which can only be realized by employing smart sensors, are denoted by VoI (Section IV) and CoIL $\overline{P}$. The later refers to a scenario in which sensors preprocess their measurements similar to VoI, while the priority measure for contention resolution is variance-based, i.e., $m_{i,k}{=}\mathrm{CoIL}_{i,k}q_i$ in (8). Fig. 5 shows that as the communication rate decreases,

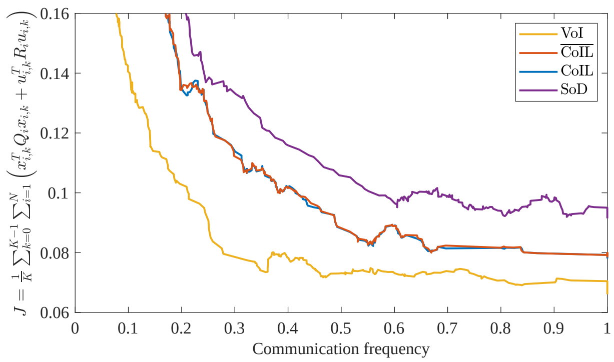

Fig. 5. Average communication rate vs. quadratic cost for conventional and smart sensors.

i.e., higher triggering threshold, performance deteriorates in all setups. Nevertheless, best performance is achieved when real-time processed measurements are utilized for event triggering and channel access as in VoI. Using real-time raw measurements, however, results in worst performance as indicated by SoD. Interestingly, when contention resolution is variance-based as in CoIL and $\overline{\text{CoIL}}$, transmission of the *a posteriori* state estimate instead of the raw measurement by the sensor does does not lead to significant gain in terms of the average quadratic cost. Moreover, the effect of using smart sensors when considering scalar sensors communicating over a perfect channel is more significant in terms of the estimation error as shown in [16]. However, when the effect of packet dropouts and channel access decisions are explicitly considered, the improvements are less significant.

VI. Conclusions

We investigated prioritized transmission schemes for event-triggered WNCSs with conventional and smart sensors. First, we brought together several approaches proposed in the literature, drew the connection between them and extended their operation for vector systems in unreliable wireless multi-channel environments. More specifically, we proposed *i)* the event-triggered version for CoIL for both conventional and smart sensors and *ii)* the event-triggered version for VoI which allows for distributed implementation and takes into account the channel conditions. For all scenarios, in order to be able to consider the coupling arising due to the shared communication resources, we proposed prioritization schemes compatible with TOD protocol. The results showed that when smart sensors are used, utilizing measurement-based triggering and VoI as the priority measure leads to the best performance. However, when no computational power is available at the sensor side, as in case of conventional sensors, VBT and CoIL for prioritizing channel access offers the best performance.

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