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Finite-Time and Adaptive Observer based Fully Distributed Synchronization of Heterogeneous Linear Systems with Delays

Wei Jiang and Themistoklis Charalambous

Abstract— **In this paper, the output synchronization (OS) problem of heterogeneous linear multi-agent systems (MASs) with input delays is addressed. Agents may have different state dimensions and different dynamics. A finite-time observer (FO) is firstly proposed to estimate the uncertain leader's system dynamics. Then, based on the above FO, an adaptive observer (AO) is designed to estimate leader's state information. Thirdly, a novel state predictor is proposed to tackle the input delay effect based on the above AO and output regulation theory. After that, a third observer is designed to estimate the above state predictor so that the controller can be implemented in reality. The stability analysis is performed via Lyapunov stability theory with sufficient conditions derived in terms of an algebraic Riccati equation. The main achievement of this work is the construction of an observer-based fully distributed controller (FDC) which relies on local information only and does not require knowledge of the leader's dynamics or global graph information. As a result, such an approach can be implemented to large-scale systems. Finally, the effectiveness of the proposed FDC is verified via simulations and the influence of the system's graph structure on the convergence rate of the FDC is discussed.**

Index Terms— **Heterogeneous multi-agent systems, input delays, output synchronization, finite-time observer, adaptive observer, fully distributed controller.**

I. INTRODUCTION

Synchronization problems of linear MASs have been studied extensively during the past decades varying from first order, second order, to general linear dynamics [1]. The aforementioned works deal with homogeneous dynamics. However, heterogeneous MASs consisting of different state dimensions and different dynamics can be encountered (e.g., formation-containment control of multi-vehicle systems [2] with different state dimensions). That is why there has been an ongoing surge of investigation of heterogeneous MASs, varying from the first-order and second-order systems [3], non-identical double-integrator [4] to heterogeneous high-order [5] and general liner systems [6].

Since state synchronization is not feasible for heterogeneous MASs if the state dimensions are different, the OS problem is investigated, e.g., by adopting output regulation theory. Note that it might be the case that the output dimensions are still different. Nevertheless, the output synchronization can be done on certain common components by setting the corresponding output matrix of heterogeneous MASs to achieve, e.g., position synchronization or velocity synchronization. Nowadays, more complicated situations related to OS for heterogeneous MASs are considered, e.g., communication delays [3], [5], uncertain linear dynamics [7] and uncertain leader [8], to name a few. However, the controllers in [6]–[8] are related to the eigenvalue information of the Laplacian matrix of communication topology (a piece of global information), thus those controllers are not fully distributed. The main drawback is that in a large-scale system, it is nearly impossible for each agent to know this eigenvalue information to design its controller. Therefore, designing an FDC for heterogeneous MASs is important, necessary and challenging.

FDCs have been investigated and designed systematically during the last ten years in relation to the evolution of dynamics and graphs from constrained cases to the most general cases. For a literature review about FDCs, see [9] for further elaboration. Most works related to FDCs do not consider time delays which are usually inherent in sensors or actuators that are involved in feedback loops of networked control systems. This is one of our motivations to investigate FDC synthesis in the presence of time-delays. Specifically, there are four kinds of delay formats occurring in MASs; namely, state, output, communication and input. This paper covers the constant input delay which is related to the processing and connecting queuing time for the packets arriving at each agent [10]. The state synchronization with communication and input delays was investigated in [10] where the open-loop dynamics of agents need to be not exponentially unstable. More recent work [11] embraced the discrete-time predictor feedback method to achieve the synchronization without a leader. However, those three works [10], [11] are not fully distributed and focus on MASs with homogeneous dynamics. In terms of input delay for heterogeneous MASs, authors in [3] adopted a dynamical synchronization algorithm for the first-order and second-order system based on frequency-domain analysis. In the time-domain, constant input delay is considered for heterogeneous MASs for leaderless consensus control in [12] where the controller is not fully distributed either. To deal with the input delay effect, the model reduction technique [13] is usually utilized with the introduction of a state predictor to transform the input-delayed homogeneous system into a delay-free one. But for heterogeneous MASs, given each agent's state could be of any state dimension, how to design the corresponding state predictor to construct an FDC to achieve OS with an explicit leader is one of the main challenges.

