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Ultra-high harmonic mode-locking with a micro-fiber knot resonator and Lyot filter

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Abstract: We report on ultra-high harmonic mode-locking with a repetition rate of up to ~1 THz by combining a microfiber knot resonator (MKR) and a Lyot filter. The harmonic mode-locked pulses are tunable by changing the diameter of MKR, which agrees well with the theoretical calculation. Our results indicate that the ultrafast pulse generation mechanism is due to the dissipative four-wave mixing mode-locking technique. This work provides a simple and efficient scheme to generate tunable ultrafast pulses with a high repetition rate for various applications, such as THz generation and ultrafast data communication.

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1. Introduction

High-repetition-rate (HRR) lasers have wide applications in many fields, including optical communication [1,2], optical frequency comb [3], laser processing [4], nonlinear optics [5,6], and data storage [7] and so on. Particularly, there is an urgent need to develop ultrafast fiber lasers with pulse repetition rates higher than gigahertz, such as nonlinear bioimaging and microwave photonics [8,9]. The all-fiber lasers are among the most attractive candidates for generating HRR pulses thanks to their excellent dissipation, compact design, and low-cost [10]. So far, many schemes have been proposed. For example, 10 GHz pulses were reported based on active mode-locking fiber lasers, but the repetition rate is mainly limited by the working bandwidth of the modulator and the driving electronic devices [11,12]. Compared with active mode-locking, the passive mode-locking configuration can achieve a higher repetition rate by reducing the cavity length. Pulse repetition rates in a range of 10-20 GHz has been obtained from short cavities [13]. However, the output power is limited by the reduced gain in the short cavity. By increasing the number of mode-locked pulses evenly distributed along the laser cavity, harmonic mode-locking can use a longer laser cavity and achieve a higher repetition rate [14,15]. Reference [16] has reported the generation of pulses with a repetition rate of ~22 GHz and the harmonic order of ~928. However, the harmonic order and pulse stability are difficult to control [14,15]. Although it has been reported that all-polarization-maintaining harmonic mode-locked erbium-doped fiber laser has good stability [17], it is not sensitive to bending and squeezing due to the high intracavity birefringence, which is not advantaged to enable tunable operation. At the same time, the all-polarization-maintaining fiber devices are not cost-effective.

Since the last decade, another mode-locking scheme, the dissipative four-wave mixing (DFWM) has been proposed to increase the repetition rate by adding multi-wavelength selective elements (such as comb filters) and nonlinear elements with reasonable dispersion into the cavity. The comb filters are used to generate equal-phase coherent sidebands, including Fabry-Perot filters.
Mach-Zehnder interferometer \cite{19}, and high-Q microring resonator (MRR) \cite{20,21}. The DFWM mode-locking can generate an ultra HRR (over 100 GHz or even THz) because all sidebands are phase-locked \cite{22,23}. Moreover, the comb filters as a multi-wavelength selective element can effectively suppress background noise \cite{24,25}. Among those filters, MRR is of great potential because it can serve as both a filter and a nonlinear element. For example, a stable repetition rate of 200 GHz has been reported using a silicon-based MRR \cite{20}, called filter-driven FWM (FD-FWM). However, the silicon-based MRR has the disadvantages of high cost and low coupling efficiency \cite{20,26}. Micro-fiber based MRR is therefore proposed to overcome those disadvantages. However, the repetition rate of the mode-locked lasers based on FD-FWM equals the free spectral range (FSR), which depends on the diameter of the MRR \cite{27}. Therefore, smaller the ring typically results in a higher repetition rate. As the MRR would easily be fractured when the diameter is reduced, the reported repetition rates with micro-fiber based MRRs are in the range of 100-200GHz \cite{28,29}.

