
This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Volovik, G. E.

Negative Newton constant may destroy some conjectures

Published in:
Modern Physics Letters A

DOI:
[10.1142/S0217732322500341](https://doi.org/10.1142/S0217732322500341)

Published: 28/02/2022

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Volovik, G. E. (2022). Negative Newton constant may destroy some conjectures. *Modern Physics Letters A*, 37(6), Article 2250034. <https://doi.org/10.1142/S0217732322500341>

1 Modern Physics Letters A
 2 2250034 (7 pages)
 3 © World Scientific Publishing Company
 5 DOI: 10.1142/S0217732322500341



6 Negative Newton constant may destroy some conjectures

7 G. E. Volovik
 8 *Low Temperature Laboratory, Aalto University,*
 9 *P. O. Box 15100, FI-00076 Aalto, Finland*
 10 *Landau Institute for Theoretical Physics, Acad. Semyonov Av.,*
 11 *1a, 142432, Chernogolovka, Russia*
 12 *grigori.volovik@aalto.fi*

Please check author
name, email and
affiliation

Please check
throughout the text for
spelling errors, figures
and tables

13 Received 16 February 2022
 14 Accepted 22 February 2022
 15 Published

16 The magnitude and sign of the gravitational coupling $1/G$ depend on the relations be-
 17 tween different contributions from scalar, fermionic and vector fields. In principle, this
 18 may give the zero and negative values of $1/G$ in some hypothetical Universes with the
 19 proper relations between the fermionic and bosonic species. We consider different con-
 20 jectures related to gravity, such as the wave function collapse caused by gravity, entropic
 21 gravity and maximum force. We find that some of them do not work in the Universes
 22 with zero or negative $1/G$, and thus cannot be considered as universal. Thus, the ex-
 23 tension of the gravitational coupling to $1/G \leq 0$ provides the test on the universality of
 24 the proposed theories of gravity.

25 *Keywords:*

gravitational coupling,
maximum force, wave
function collapse,
entropic gravity

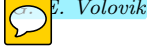
26 1. Introduction

27 Both in the fundamental gravity and in the induced Sakharov gravity,^{1–3} the grav-
 28 itational coupling G^{-1} depends on the fluctuating vacuum quantum fields. The
 29 contributions of massless fields at zero temperature are as follows:

$$30 \quad G^{-1} = G_0^{-1} + \frac{\Lambda_{uv}^2}{12\pi}(n_0 + n_{1/2} - 4n_1). \quad (1)$$

31 Here, Λ_{uv} is the UV cutoff (for the asymptotically safe scenario see review⁴);
 32 n_0 , $n_{1/2}$ and n_1 are the numbers of correspondingly scalar, Weyl and vector fields;
 33 and $1/G_0$ contains the other possible contributions, which may include the “funda-
 34 mental” value.

35 For massive fields, Eq. (1) is modified by the m^2 terms, while for the temperature
 36 T , which is larger than the masses of the fields but smaller than the UV cutoff, the
 37 G^{-1} contains the T^2 contributions. For example, in effective gravity emerging in



the superfluid ${}^3\text{He-A}$ with Weyl fermionic quasiparticles, the temperature correction to the inverse Newton “constant” contains the conventional contribution from the Weyl fermions^{5,6}:

$$G^{-1}(T) - G^{-1}(T = 0) = -\frac{\pi}{18}n_{1/2}T^2. \quad (2)$$

Since the thermal contribution does not contain the UV cutoff, it is universal, i.e. it is the same for the fermionic relativistic quantum fields and for the Weyl-like fermions in condensed matter. The same universality takes place for scalar fields: the $(\pi/9)n_0T^2$ contribution to $G^{-1}(T)$ comes both from the relativistic scalars in gravity⁷ and from phonons in the effective gravity emerging in superfluid ${}^4\text{He}$.⁶