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Another important observation is that among the above FDCs, the leader-follower approach is adopted only in [2], [14]–[16]. Among them, [14], [16] considered MASs with homogeneous dynamics. For the heterogeneous case, the leader's system dynamics is supposed to be known to all followers in [2]. To remove this assumption, authors in [15], [17], [18] used the cooperative distributed observer to estimate leader's dynamics. However, this kind of observer cannot be applied to deal with the input delay problem. This is another motivation to study the FDC synthesis problem considering the input delay and a leader with unknown dynamics.

Based on the discussion above, the objective of this paper is to design an FDC for heterogeneous linear MASs to address the OS problem considering input delays and an uncertain leader simultaneously. More specifically, in the problem under consideration, the uncertain leader is directly connected to some of the followers, herein called the *informed followers* (IFs), which have access to the leader's dynamics. The rest of the followers, herein called *uninformed followers* (UFs), can be connected to either IFs or UFs and have no direct access to the leader; as a result, the leader's dynamics are unknown to them. To be able to construct an FDC for both the IFs and UFs, a set of different observers is required. A detailed description of how to design the observers required to construct the FDC (shown in Fig. 1) is presented in Sec. III. Contributions are the following:

- A finite-time observer in Sec. III-A is designed so that UFs can estimate the leader's dynamics. The fact that the observer converges in finite time makes it easier to build other observers on top and improve the convergence rate.
- An adaptive observer in Sec. III-B is constructed for UFs to estimate the leader's state.
- A state predictor is proposed in Sec. III-C.

II. Preliminaries and Problem Statement

A. Mathematical Preliminaries

Let $\mathbb{R}^n, \mathbb{R}_+$ and $\mathbb{R}^{m\times n}$ be the n-dimensional Euclidean vector space, positive real number and $m \times n$ real matrix space, respectively. The symbol $\mathbf{1}$ denotes a column vector with all entries being 1. Matrix dimensions are supposed to be compatible if not explicitly stated. The symbol $\otimes$ denotes the Kronecker product and $\text{diag}(\cdot)$ represents a diagonal matrix of its argument. A matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is called a nonsingular M-matrix if $a_{ij} \le 0, \forall i \ne j$, and all eigenvalues of A have positive real parts. The spectrum of square matrix $A = [a_{ij}]$ is represented as $\sigma(A)$ which describes the multi-set of its eigenvalues. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent the minimal and maximal eigenvalues of A, respectively. $\forall x_i \in \mathbb{R}^{n_i}, i \in \mathbf{I}_1^N, \text{col}(x_1,\ldots,x_N) = [x_1^T,\ldots,x_N^T]^T$. $\forall A \in \mathbb{R}^{m\times n}, \text{vec}(A) = \text{col}(A_1,\ldots,A_n) \in \mathbb{R}^{mn}$ with $A_i \in \mathbb{R}^m$ being the i-th column of A. For $b = [b_{ij}] \in \mathbb{R}^{m\times n}$, introduce the involution matrix operation without loss of the element's sign by $b^{[q]} = [|b_{ij}|^q \,\text{sign}(b_{ij})] \in \mathbb{R}^{m\times n}$ with $q \in \mathbb{R}$ and $|\cdot|$ being the absolute operation. $\|x\|$ denotes the 2-norm of a vector x. For any integers $a_1 < a_2$, define $\mathbf{I}_{a_1}^{a_2} = \{a_1, a_1+1, \ldots, a_2\}$. When there is no confusion, the argument t will be omitted, e.g., $x \equiv x(t)$. A square matrix $P \succ 0$ means P is symmetric and positive definite.