In addition, although microfibers exhibit higher nonlinearity than single-mode fiber (SMF) \cite{30}, their nonlinearity is lower than that of silicon waveguides and not able to provide enough nonlinearity required to trigger ultrashort pulse generation \cite{20}. Therefore, microfiber needs to be combined with other higher nonlinear elements. In recent years, various two-dimensional (2D) materials have attracted intense attention for their excellent optical, electronic, and electrochemical characteristics. The 2D materials possess large saturable absorption coefficients and nonlinear refractive indexes, which are excellent candidates for saturable absorbers (SA) and nonlinear elements \cite{31–35}. Graphene is one of the most studied and well-known 2D materials, which can not only provide nonlinearity to the laser cavity as a nonlinear element, but also can be used as a saturable absorber to narrow pulse duration to obtain higher repetition rate. Reference \cite{29} has used micro-fiber knot resonators (MKR) with the assistance of graphene and obtained pulse repetition rates of ~162 GHz.

In this work, an MKR comb filter is taken as the key element to achieve the DFWM effect, and a graphene film (GF) is placed between two fiber connectors to form a SA. Furthermore, using the Lyot filtering effect in a fiber laser based on nonlinear polarization rotation (NPR) technique, we obtained the harmonic mode-locking with a pulse repetition rate of > 1 THz, the highest repetition rate reported with MRR to the best of our knowledge. The mechanism of harmonic mode-locking was analyzed through theoretical simulation. Our work provides an effective method for developing ultra-high repetition rate lasers with compact structure, low cost, and simple operation.

2. Experimental details

2.1. HRR Erbium-doped fiber laser setup

Figure 1 shows the experimental setup of the HRR erbium-doped fiber laser. Two 980-nm laser diodes (LDs) (power-adjustable in the range of 0-0.9 W) are used to pump a 3-m erbium-doped fiber (LIEKKI, Er110-4-125, group velocity dispersion of ~0.012 ps$^2$/m) through a wavelength division multiplexer (WDM). The Er doped fiber CW laser spectrum as depicted in the red illustration on the upper right. The other fibers involved in the setup are single-mode fibers (SMF-28, group velocity dispersion of -0.023 ps$^2$/m). The total cavity length is ~17.6 m, corresponding to a longitudinal mode spacing of ~11.7 MHz. The total dispersion of the main cavity is calculated to be ~0.3 ps$^2$. Two polarization controllers (PCs) are used to adjust the polarization state of modes. A polarization-dependent isolator (PD-ISO) ensures the unidirectional laser propagation and provides polarization selectivity, which forms a Lyot filter with the intra-cavity fiber birefringence \cite{36}. An MKR is placed on a magnesium fluoride substrate, and the diameter of the MKR can be adjusted by the motors underneath the holders at both sides. The laser output through a 10% optical coupler (OC) was measured by an optical spectrum analyzer (Agilent, HP70951B) and an autocorrelator (APE, pulse Check).
Fig. 1. Setup of the HRR fiber laser. LD: laser diode; WDM: wavelength division multiplexer; EDF: erbium-doped fiber; PC: polarization controller; OC: optical coupler; PD-ISO: Polarization-dependent isolator; SA: saturable absorber (graphene film).

2.2. Preparation and spectral characterization of MKR

A microfiber with a diameter of \( \sim 5 \mu m \) is fabricated with the standard single-mode fiber (SMF) using the flame-stretching technique. Then the microfiber knot is manually knotted at the waist of the microfiber. In this work, two MKRs with an initial diameter of \( \sim 630 \mu m \) and \( \sim 1.6 mm \) are involved. The morphology and spectral response of both MKRs are characterized. The morphology of MKRs is observed under a microscope. And the spectral response curve is measured by injecting an amplified spontaneous emission (ASE) light (1530 nm-1570 nm) into the MKR, and detecting the output by an optical spectrum analyzer. The characterization of the MKR with a diameter of \( \sim 630 \mu m \) is shown in Fig. 2 as an example. The solid blue line in Fig. 2(c) shows the measured spectral response of the MKR. The estimated free spectrum range of the MKR (denoted as \( FSR_{MKR} \)) is \( \sim 0.83 nm \), which is in good agreement with the theoretical calculation with a diameter of \( \sim 630 \mu m \). \( FSR_{MKR} \) is determined by the circumference of MKR with the equation of \( FSR_{MKR}=\frac{\lambda^2}{nL} \) (\( n \) is the refractive index, \( \lambda \) is the wavelength, \( L \) is the circumference of MKR). The Q-factor of our MKR is calculated to be \( \sim 8620 \).