So, we have two separate quantities: the UV cutoff Λ_{uv} , which, in principle, can be fundamental and can be called the Planck energy scale; and the gravitational coupling $1/G$, which cannot be fundamental, since it depends on temperature, species and their masses. In particular, while Λ_{uv}^2 is always positive, in the Universes with the proper relations between the fermionic and bosonic fields, the gravitational coupling $1/G$ can be zero and even negative. The negative and zero gravitational coupling $1/G$ have been discussed in theories with spontaneous breaking of scale invariance by scalar field.⁸ The possibility of the negative value of G during the cosmological time evolution was also debated, see, e.g. Refs. 9 and 10. The negative G naturally appears in the effective gravity emerging in the superfluid ${}^3\text{He}$ with massless Dirac quasiparticles,¹¹ where it is responsible for the solid angle excess in the effective metric of the global monopole.

The zero value of $1/G$ may take place in the Big Bang, if it is considered as the topological quantum phase transition.¹² In this scenario $1/G$ is always positive, and become zero only at the point of the Big Bang. The reason for that is that the Big Bang is considered as the intermediate state between two topological vacua with broken conformal symmetry, while in the topologically trivial intermediate state, the conformal symmetry is restored, and $1/G = 0$.

Some conjectures related to gravity, which are based on the positive value of the gravitational coupling, $G^{-1} > 0$, do not work in the Universes with $G^{-1} \leq 0$. That is why such conjectures cannot be considered as universal. Example is provided by the Diosi–Penrose scenario of the collapse of the wave function induced by gravity,^{13–15} which is discussed in Sec. 2. This scenario does not work in the Universe with $G^{-1} < 0$, and thus cannot be considered as fundamental. Verlinde conjecture of the entropic gravity in Sec. 3 and the conjecture of the maximum force in Sec. 5 do not work in the Universe with $1/G = 0$.

Of course, if $1/G < 0$, the vacuum can be unstable, since the gravitons have negative kinetic energy. But in a given context, it is important that $1/G < 0$ is, in principle, possible. If some conjecture is fundamental and universal, it should be valid irrespective of the stability or instability of a given state of the quantum vacuum. That is why the extension of the conjecture to the vacua with $1/G \leq 0$ provides the test on the universality of the conjecture.



ative Newton constant may destroy some conjectures

2. Diosi–Penrose Scenario of Gravitationally Induced Collapse

The physical origin of the collapse of the wave function is a fundamental problem of quantum mechanics. Here, we discuss the scenario in which the wave function collapse is caused by gravity.^{13,15} In the semiclassical approach, which uses the nonlinear Schrödinger equation, there is the gravitational self-interacting term in the energy:

$$U = -\frac{GM^2}{4} \iint \frac{d^3\mathbf{r}d^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} (|\Psi_R(\mathbf{r})|^2 - |\Psi_L(\mathbf{r})|^2)(|\Psi_R(\mathbf{r}')|^2 - |\Psi_L(\mathbf{r}')|^2). \quad (3)$$

Here, M is the mass of the body; Ψ_R and Ψ_L are two wave packets (or in the other similar scenario, the wave functions in the two potential wells, left and right). For $G > 0$ the localization of the wave packet (or the localization of the body in one of the potential wells) is energetically favorable. This according to Penrose¹⁵ and Diosi¹³ leads to the collapse of the quantum superposition, with the collapse time $\Delta t \sim 1/U$.

However, for $G < 0$ this conjecture does not work: the quantum superposition of the wave packets has lower energy, thus leading to the “anti-collapse”, i.e. the superposition “wins” the game. Since the Diosi–Penrose scenario depends on the sign of the gravitational coupling, and thus on the relations between the quantum fields, it cannot be the fundamental source of the wave function collapse.

The criticism of this scenario of collapse can be found for example in Ref. 16, where in particular the application of nonlinear Schrödinger–Newton equation is criticized, since it violates the linear character of quantum mechanics.