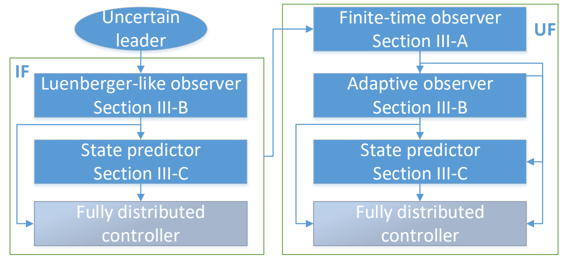

Fig. 1. The construction of fully distributed controller. IF: informed followers (with leader as neighbor); UF: uninformed followers (without leader as neighbor). See more details about IF and UF in Sec. III-A.

The interaction among agents is represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{0, 1, \ldots, N\}$ denotes the finite nonempty set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. Adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined such that $a_{ij} = 1$ if $(j,i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ are defined as $l_{ii} := \sum_{j\ne i} a_{ij}$ and $l_{ij} := -a_{ij}$ for all $i \ne j$. A graph is said to be undirected if $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$ for any $i,j \in \mathcal{V}$. An undirected graph is connected if there exists a path between each pair of distinct nodes. A directed path from node i to node j is a sequence of edges linking node i to node j as $(i,i_1), (i_1,i_2), \ldots, (i_k, j)$ with different nodes $i_s, s = 1, 2, \ldots, k$.

B. Problem Formulation

The dynamics of N followers are

$$\begin{aligned} \dot{x}_i(t) =& A_i x_i(t) + B_i u_i(t - \tau_i), \\ y_i(t) =& C_i x_i(t), \quad i \in \mathbf{I}_1^N \end{aligned} \tag{1}$$

where $x_i(t) = [x_{i1}(t), \ldots, x_{in_i}(t)]^T \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{p_i}$ and $y_i(t) \in \mathbb{R}^q$ are respectively the state, input and measured output of the i-th follower which can have different state dimension n_i. The output dimension being the same as q is for the purpose of achieving OS. $A_i \in \mathbb{R}^{n_i\times n_i}, B_i \in \mathbb{R}^{n_i\times p_i}$ and $C_i \in \mathbb{R}^{q\times n_i}$ are constant matrices. Here, $\tau_i \ge 0, i \in \mathbf{I}_1^N$ are constant input delays and can be heterogeneous.

The leader is indexed by 0 and its dynamics is

$$\dot{x}_0(t) = A_0 x_0(t), \quad x_0(0) = \bar{x}_0, \quad y_0(t) = C_0 x_0(t), \tag{2}$$

where $A_0 \in \mathbb{R}^{n\times n}, C_0 \in \mathbb{R}^{q\times n}$ and $x_0(t) \in \mathbb{R}^n$, $y_0(t) \in \mathbb{R}^q$ are the state and output of the leader, respectively. Note that A_0 is not necessarily Hurwitz and only a subset of followers can get leader's information. The leader is considered without a control input $u_0(t)$, which is a common assumption in synchronization problems; see, for example, [7], [8], [17].

Assumption 1: (A_i, B_i) is controllable. (C_i, A_i) is detectable.

We separate followers as informed followers (IFs), indexed from 1 to $M, M \ge 1$, and uninformed followers (UFs), indexed from $M+1$ to $N, M \le N-1$. For the convenience of presentation, in the following of this paper, denote sets $\mathbb{IF} = \{i, |i = 1, 2, \ldots, M\}$ and $\mathbb{UF} = \{i, |i = M + 1, \ldots, N\}$.

Assumption 2: There is a direct link to each IF from the leader which acts as the root node in graph $\mathcal{G}$. The subgraph describing the connectivity among all UFs is undirected and connected, and there exists at least one UF to which there is a directed link from at least one of the IFs.

