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Calculation of Far-Field Scattering from Nonuniform Reflective Metasurfaces: A Critical Perspective

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***Abstract*—To design and characterize radio environments with metasurfaces, it is necessary to know the scattering properties of finite-size anomalously reflecting (phase-gradient) metasurfaces. Recently, different approaches have been proposed to tackle this problem. In this presentation, we will present a comparison between alternative analytical models. We will first derive a general formulation for the far-field scattering by reflective metasurfaces based on the Huygens principle. Then we will specify the assumptions of known models. The first model that we will study is based on the assumption that at each point of the metasurface the reflected field equals the incident field multiplied by a controllable reflection coefficient. Then we will analyze the same structure with an analytical model based on the use of macroscopic reflection coefficients following the theory of diffraction gratings. Finally, we will numerically analyze the structure and compare the results.**

I. Introduction

Recently, researchers have been actively working on development of approximate models for communication channels in the presence of metasurfaces. Vast majority of estimations of the far-field scattering are based on the assumption of coherent summation of waves reflected by each individually controllable meta-atom, e.g. [1]–[3]. In this work, we refer to this model as the *local reflection coefficient model*. The applicability of this model is restricted to some classes of reflectarrays, and, as we will discuss later, even in this case the accuracy of the method is strongly dependent on the desired functionality of a specific metasurface. Recently, an alternative approximate analytical method for calculation of scattered fields in the far zone that merges approaches based on the physical optics and on the theory of diffraction gratings was introduced in [4]. We will refer to this method as the *macroscopic reflection coefficient model*. This model takes into account parasitic, multi-beam scattering, and is applicable for arbitrary illumination angles.

Because this field of research is evolving fast, it is important to provide a comprehensive comparison between the two methods and define the conditions for which each model can be applied with reasonable accuracy. In this work, we will provide a general framework for calculating the far-field scattering of metasurfaces based on the Huygens principle that is compatible with both methods. Later, we will particularize the applicability conditions for the local reflection coefficient model and the macroscopic reflection coefficient model. Finally, we will compare analytical calculations with numerical results to support our conclusions.

II. Far-field scattering by reflective finite-sized metasurfaces

In this section, we will study the scattering produced by an anomalously reflecting metasurface with the size $2a\times 2a$. The metasurface is illuminated by external fields $\mathbf{E}_\mathrm{i}$ and $\mathbf{H}_\mathrm{i}$. The electric and magnetic fields reflected by the metasurface are denoted by $\mathbf{E}_\mathrm{r}$ and $\mathbf{H}_\mathrm{r}$. Using a similar approach as in our recent work [4], we use the Huygens principle to calculate the far fields scattered by the metasurface as functions of the fields created by the metasurface right at its surface. Here, we consider metasurfaces illuminated by a TE-polarized plane wave $\mathbf{E}_\mathrm{i}=E_0e^{-jk(\sin\theta_\mathrm{i}x-\cos\theta_{\mathrm{in}}y)}\hat{\mathbf{z}}$ and assume that the metasurface does not introduce any change in the polarization state of the incident fields. For a clear representation, it is also convenient to normalize the pattern so that its maximum value for the PEC plate of the same size is unity. The normalized radiation patterns in the incidence plane ($\phi=\pi/2$) can be calculated as the following integral:

$$F_\mathrm{sc}^{(z)}\approx\frac{a}{S\cos\theta_\mathrm{i}}\int_{-a}^{a}e^{jk\sin\theta x'}[Z_0H_\mathrm{tot}^{(x)}(x')+\cos\theta E_\mathrm{tot}^{(z)}(x')]dx' \tag{1}$$

where S is the area of the metasurface. In this expression the terms $E_\mathrm{tot}^{(z)}$ and $H_\mathrm{tot}^{(x)}$ represent the tangential components of the total field and contain the information of both the equivalent surface currents defined by the reflected fields and the shadow currents defined by the incident fields, $F_\mathrm{sc}=F_\mathrm{r}+F_\mathrm{sh}$. For a given illumination, the differences between the two

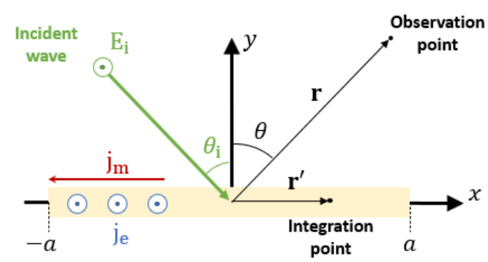

Fig. 1. Schematic representation of a reflective metasurface with the size $2a\times 2a$ under TE-illumination.

models appear in the definition of the reflected fields and consequently in the radiation pattern F_r.