3. Verlinde Entropic Gravity

Let us first mention that when the entropy of horizon is considered, the contribution of the fluctuating n_1 vector fields can be different from the negative contribution of these fields in Eq. (1).^{17–19} Nevertheless, the negative or zero value of G^{-1} may come from the negative value of G_0^{-1} .

In the entropic gravity by Verlinde²⁰ it is assumed that the number of bits of information on the holographic screen is ($\hbar = c = 1$):

$$N = \frac{A}{G}. \quad (4)$$

This entropic scenario of emergent gravity does not work in the Universe with $1/G = 0$, since the information is simply absent, $N = 0$. Nevertheless, in such Universe, gravity does exist, since $1/G = 0$ simply means that gravitational action starts with the quadratic terms. Equation (4) does not make sense for $1/G < 0$. All this demonstrates that this entropic approach to gravity is not universal.

If the holographic principle is valid in the Universe with $G^{-1} \leq 0$, it would be natural if it is determined by the UV cutoff scale: $N \sim A\Lambda_{\text{uv}}^2$. But this has no relation to gravity in the Universe with $G^{-1} < 0$, where Newton’s law has the opposite sign.

G. E. Volovik

4. Jacobson Gravity from the First Law of Thermodynamics

In the Jacobson conjecture²¹ of the thermodynamic origin of Einstein equations (see also review in Ref. 22), the “species problem” (dependence of G on species and their masses) is taken into account. According to Ref. 23 the Bekenstein–Hawking entropy $A/4G$ contains the same renormalized $1/G$, which comes from the fluctuating vacuum fields (the possible exception from this rule may come from the vector fields^{17–19}).

At first glance the extension to the negative $1/G$ looks problematic. This is because the black hole horizons do not exist in the Universe with $1/G < 0$. However, in this Universe, the de Sitter state can exist if the vacuum energy density Λ (the cosmological constant) is negative, with the Hubble parameter $H^2 = (8\pi/3)|G||\Lambda|$. Thus, at least the cosmological horizon at $R = 1/H$ may exist, which shows that the local Rindler horizon can be constructed and can be used for the derivation of the Einstein action.

There are two possible versions for the entropy of the horizon in the Universe with $1/G < 0$: it can be either $S = A/4|G|$, or $S = -A/4|G|$. The negative entropy of horizon was discussed for a white hole obtained from the black hole with the same mass by quantum tunneling,^{24,25} and in Einstein–Gauss–Bonnet gravity,^{26,27} see also Ref. 28 and references therein. It is possible that if there is a meaningful notion of negative entropy, then the Jacobson approach may produce gravity with negative G . It would be interesting to consider the connection between the negative G and the negative S .

In the Universe with $1/G = 0$, the action for gravity consists of the R^2 terms, such as in the fourth-order conformal Weyl gravity. In this case, the entropy is not proportional to area, and the Jacobson approach should be modified to obtain the 4th order gravity.²⁹

All this would mean that in this approach the entanglement entropy per unit area is not a universal constant, which is the same for all Universes, but it is the constant which characterizes a given Universe. Note that in the tetrad gravity theory, where the tetrad fields emerge as bilinear combinations of the fermionic fields,^{30–34} the tetrads have dimension of inverse length. Similar dimensional tetrads emerge in the elasticity theory of solids.^{35,36} In this case, the gravitational coupling $1/G$ and the area A become dimensionless.³⁷ In principle, the dimensionless parameter $1/(4G)$ can be a kind of topological invariant which characterizes the Universes with different topology. Such invariant can take any integer value including the zero and negative values, $1/(4G) = \dots - 2, -1, 0, +1, +2, \dots$

5. Maximal force

The maximum force was conjectured by Gibbons,³⁸ see also recent papers^{39,40} and references therein:

$$F_{\max} = \frac{1}{4G}. \quad (5)$$

1 This is the force between two equal mass static uncharged Schwarzschild black holes
 2 touching each other at the horizon. It was suggested that this is the maximum
 3 value of a force between any two objects. However, in the Universe with $1/G = 0$,
 4 any force between the two objects exceeds this limit, and thus this limit is not
 5 universal.