Based on Assumption 2, only the IFs have the leader as their neighbor. So, IFs can get access to the leader's output $y_0(t)$ and dynamics (A_0, C_0) directly, while UFs cannot. Then, the Laplacian matrix of graph $\mathcal{G}$ can be partitioned as $\mathcal{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{L}_4 & \mathcal{L}_3 \end{bmatrix}$, where $\mathcal{L}_3 = \begin{bmatrix} I_{M\times M} & 0_{M\times(N-M)} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}$, $\mathcal{L}_1 \in \mathbb{R}^{(N-M)\times(N-M)}$, and $\mathcal{L}_2 \in \mathbb{R}^{(N-M)\times M}$.

Problem 1: Denote the OS error for each follower i by $e_i(t) \triangleq y_i(t) - y_0(t)$. Given systems (1), (2) and a graph $\mathcal{G}$, design the FDC for each follower, such that $\lim_{t\to\infty} e_i(t) = 0$ for any initial conditions $x_i(0) \cup x_0(0), i \in \mathbf{I}_1^N$.

III. MAIN RESULTS

In Sec. III-A, an FO is proposed for the UF to estimate the uncertain leader's dynamics (A_0, C_0). Then, a Luenberger-like observer and an AO are designed in Sec. III-B for the IF and UF, respectively, so that they can estimate the uncertain leader's state $x_0(t)$. Subsequently, a state predictor $z_i(t)$ is proposed to transform the input-delayed system (1) into a delay-free one in Sec. III-C. Finally, the FDC input design procedure is presented in detail in Sec. III-D.

A. Finite-Time Observer for UFs

In what follows, for each UF i, we design the FO $R_i(t) = \mathrm{col}(A_0^i(t), C_0^i(t)) \in \mathbb{R}^{(n+q)\times n}$ with $A_0^i(t) \in \mathbb{R}^{n\times n}$ and $C_0^i(t) \in \mathbb{R}^{q\times n}$ to estimate the leader's dynamics $R_0 = \mathrm{col}(A_0, C_0)$. Towards this end, we propose the following FO:

$$\dot{R}_i = -\chi_1 \xi_i^{[\phi]} - \chi_2 \xi_i^{[\kappa]}, i \in \mathbb{UF}, \tag{3}$$

where $\xi_i = \sum_{j\in\mathbb{UF}} a_{ij}(R_i - R_j) + \sum_{k\in\mathbb{IF}} a_{ik}(R_i - R_0)$, $\chi_1 > 0$, $\chi_2 > 0$, $\phi > 1$, $\kappa \in (0,1)$ are constant parameters, and a_{ij}, a_{ik} are elements of the adjacency matrix $\mathcal{A}$.

Denote the estimating error as $\tilde{R}_i(t) = R_i(t) - R_0$. Then, after some algebraic manipulations we get $\xi_i = \sum_{j\in\mathbb{UF}} a_{ij}[(R_i - R_0) - (R_j - R_0)] + \sum_{k\in\mathbb{IF}} a_{ik}(R_i - R_0) = \sum_{j\in\mathbb{UF}} l_{ij}\tilde{R}_j, i \in \mathbb{UF}$. So, based on $\dot{\tilde{R}}_i = \dot{R}_i - \dot{R}_0 = \dot{R}_i$, we have $\dot{\tilde{R}}_i = -\chi_1(\sum_{j\in\mathbb{UF}} l_{ij}\tilde{R}_j)^{[\phi]} - \chi_2(\sum_{j\in\mathbb{UF}} l_{ij}\tilde{R}_j)^{[\kappa]}$. Denote $\tilde{R} = \mathrm{col}(\tilde{R}_{M+1}, \ldots, \tilde{R}_N) \in \mathbb{R}^{(N-M)(n+q)\times n}$, then

$$\dot{\tilde{R}} = -\chi_1[(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}]^{[\phi]} - \chi_2[(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}]^{[\kappa]}. \tag{4}$$