2.3. Preparation and spectral characterization of GF-SA

The GF is made by spin-coating the graphene polyvinyl alcohol (PVA) water solution, which is obtained by dissolving 2 mg graphene powder (purchased from Aladdin) and polyvinyl alcohol (PVA) into 40 ml deionized water and heating the mixture to appropriate viscosity. The Raman spectrum of the fabricated GF is shown in the inset of Fig. 3(a). Three typical peaks can be observed, including the D peak at \( \sim 1321 \) cm\(^{-1} \), the G peak at \( \sim 1581 \) cm\(^{-1} \), and the 2D peak at \( \sim 2645 \) cm\(^{-1} \). The D peak verifies the existence of weak defects in graphene. And the intensity ratio of the 2D peak to G peak is calculated to be \( \sim 0.51 \), which indicates that the GF is multi-layered (about 4 layers) [37]. A scanning electron microscope (SEM) is used to measure the thickness of the film, as shown in the inset of Fig. 3(a), which can be seen that the thickness of the film is about 12.82 \( \mu m \). Then, the GF was placed between two fiber connectors to form a GF-SA, as shown in Fig. 3(a).

The nonlinear absorption characteristics of the GF-SA was measured by using an ultrafast fiber laser with a repetition rate of \( \sim 28.8 \) MHz and a pulse duration of \( \sim 11 \) ps (Fig. 3(b)). The figure shows that the modulation depth of the GF-SA is \( \sim 6.4\% \).
3. Experimental results

3.1. MKR filter mode-locking

Firstly, the self-starting mode-locking pulse trains are obtained by carefully adjusting the PCs when the pump power is higher than \(~700\) mW. Similar results have been reported using the MKR with graphene [31,32]. The spectral response curve and pulse trains of the MKR filter mode-locking are shown in Fig. 4. It is seen from Fig. 4 (a) that the laser is in the DFWM mode-locking state. It is noted that stable multiple lasing modes are not achieved without GF, indicating GF provides the nonlinearity for the generation of the FWM effect to realize DFWM mode-locking. The spectrum exhibits comb profiles with a spectral spacing of \(~0.83\) nm, which is in accordance to the \(FSR_{MKR}\) of the MKR comb filter shown in Fig. 2(c). We can then estimate the pulse repetition rate \(\Delta \nu\) to be \(~104\) GHz based on the relationship between the repetition rate \(\Delta \nu\) and \(FSR_{MKR}\) as \(\Delta \nu = c \cdot FSR_{MKR} / \lambda^2\) (\(c\) is the speed of light in vacuum, \(\lambda\) is the wavelength). The pulse trains measured by the autocorrelator confirm our estimation. Figure 4(b) shows that
the laser pulse train with a temporal spacing ($\Delta T$) of $\sim 9.53$ ps is generated, and the corresponding $\Delta \nu$ is $\sim 105$ GHz.

Fig. 4. Spectra and autocorrelation curves of the laser outputs with tunability. (a-c) The spectra of the pulses and (d-f) the corresponding autocorrelation traces with the MKR diameter of $\sim 630\, \mu m$, $\sim 460\, \mu m$ and $\sim 200\, \mu m$. The diameter of the MKR is tuned by the motors underneath the holders on both sides (Fig. 1).