6 On the other hand, in the Universe with $1/G < 0$ the black holes are not formed,
 7 and thus the force between the gravitational bodies is not limited, i.e. the maximum
 8 force does not exist. Also, the cosmic strings and global monopoles may exist in the
 9 Universe with $1/G < 0$. Instead of the angle deficit in metric outside these defects,
 10 there will be correspondingly the angle excess and solid angle excess.¹¹ As a result
 11 the arguments by Gibbons,³⁸ which are based on the maximum angle deficit, are
 12 not applicable.

13 In principle, it is possible, that there exists the maximum force, which is deter-
 14 mined, say, by the UV cutoff, $F_{\max} \sim \Lambda_{\text{uv}}^2$. But while this maximum force can be
 15 fundamental, it is not related to the gravitational coupling $1/G$, which depends on
 16 many details and can be made arbitrarily small. Anyway, one should very clearly
 17 specify what type of force is considered.⁴¹

18 6. Conclusion

19 In our Universe the gravitational coupling $1/G$ is positive. However, there is no
 20 rule at all prohibiting the existence of a Universe with a negative or zero value of
 21 $1/G$, even if it may be short-lived due to instability. That is why this possibility
 22 should be taken into account, when the general principles are introduced. Some
 23 conjectures related to gravity do not work in the Universes with $1/G < 0$ or with
 24 $1/G = 0$. That is why such conjectures cannot be considered as the universal
 25 principles.

26 The extension of the gravitational coupling to $1/G \leq 0$ provides the test on the
 27 universality of the proposed theories of gravity. In particular, the Diosi–Penrose
 28 scenario of gravitationally induced collapse, the Verlinde entropic gravity and max-
 29 imum force conjecture cannot be extended to $1/G \leq 0$, while the Jacobson approach
 30 requires modification.

31 It will be interesting to test the other theories,^{42,43} and to exploit the condensed
 32 matter systems, where there are different types of emergent gravity (acoustic,^{44,45}
 33 from Weyl point,⁵ from bilinear combinations of the fermionic fields,³⁷ etc.), and
 34 different types of event horizons can be simulated.^{46,47}

35 Acknowledgments

36 I thank V. Faraoni, T. Jacobson and A. Zelnikov for discussions and critical
 37 comments. This work has been supported by the European Research Council (ERC)
 38 under the European Union’s Horizon 2020 research and innovation programme
 39 (Grant Agreement No. 694248).