Lemma 1: Under Assumption 2, for UF i, there exists a settling time t_1 such that $\lim_{t\to t_1} \tilde{R}(t) = 0$, i.e., $\lim_{t\to t_1} A_0^i(t) = A_0, \lim_{t\to t_1} C_0^i(t) = C_0$ if $\chi_1 > 0, \chi_2 > 0$, $\phi > 1$ and $\kappa \in (0,1)$. The upper bound of t_1 is fixed as

$$t_1 \leq \frac{(n+q)^{\phi}(N-M)^{\frac{\phi-1}{2}}\lambda_{\min}^{-\frac{\phi+1}{2}}(\mathcal{L}_1)}{\chi_1(\phi-1)} + \frac{\lambda_{\min}^{-\frac{\kappa+1}{2}}(\mathcal{L}_1)}{\chi_2(1-\kappa)}. \tag{5}$$

Proof: See the Appendix A. ■

The FO is designed so that UFs are able to estimate the uncertain leader's system dynamics in finite time, thus, making it easier to build other observers on top (e.g., the AO, presented in Sec. III-B). Note that the upper bound of t_1 is independent of the initial conditions. Recovering the leader's dynamics from its output in an adaptive manner is challenging and interesting and is the future direction.

B. Estimation of Uncertain Leader's State

1) Luenberger-like observer design for IF: Since IFs can get leader's dynamics (A_0, C_0) directly by communication with the leader, there is no need to use an FO. However, for estimating the leader's state, for each IF, we design a Luenberger-like observer $v_{1,i} \in \mathbb{R}^n$ by

$$\dot{v}_{1,i} = A_0 v_{1,i} + F_i(y_0 - C_0 v_{1,i}), i \in \mathbb{IF}. \tag{6}$$

Denote the observer error as $\tilde{v}_{1,i} = v_{1,i} - x_0, i \in \mathbf{I}_1^N$. After some calculations, we have $\dot{\tilde{v}}_{1,i} = (A_0 - F_i C_0)\tilde{v}_{1,i}$. Based on the pole placement method, we design $F_i \in \mathbb{R}^{n\times q}$ such that $A_0 - F_i C_0$ is Hurwitz and, hence, $\lim_{t\to\infty} \tilde{v}_{1,i} = 0, i \in \mathbf{I}_1^M$.

2) Adaptive observer design for UF: Now, since UFs cannot have the leader's output information, the idea is to design the observer $v_{1,i} \in \mathbb{R}^n$ to estimate x_0. For each UF, the uncertain leader's dynamics have been estimated by the FO (3). Then, the AO can be designed as follows:

$$\begin{aligned} \dot{v}_{1,i} &= A_0^i v_{1,i} - (c_i + \varrho_i^T P_i \varrho_i) P_i \varrho_i, \\ \dot{c}_i &= \varrho_i^T P_i^2 \varrho_i, i \in \mathbb{UF} \end{aligned} \tag{7}$$

where $\varrho_i = \sum_{j=1, j\neq i}^{N} a_{ij}(v_{1,i} - v_{1,j})$ and $c_i(t)$ is the coupling weight associated with the i-th UF with the initial condition $c_i(0) > 0$. $P_i \in \mathbb{R}^{n\times n}$ with $P_i \succ 0$ is the solution to the algebraic Riccati equation (ARE):

$$A_0^{iT}(t) P_i + P_i A_0^i(t) - P_i^2 + I_n = 0. \tag{8}$$

Remark 1: Though $A_0^i(t)$ in Sec. III-A is time-varying, ARE (8) always has a unique solution P_i as $(A_0^i(t), I_n), i \in \mathbb{UF}$ is always controllable. The reason of designing AO instead of Luenberger observer using $(A_0^i(t), C_0^i(t))$ is that UFs cannot have the leader's output information.