Then, the diameter of the MKR is slowly reduced to $\sim 460\, \mu m$ and $\sim 200\, \mu m$ by controlling the stepping motor (Fig. 1). The $FSR_{MKR}$ of $\sim 1.15$ nm and $\sim 2.64$ nm is obtained, respectively (Figs. 4(b-c)). The $\Delta T$ of the two cases is further measured as $\sim 7.09$ ps and $\sim 3.10$ ps, corresponding to $\Delta \nu$ of $\sim 141$ GHz and $\sim 323$ GHz, respectively. The bandwidths of the spectral envelopes in Fig. 4(a-c) are estimated as $\sim 1.02$, $1.39$, and $3.07$ nm. If a Gaussian profile is assumed for fitting, the pulse durations in Fig. 4(d-f) are estimated as $\sim 3.90$, $2.80$, and $1.23$ ps, respectively. Then, the time-bandwidth products (TBPs) of the pulses are calculated as $\sim 0.50$, $0.51$, and $0.49$, indicating a slightly chirped pulse.

The above experimental spectra indicate that the intensity of lasing lines decreases faster than the typical DFWM mode-locked spectrum from the center to the edge, which is attributed to the saturation absorption effect of graphene [23]. The slight asymmetry in the spectra is the consequence of the uneven gain characteristics of the EDF. In addition, the supermode noise can be seen in the background of the autocorrelation traces, which is since the large bandwidth of each MKR spectral line allows multiple-cavity longitudinal modes lasing. For example, the $\Delta \lambda_{FWHM}$ of an individual MKR spectral line in Fig. 4(a) is about $\sim 0.087$ nm. And the longitudinal mode spacing of the main cavity is $\sim 1.7$ MHz, corresponding to a spectral separation $\Delta \lambda_{LM}$ of $\sim 9.37 \times 10^{-5}$ nm. Therefore, there are about $\sim 929$ longitudinal modes lasing within each MKR spectral line, leading to the problem of supermode instability [38]. The supermode noise can be suppressed by shortening the laser cavity or adopting a comb filter with higher fineness [39].
3.2. **Lyot filter mode-locking**

Furthermore, we find that the pulse repetition rate would be multiplied by precisely adjusting the PCs, thanks to the Lyot filtering effect in fiber lasers based on NPR technology. We take an MKR with a diameter of ~1.6 mm as an example for demonstration. The MKR (the intrinsic $FSR_{MKR}$ was measured to be 0.33 nm) was placed in the laser cavity, and the spectral response and autocorrelation trace are measured under different polarization states of the PCs. Figures 5(a-c, f-h) show the three sets of typical results. From the spectra, we can see there are two main peaks, with the spacing ($\Delta \lambda$) of ~2.69 nm, ~8.04 nm, and 11.3 nm respectively, which are about 8, 24

![Fig. 5](image)

**Fig. 5.** Experimental results of the HRR laser under different polarization states. The spectra of the pulses with an MKR diameter of (a-c) ~ 1.6 mm and (d-e) ~ 1.05 mm, and corresponding autocorrelation traces with an MKR diameter of (f-h) ~ 1.6 mm and (i-j) ~ 1.05 mm. The diameter of the MKR is tuned by the motors underneath the holders on both sides (Fig. 1).
and 34 times of the intrinsic $FSR_{MKR1}$ of this MKR. From the corresponding pulse trains, we can see $\Delta T$ of $\sim 3.00$ ps, $\sim 1.01$ ps, and $\sim 0.72$ ps were obtained, which give the $\Delta \nu$ of about 333 GHz, 990 GHz, and 1388 GHz, respectively. It is also 8, 24, and 34 times the intrinsic fundamental repetition rate ($\Delta \nu_{MKR1} = 41$ GHz).