G. E. Volovik

References

1. A. D. Sakharov, *Sov. Phys. Dokl.* **12**, 1040 (1968); [Reprinted in *Gen. Relat. Gravit.* **32**, 365 (2000); *Theor. Math. Phys.* **23**, 435 (1976)].
2. V. P. Frolov, D. V. Fursaev and A. I. Zelnikov, *Nucl. Phys. B* **486**, 339 (1997).
3. M. Visser, *Mod. Phys. Lett. A* **17**, 977 (2002).
4. J. M. Pawłowski and M. Reichert, *Front. Phys.* **8**, 551848 (2021).
5. G. E. Volovik, *The Universe in a Helium Droplet* (Clarendon Press, 2003).
6. G. E. Volovik and A. I. Zelnikov, *JETP Lett.* **78**, 751 (2003).
7. Y. V. Gusev and A. I. Zelnikov, *Phys. Rev. D* **59**, 024002 (1999).
8. N. N. Khuri, *Phys. Rev. D* **26**, 2664 (1982).
9. A. A. Starobinskij, *Sov. Astron. Lett.* **7**, 36 (1981).
10. I. Ayuso, J. P. Mimoso and N. J. Nunes, *Galaxies* **7**, 38 (2019).
11. G. E. Volovik, *JETP Lett.* **112**, 505 (2020).
12. F. R. Klinkhamer and G. E. Volovik, arXiv:2111.07962.
13. L. Diosi, *Phys. Lett. A* **105**, 199 (1984).
14. L. Diosi, *Phys. Rev. A* **40**, 1165 (1989).
15. R. Penrose, *Gen. Relat. Gravit.* **8**, 581 (1996); R. Penrose, in *Mathematical Physics 2000*, ed. A. Fokas *et al.* (Imperial College), pp. 266–282.
16. S. L. Adler, *J. Phys. A: Math. Theor.* **40**, 755 (2007).
17. D. Kabat, *Nucl. Phys. B* **453**, 281 (1995).
18. W. Donnelly and A. C. Wall, *Phys. Rev. Lett.* **114**, 111603 (2015).
19. W. Donnelly and A. C. Wall, *Phys. Rev. D* **94**, 104053 (2016).
20. E. Verlinde, *J. High Energy Phys.* **4**, 29 (2011).
21. T. Jacobson, *Phys. Rev. Lett.* **75**, 1260 (1995).
22. T. Padmanabhan, *Rep. Prog. Phys.* **73**, 046901 (2010).
23. T. Jacobson, arXiv:gr-qc/9404039.
24. G. E. Volovik, *Mod. Phys. Lett. A* **36**, 2150117 (2021).
25. G. E. Volovik, arXiv:2108.00419.
26. M. Cvetič, S. Nojiri and S. D. Odintsov, *Nucl. Phys. B* **628**, 295 (2002).
27. S. I. Kruglov, *Symmetry* **13**, 944 (2021).
28. Y. Li, K. A. Milton, P. Parashar and L. Hong, *Entropy* **23**, 214 (2021).
29. R. Guedens, T. Jacobson and S. Sarkar, *Phys. Rev. D* **85**, 064017 (2012).
30. D. Diakonov, arXiv:1109.0091.
31. A. A. Vladimirov and D. Diakonov, *Phys. Rev. D* **86**, 104019 (2012).
32. A. A. Vladimirov and D. Diakonov, *Phys. Part. Nucl.* **45**, 800 (2014).
33. Y. N. Obukhov and F. W. Hehl, *Phys. Lett. B* **713**, 321 (2012).
34. C. Wetterich, *Phys. Lett. B* **712**, 126 (2012).
35. J. Nissinen and G. E. Volovik, *ZhETF* **154**, 1051 (2018) [*JETP* **127**, 948 (2018)].
36. J. Nissinen and G. E. Volovik, *PRResearch* **1**, 023007 (2019).
37. G. E. Volovik, *ZhETF* **159**, 815 (2021) [*JETP* **132**, 727 (2021)].
38. G. W. Gibbons, *Found. Phys.* **32**, 1891 (2003).
39. V. Faraoni, *Phys. Rev. D* **103**, 124010 (2021).
40. C. Schiller, *Phys. Rev. D* **104**, 124079 (2021).
41. A. Jowsey and M. Visser, *Universe* **7**, 403 (2021).
42. R. Bousso, X. Dong, N. Engelhardt, T. Faulkner, T. Hartman, S. H. Shenker and D. Stanford, arXiv:2201.03096.
43. D. Harlow, B. Heidenreich, M. Reece and T. Rudelius, arXiv:2201.08380.
44. W. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981).

1st Reading

Negative Newton constant may destroy some conjectures

- ¹ 45. C. Gooding, S. Piarmann, S. Erne, J. Louko, W. G. Unruh, J. Schmiedmayer and
- ² S. Weinfurtner, [arXiv:2007.07160](#).
- ³ 46. G. E. Volovik, *Pis'ma ZhETF* **104**, 660 (2016) [*JETP Lett.* **104**, 645 (2016)].
- ⁴ 47. Y. Kedem, E. J. Bergholtz and F. Wilczek, *Phys. Rev. Res.* **2**, 043285 (2020).