Lemma 2: Under Assumption 2, for UF i with the FO (3) and AO (7), $\lim_{t\to\infty} \tilde{v}_{1,i}(t) = 0$ and $c_i(t)$ converges to a finite steady-state value.

Proof: See the Appendix B. ■

C. State Predictor and Its Derivative

For the existence of our proposed state predictor, the following assumption is necessary, which is also common in [17], [19].

Assumption 3: There exist solutions $X_i \in \mathbb{R}^{n_i\times n}$ and $U_i \in \mathbb{R}^{p_i\times n}$ for the following output regulation equation:

$$X_i A_0 = A_i X_i + B_i U_i, \tag{9a}$$

$$C_0 = C_i X_i, \ i \in \mathbb{IF} \cup \mathbb{UF}. \tag{9b}$$

The solvability of Eq. (9) is guaranteed if $\forall\lambda \in \sigma(A_0)$, there exists rank $\begin{bmatrix} A_i - \lambda I & B_i \\ C_i & 0 \end{bmatrix} = n_i + q$ (full rank) with A_i, B_i and C_i being the dynamics of agent i (see Theorem 1.9 in [19]). As it will be seen later, X_i and U_i are part of the solution of the state predictor for each follower $i \in \mathbf{I}_1^N$.

Remark 2: When the information of (A_0, C_0) is known, to get the solution (X_i, U_i), X_i is firstly calculated by Eq. (9b) and then U_i. However, if U_i is firstly set, then, Eq. (9a) becomes a Sylvester equation and can be solved. Remember that the solution (X_i, U_i) is constant. The detailed steps of how to find (X_i, U_i) is presented in simulations in Sec. IV.

In the same way, as the IFs can get the information of (A_0, C_0) directly from the leader, similarly each IF can calculate its own (X_i, U_i) based on (9) directly.

However, since the UFs cannot get (A_0, C_0) directly from the leader, each UF does not know the precise information of (X_i, U_i) from (9). Based on the FO (3), however, UFs could use the following equation to calculate $(\hat{X}_i(t), \hat{U}_i(t))$ which will converge to $(X_i, U_i), i \in \mathbb{UF}$:

$$\begin{aligned} \hat{X}_i(t)A_0^i(t) &= A_i\hat{X}_i(t) + B_i\hat{U}_i(t), \\ C_0^i(t) &= C_i\hat{X}_i(t), i \in \mathbb{UF}. \end{aligned} \tag{10}$$

Denote $\hat{\zeta}_i(t) = \text{vec}\left(\begin{bmatrix}\hat{X}_i(t)\\ \hat{U}_i(t)\end{bmatrix}\right)$, $\beta_i(t) = \text{vec}\left(\begin{bmatrix}0_{n_i\times n}\\ -C_0^i(t)\end{bmatrix}\right)$. Then, to solve the output regulation equation (10) on-line, based on $A_0^i(t)$ in FO (3) and $\epsilon_i > 0$ being a constant, inspired by [17], the following adaptive law is proposed:

$$\begin{aligned} \dot{\hat{\zeta}}_i(t) &= -\epsilon_i E_i^T(t)[E_i(t)\hat{\zeta}_i(t) - \beta_i(t)], \qquad (11)\\ E_i(t) &= A_0^{iT}(t) \otimes \begin{bmatrix} I_{n_i} & 0_{n_i\times p_i}\\ 0_{q\times n_i} & 0_{q\times p_i}\end{bmatrix} - I_n \otimes \begin{bmatrix} A_i & B_i\\ C_i & 0_{q\times p_i}\end{bmatrix}. \end{aligned}$$

Lemma 3: Considering Assumption 3, based on FO (3) and adaptive law (11), one gets $\lim_{t\to\infty}\hat{\zeta}_i(t) = \zeta_i$, where $\zeta_i = \text{vec}\left(\begin{bmatrix}X_i\\ U_i\end{bmatrix}\right), i \in \mathbb{UF}$ are the solutions in (9).