We further reduce the diameter of the MKR to $\sim 1.05$ mm with the intrinsic $FSR_{MKR2}$ of $\sim 0.51$ nm and adjusted the PCs. Typical two sets of spectral responses and autocorrelation traces are shown in Figs. 5(d-e, i-j). We can see that the $\Delta \lambda$ are $\sim 2.55$ nm and $\sim 7.16$ nm respectively, which are about 5 and 14 times of the intrinsic $FSR_{MKR2}$ of this MKR. Accordingly, the temporal spacings $\Delta T$ of $\sim 3.22$ ps and $\sim 1.14$ ps are obtained, which give the $\Delta \nu$ of about 311 GHz, and 877 GHz, respectively. The bandwidths of the spectral envelopes in Fig. 5(a-e) are estimated as $\sim 2.85$, 8.13, 11.4, 2.65, and 7.29 nm. The pulse durations in Fig. 5(f-j) are estimated as $\sim 1.35$, 0.46, 0.34, 1.38, and 0.51 ps, respectively. Then, the TBPs of the output pulses are calculated as $\sim 0.49$, 0.47, 0.48, 0.47, and 0.47, indicating that the output pulses are slightly chirped. Here, the tuning range of $\Delta \lambda$ is limited by the loss of the laser cavity and the gain bandwidth of the EDF.

To examine the stability of the fiber laser, we employed an MKR with a diameter of $\sim 1.05$ mm, got a typical DFWM mode-locked spectrum by adjusting PCs, and afterwards collected the spectrum in 100 minutes with a 10-min interval, as shown in Fig. 6(a). It is seen that no significant wavelength drifts and evident intensity fluctuations were observed. To better check the power fluctuation, we plotted the power of each spectral line over time in Fig. 6(b). The maximum power fluctuation is less than 1.7 dB, indicating that our DFWM mode-locking fiber laser is quite reliable.

![Fig. 6. Stability of the DFWM mode-locked fiber laser. (a) Spectra collected at various times and (b) power fluctuation of each peak within 100 minutes.](image-url)

### 4. Theoretical simulation and analysis

During experiments, we find that when the polarization-dependent isolator is replaced by a polarization-independent isolator, the pulse trains with the multi-wavelength spectrum, as shown in Fig. 4, can still be achieved. But the repetition rate multiplication pulsed trains as shown in Fig. 5 cannot be obtained. Considering the fact that other intra-cavity components are insensitive to polarization, we suspect that the pulse multiplication is due to the change of weak birefringence based on NPR technology.

To further confirm and illustrate the mechanism of the experimental observations, we numerically simulated the evolution of the HRR pulses, considering the pulse circulation through various parts in the laser cavity and finally evolves to a stable state with the increase of the round-trip times.
According to the nonlinear Schrödinger equation (NLSE) [40], the propagation of pulse in fiber is modelled by [19,41]:

\[
\frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = \frac{g}{2} |A|^2 A + \frac{g}{2 \Omega_g^2} \frac{\partial^2 A}{\partial t^2}
\]

(1)

in which, \( A \) represents the slowly varying amplitude of the light field, \( t \) and \( z \) represent transmission time and distance, \( \beta_2 \) is the second-order dispersion coefficient, \( \gamma \) is the third-order nonlinear coefficient, \( \Omega_g \) is the gain bandwidth of the laser, and \( g \) is the saturation gain of the fiber. Saturation gain \( g \) is zero in the single-mode fiber, and expressed as follows for EDF:

\[
g = g_0 \exp(-E_p/E_s)
\]

(2)

Where \( g_0 \) is the small-signal gain coefficient, \( E_p \) is pulse energy, \( E_s \) is the gain saturation energy. The pulse energy and gain saturation energy depend on the pump power.

During pulse circulation in the cavity, the field amplitude is modulated mainly by the GF, MKR and Lyot filtering. GF works as a simplified SA and the relationship between the absorption coefficient \( \alpha(I) \) and the light intensity \( I \) is:

\[
\alpha(I) = \alpha_{ns} + \alpha_0/(1 + I/I_{sat})
\]

(3)

Where \( \alpha_{ns} \) is the unsaturated absorption loss, \( \alpha_0 \) is the modulation depth, \( I_{sat} \) is the saturation strength. The transfer function of the MKR filtering effect is expressed as [42]:

\[
T = \frac{e^{-\frac{2}{3}\pi e^{i\phi}} - \sin(K)}{1 - \sin(K)e^{-\frac{2}{3}\pi e^{i\phi}}}
\]

(4)

Where \( K \) is the coupling coefficient, \( \alpha \) is the attenuation coefficient, and \( \psi \) is the phase change of MKR.