A. Local reflection coefficient model

It is well known that anomalous reflectors can be implemented using the phased-array principle, also known a generalized reflection law. For a metasurface sending the energy impinging from θ_i into θ_r, the desired local reflection coefficient is defined by $\Gamma = e^{jk(\sin\theta_\mathrm{i}-\sin\theta_\mathrm{r})x}$. According to this definition, the local reflection coefficient defines the relation between the incident and reflected fields for each point as if the metasurface were locally homogeneous. In the theory of diffraction this approximation is called physical optics.

Using this local reflection coefficient assumption, the electric and magnetic fields reflected by the metasurface (right at the surface) are defined as $E_{rz} = E_0 e^{-jk\sin\theta_\mathrm{r}x'}$ and $H_{rx} = \frac{E_0}{Z_0}\cos\theta_\mathrm{i} e^{-jk\sin\theta_\mathrm{r}x'}$. It is important to stress that these reflected fields do not describe a plane wave traveling into θ_r. With this definition, the scattering pattern in the far zone can be written as

$$F_\mathrm{r}(\theta) = \frac{1}{2\cos\theta_\mathrm{i}}(\cos\theta_\mathrm{i} + \cos\theta)\mathrm{sinc}(ka_{\mathrm{ef}0}) \quad (2)$$

where $a_{\mathrm{ef}0} = (\sin\theta - \sin\theta_\mathrm{r})a$.

B. Macroscopic reflection coefficient model

A different way of modeling the far-field scattering by anomalous reflectors comes from the idea that the operational principle of metasurfaces capable of reflecting incident waves into anomalous directions is similar to that of diffraction gratings, as these surfaces are periodical structures. According to the Floquet theory, the spatial periodicity of the structure, D, defines the propagation directions of reflected modes as $\theta_{\mathrm{r}n} = \arcsin\left(\sin\theta_\mathrm{i} + \frac{2\pi}{kD}n\right)$.

The reflected electric field at a certain point of the metasurface and the tangential component of the magnetic field can be written as $E_{\mathrm{r}z} = E_0\sum_n r_n e^{-jk\sin\theta_{\mathrm{r}n}x'}$ and $H_{\mathrm{r}x} = \frac{E_0}{Z_0}\sum_n r_n\cos\theta_{\mathrm{r}n}e^{-jk\sin\theta_{\mathrm{r}n}x'}$, where r_n are the ratios of the complex amplitudes of the tangential components of the electric fields of the propagating Floquet harmonics of the reflected field and the tangential component of the incident electric field at the metasurface plane. The complex amplitudes of the reflection coefficients can be calculated using Floquet's expansion of the fields and enforcing the boundary conditions for an infinite metasurface [5]. Finally, the radiation pattern of the reflected fields reads

$$F_\mathrm{r}(\theta) = \frac{1}{2\cos\theta_\mathrm{i}}\sum_n r_n(\theta_\mathrm{i})(\cos\theta_{\mathrm{r}n} + \cos\theta)\mathrm{sinc}(ka_{\mathrm{ef}n}) \quad (3)$$

where $a_{\mathrm{ef}n} = (\sin\theta - \sin\theta_{\mathrm{r}n})a$.

III. Numerical Results and Discussion

To compare the two models, we analyze the scattering properties of an anomalous reflector with $\theta_\mathrm{i} = 0°$, $\theta_\mathrm{r} = 70°$, and $a = 5D$ under normal illumination. In the local reflection coefficient model, the radiation pattern is shown in Fig. 2 in red. As expected, this model predicts a maximum of the radiation pattern in the desired direction. For comparison, the radiation pattern calculated using the macroscopic reflection coefficient model is represented on the same picture (blue curve). We can see that this model predict a maximum of radiation in the desired direction, but with a significantly smaller amplitude, and additional parasitic beams that correspond to other propagating Floquet harmonics. This behaviour is in agreement with the analysis made in [6]. To verify the validity of approximate models, we conducted numerical simulations of finite-sized metasurfaces modeled with the same surface impedance profile. The results of the numerical simulations are in good agreement with the analytical results based on the macroscopic reflection coefficients, especially in the main lobes of the beams. This result highlights the limitations of the local reflection coefficient model. This approximation provides accurate predictions for anomalous reflectors only with small transformations between incident and reflected directions.

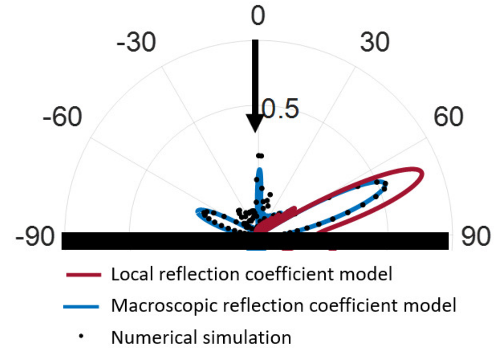

Fig. 2. Far-field scattering patterns calculated using the local reflection coefficient model and the macroscopic reflection coefficients model.

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