Proof: Specifically, we have $\lim_{t\to\infty}\hat{X}_i(t) = X_i$, $\lim_{t\to\infty}\hat{U}_i(t) = U_i$. Furthermore, we can get the following property that $\dot{\hat{X}}_i(t) = 0, \dot{\hat{U}}_i(t) = 0, i \in \mathbb{UF}$ as $t \to \infty$. The proof follows a similar procedure *mutatis mutandis* to that of [17, Lemma 4] and, hence, is omitted here. The main difference is that we propose the finite-time observer to estimate the leader's dynamics while the authors in [17] use asymptotic observer with exponential convergence rate to estimate the leader's dynamics. Additionally, [17] cannot consider the input delay effect which will be investigated in detail in the following and also presented in Remark 3. ■

Now, in terms of the input delay $u_i(t-\tau_i)$, inspired by the Artstein's model reduction technique [13] a novel state predictor $z_i(t) \in \mathbb{R}^{n_i}$ for each follower is proposed as follows:

$$\begin{aligned} z_i(t) &= \int_{t-\tau_i}^{t} e^{A_i(t-s)}B_i(u_i(s) - U_i e^{A_0\tau_i}v_{1,i}(s))ds\\ &\quad + e^{A_i\tau_i}(\hat{x}_i(t) - X_i v_{1,i}(t)), i \in \mathbb{IF},\\ z_i(t) &= \int_{t-\tau_i}^{t} e^{A_i(t-s)}B_i(u_i(s) - \hat{U}_i(t)e^{A_0^i(t)\tau_i}v_{1,i}(s))ds\\ &\quad + e^{A_i\tau_i}(\hat{x}_i(t) - \hat{X}_i(t)v_{1,i}(t)), i \in \mathbb{UF}, \end{aligned} \tag{12}$$

where $\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_iu_i(t) + L_i(y_i(t) - C_i\hat{x}_i(t)), i \in \mathbb{IF}\cup\mathbb{UF}$ is a Luenberger observer. Here, the link between follower's output $y_i(t)$ and state predictor $z_i(t)$ is established through the historical and current value of $v_{1,i}(\theta), \theta \in [t-\tau_i, t]$ and the solution $(X_i, U_i), i \in \mathbb{IF}$ of regulation function (9) or $(\hat{X}_i(t), \hat{U}_i(t)), i \in \mathbb{UF}$ of (10), which is one of the main challenges in this paper. To the authors' best knowledge, this is the first work solving the OS problem for heterogeneous general linear MASs considering input delays and uncertain leader combined. Denote

$$\begin{aligned} \bar{x}_i(t) &= \hat{x}_i(t) - x_i(t), i \in \mathbb{IF}\cup\mathbb{UF},\\ \tilde{x}_i(t) &= \begin{cases} \hat{x}_i(t) - X_i v_{1,i}(t), i \in \mathbb{IF},\\ \hat{x}_i(t) - \hat{X}_i(t)v_{1,i}(t), i \in \mathbb{UF}. \end{cases} \end{aligned} \tag{13}$$

Lemma 4: Under Assumption 3, if $\lim_{t\to\infty}\tilde{x}_i(t) = 0, i \in \mathbb{IF}\cup\mathbb{UF}$, Problem 1 is solved.

Proof: For Luenberger observer $\hat{x}_i(t) \in \mathbb{R}^{n_i}$ in (12), based on Assumption 1, It is obvious that $\lim_{t\to\infty}\bar{x}_i(t) = 0$ if $A_i - L_iC_i, i \in \mathbb{IF}\cup\mathbb{UF}$ are Hurwitz.