The NPR-based ring cavity can be regarded as a section of birefringent fiber with two polarizers at both ends, which can be treated as a tunable coarse comb filter, i.e. Lyot birefringence filter [36,43]. In this case, the position and spectral spacing of transmission peaks can be adjusted by rotating the PCs. The transmission function of the Lyot filter induced by intra-cavity birefringence can be described as [44,45]:

\[
T = \cos^2 \theta_1 \cos^2 \theta_2 + \sin^2 \theta_1 \sin^2 \theta_2 + \frac{1}{2} \sin(2\theta_1) \sin(2\theta_2) \cos \left( \frac{2\pi L \Delta n}{\lambda} \right)
\]

(5)

Where \( \theta_1 \) and \( \theta_2 \) are the angles between the fast axis of the fiber and the polarization direction of the two polarizers respectively (controlled the PCs), \( \lambda \) is the operating wavelength, \( L \) is the length of the laser cavity, \( \Delta n \) is the refractive index difference between the fast and slow axis of the fiber (generally with an estimation of \( 5 \times 10^{-6} \) [46]). It is seen that the transmission coefficient changes periodically with respect to the wavelength. The free spectral range of the Lyot filter \( FSR_{Lyot} \) mainly depends on the intracavity birefringence, which is expressed as \( FSR_{Lyot} = \lambda^2/\Delta n L \). In addition, the cavity transmission function is closely related to \( \theta_1 \) and \( \theta_2 \), and therefore, the transmission can be adjusted by the states of PCs.

We use the following parameters to match the experimental conditions: \( c = 3 \times 10^8 m/s^2 \), \( g_0 = 3 \text{dB} \cdot m^{-1} \), \( n = 1.45 \), \( \Omega_g = 40 \text{nm} \), \( E_s = 80 \text{pJ} \), \( \alpha_{ns} = 37.3\% \), \( I_{sat} = 4.97 \text{MW} \cdot \text{cm}^{-2} \), the linear loss of 0.41 \( L = 3 \text{m} \), \( \gamma = 3 \text{W}^{-1} \cdot \text{km}^{-1} \) and \( \beta_2 = 12 \text{ps}^2 \cdot \text{km}^{-1} \) for EDF, \( L = 14.6 \text{m} \), \( \gamma = 3.3 \text{W}^{-1} \cdot \text{km}^{-1} \), and \( \beta_2 = -23 \text{ps}^2 \cdot \text{km}^{-1} \) for SMF; \( \gamma = 40 \text{W}^{-1} \cdot \text{km}^{-1} \) and \( K = 0.77 \) for MKR. The other parameters are the same as that of experiments.

The simulated spectra and pulse train of laser output for the case with the MKR diameter of 1.05 mm are shown in Fig. 7, in which only the MKR filter is taken to consideration. The optical
field reaches a stable state after \( \sim 200 \) round trips. As shown in Fig. 7 (a), the multi-wavelength comb spectrum is gradually stabilized due to the spectral filtering effect of MKR, and the energy of the main mode is transferred to the side mode through the FWM effect. From the corresponding time-domain evolution diagram [Fig. 7(b)], we can see that the continuous light field evolves into several pulses after \( \sim 100 \) rounds. From the steady spectrum and pulse train after 500 rounds [Figs. 7(c-d)], we got the spectral interval and pulse interval as 0.51 nm and 15.6 ps, respectively. The spectral interval agrees well with the experimental results [Fig. 5(d-e)], but the spectral envelope is simulated pulse interval is much larger than the experimental results [Fig. 5(i-j)].

