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Barbalinardo *et al.* Reply: Comment [1] shows that the thermal conductivity (κ) of a (10,0) carbon nanotube (CNT), obtained by inversion of the linearized Boltzmann transport equation (BTE), may not converge when the third-order interatomic force constants (IFC) are computed analytically. Bruns *et al.* [1,2] showed that the lifetimes $\tau_{\text{RTA}}(q)$ of the acoustic branches, computed in the relaxation time approximation (RTA), obey precise power laws for $q \rightarrow 0$, which are violated by the numerical IFCs in our Letter [3]. κ in Ref. [1] is much larger than that in Ref. [3], and does not converge even when 400 q -points are used to integrate the first Brillouin zone. These new results are correct and challenge the agreement between BTE (with classical statistics) and molecular dynamics (MD) proposed in Ref. [3], and the statement that κ is finite in the bulk limit.

Here we address the accuracy of finite-differences (FD) calculations, the convergence of κ for infinitely long CNTs, and $\kappa(L)$ for CNTs of length L . Bruns *et al.* shows that the long-wavelength trends of τ_{RTA} are disrupted by numerical errors introduced in the FD calculation of the IFCs resulting in nonsymmetrized force constant matrices. We computed IFCs by FD with displacements dx ranging from 10^{-6} to 10^{-3} Å, and the correct trends for τ_{RTA} of the flexural (FA), longitudinal (LA) and torsional (TW) acoustic modes at low q are recovered for $dx > 4 \times 10^{-5}$. Figure 1 shows that $\tau_{\text{RTA}}(q)$ from numerical IFCs with $dx = 10^{-4}$ Å are nearly indistinguishable from those obtained from analytical IFCs (in Ref. [3] we used $dx > 4 \times 10^{-6}$ Å). This result is useful for future works for those cases in which it is not possible to compute IFCs analytically.

τ_{RTA} provides a compelling test of the IFC's accuracy but cannot be used to calculate κ of CNTs due to the predominance of hydrodynamic effects in low-dimensional materials [4]. We computed the lifetimes of the acoustic modes by directly inverting the scattering matrix (τ_{inv}), using both analytical and numerical IFCs [Fig. 1]. Because of the hydrodynamic effects, the values and low- q trends of τ_{inv} and τ_{RTA} are different. Fitting τ_{inv} for $q \rightarrow 0$ one may infer whether κ converges or diverges in the infinite-size limit. The modes that may lead to divergence are FA and TW, as their τ_{inv} diverge for $q \rightarrow 0$. The group velocity of the FA modes is proportional to q , and that of TW is constant; thus the onset of κ divergence is either $\tau_{\text{FA}} \propto 1/q^3$ or $\tau_{\text{TW}} \propto 1/q$. τ_{TW} has a similar low- q behavior as in RTA calculations, $\tau_{\text{inv,TW}} \propto 1/q^{0.5}$. $\tau_{\text{inv,FA}}$ from analytical IFCs diverges as $1/q^3$ or steeper, suggesting divergent κ . Conversely, using numerical IFCs we get a slower than $1/q^3$ trend, which would lead to finite κ . This calculation may be more representative of systems at finite temperature, possibly reconciling BTE with convergent κ in MD simulations. Calculations with finite temperature IFC renormalization and thicker q -point sampling will be necessary to prove the convergence of κ in CNTs.

Finally, we computed finite-length $\kappa(L)$ with analytical IFCs. We find that $\kappa(L)$ remains in agreement with our

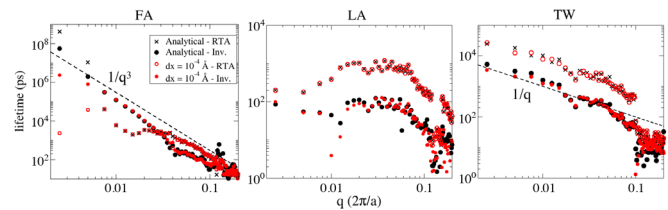


FIG. 1. Lifetimes of the FA, LA, and TW acoustic modes computed with quantum statistics. All the calculations are done by integrating the BTE over a uniform grid of 401 q points.

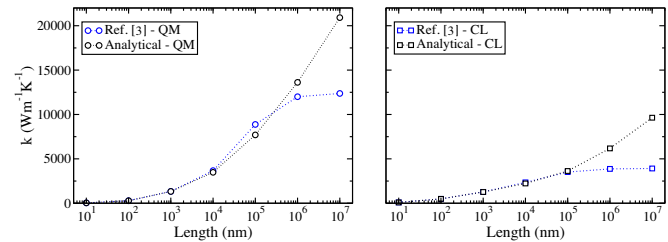


FIG. 2. Thermal conductivity as a function of length computed with analytical and the numerical IFCs from Ref. [3].

former calculations [3] up to $L = 1$ mm while it gets significantly larger for larger lengths (Fig. 2).

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