![Fig. 7. Numerical simulation results considering MKR filter only (MKR diameter of \( \sim 1.05 \) mm is used). Evolution of the spectrum (a) and pulse profile (b); Stable output spectrum (c) and pulse train (d).](image)

We then added a Lyot filter into the simulation and got the results shown in Fig. 8. The Lyot filtering curve with the period of \( FSR_{\text{Lyot}} = 5 \cdot FSR_{\text{MKR}} \) is indicated by the red sin curve in Fig. 8 (c). It is found from the spectral evolution [Fig. 8(a)] that the spectrum is gradually stabilized due to the combined effect of the MKR filtering and the Lyot filtering, and the energy transfer occurs within the modes selected by the Lyot filter. Compared with the case, only MKR filtering is considered [Fig. 7(b)], five times more sub-pulses appear in Fig. 8(b). The final spectrum in Fig. 8(c) is similar to the experimental result shown in Fig. 5 (d). Furthermore, the pulse interval \( \Delta T \) of 3.22 ps obtained from the stable pulse train [Fig. 8(d)] agrees well with the experimental result. In the same way, other results in Fig. 5 are similar to the abovementioned result. When the \( FSR_{\text{Lyot}} \) is set as an integer multiple of the \( FSR_{\text{MKR}} \), the harmonic pulses of the corresponding order will be obtained.

The simulation suggests the fact that PCs-induced birefringence together with PD-ISO imposes amplitude modulation on the spectrum. The MKR and Lyot filter’s transmission curves are similar to the optical vernier effect [47]. When the vertex of the Lyot filter curve is located at the transmission peak of MKR, the two coherent enhanced main modes experience positive gain. While the high-order modes undergo linear loss, and their side modes are suppressed by polarization-dependent loss. In this case, the pulse repetition rate is determined by the period of Lyot filtering \( FSR_{\text{Lyot}} \), and \( FSR_{\text{Lyot}} \) does not affect the \( FSR_{\text{MKR}} \), in which the ratio of \( FSR_{\text{Lyot}} \) to \( FSR_{\text{MKR}} \) determines the times of repetition rate multiplication.

Considering the above mechanism, the experimental results are clearly understood. As shown in Fig. 5, the spectrum is modulated furthermore on top of MKR comb filtering by the Lyot
Fig. 8. Numerical simulation results considering both Lyot filter and MKR filter (the MKR diameter of ∼1.05 mm is used). Evolution of the spectrum (a) and pulse profile (b); Stable output spectrum (c) and pulse train (d).

comb filter, which generates a new modulation period adjustable by the PCs, as shown in Figs. 5 (a-e) with the red curve (described by Eq. (5)). The harmonic mode-locking could only be observed when the ratio of $F_{SR_{Lyot}}/F_{SR_{MKR}}$ is an integer. And harmonic order equals this integer. Moreover, the harmonic order is independent of the diameter of the MKR. We observed harmonic mode-locking from the MKR with either ∼1.6 mm or ∼1.05 mm diameter. The saturable absorption effect of intra-cavity graphene further enhances the edge mode suppression [35,47–49]. At the same time, the high nonlinearity of graphene and MKR can also enhance the NPR effect and increase its amplitude modulation depth, which further increases the gain difference between the main mode and side mode and the stability of the laser. The laser is further self-stabilized by FWM with the help of the highly nonlinear effects of MKR and graphene.

5. Conclusion

In conclusion, we obtained DFWM mode-locked pulse trains by taking MKR as a comb filter and GF as the saturable absorber in an all-fiber cavity. A tunable HRR pulse with the maximum repetition rate of 323 GHz was generated by changing the diameter of MKR. The repetition rate is further magnified by adjusting PCs. A maximum repetition rate of 1.38 THz was finally achieved. Simulations based on MKR and Lyot filters agree well with the experimental results, indicating that the HRR generation contributes to the harmonic mode-locking. And the harmonic order equals the integer ratio of the free spectral range of the Lyot filter and MKR comb filter. Our study provides a simple and efficient method to generate a tunable HRR pulse, which is of great meaning in the applications such as applications of nonlinear bioimaging, microwave photonics and so on.

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