From (13), for IF i, the OS error in Problem 1 is $e_i(t) = C_i(\tilde{x}_i(t) + X_iv_{1,i}(t) - \bar{x}_i(t)) - C_0x_0(t)$. Due to $C_iX_i = C_0$ in (9b), then

$$e_i(t) = C_i(\tilde{x}_i(t) - \bar{x}_i(t)) + C_0\tilde{v}_{1,i}(t).$$

Due to $\lim_{t\to\infty}\tilde{v}_{1,i}(t) = 0$ in (6) and $\lim_{t\to\infty}\bar{x}_i(t) = 0$, if $\lim_{t\to\infty}\tilde{x}_i(t) = 0$, we have $\lim_{t\to\infty}e_i(t) = 0, i \in \mathbb{IF}$.

For each UF i, similar as the above calculation, from (13) and (10) we get $e_i(t) = C_i(\tilde{x}_i(t) - \bar{x}_i(t)) + C_0\tilde{v}_{1,i}(t) + C_i(\hat{X}_i(t) - X_i)v_{1,i}(t)$. Based on $\lim_{t\to\infty}\tilde{v}_{1,i}(t) = 0$ and $\lim_{t\to\infty}\hat{X}_i(t) = X_i$ in Lemma 2 and Lemma 3, respectively, we get $\lim_{t\to\infty}e_i(t) = 0, i \in \mathbb{UF}$. ■

In what follows, we will prove the convergence of $\tilde{x}_i(t) \in \mathbb{R}^{n_i}$ to zero for any initial state $x_i(0)\cup x_0(0), i \in \mathbb{IF}\cup\mathbb{UF}$.

Given the state predictor $z_i(t) = f(\tilde{x}_i(t), v_{1,i}(t))$ in (12), to deduce $\lim_{t\to\infty}\tilde{x}_i(t) = 0$, the calculation of the derivative of $z_i(t)$ is needed. Cases of calculating $\dot{z}_i(t)$ for IFs and UFs separately are presented in the Appendix C and we get

$$\begin{aligned} \dot{z}_i &= \begin{cases} A_iz_i + B_iu_i - B_iU_ie^{A_0\tau_i}v_{1,i} + \Omega_i(t), i \in \mathbb{IF},\\ A_iz_i + B_iu_i - B_i\hat{U}_i(t)e^{A_0^i(t)\tau_i}v_{1,i} + \Omega_i(t), i \in \mathbb{UF}, \end{cases}\\ &\lim_{t\to\infty}\Omega_i(t) = 0, i \in \mathbb{IF}\cup\mathbb{UF}, \end{aligned} \tag{14}$$

where $\Omega_i(t)$ is in (29) for $i \in \mathbb{IF}$ and in (32) for $i \in \mathbb{UF}$.

Remark 3: Asymptotically-stable observers in [15], [17], [18] cannot be adopted to calculate the value of $v_{1,i}(t)$ based on integrator calculation. This is the reason for designing the finite-time observer $A_0^i(t)$ in (3). The mathematical challenge of calculating $\dot{z}_i(t)$ is presented in Remark 7.

D. Control Input Design and OS Error Convergence

Based on (14), the control input could be designed as

$$u_i(t) = \begin{cases} K_iz_i(t) + U_ie^{A_0\tau_i}v_{1,i}(t), i \in \mathbb{IF},\\ K_iz_i(t) + \hat{U}_i(t)e^{A_0^i(t)\tau_i}v_{1,i}(t), i \in \mathbb{UF}, \end{cases} \tag{15}$$

such that $\dot{z}_i(t) = (A_i + B_iK_i)z_i(t) + \Omega_i(t), i \in \mathbb{IF}\cup\mathbb{UF}$. Thanks to $\lim_{t\to\infty}\Omega_i(t) = 0$ in (14), if $A_i + B_iK_i$ is Hurwitz, then $\lim_{t\to\infty}z_i(t) = 0, i \in \mathbb{IF}\cup\mathbb{UF}$. Note that $u_i(t)$ will not go to infinity even though $Re(\lambda) \geq 0, \forall\lambda \in \sigma(A_0)$ as the term $e^{A_0\tau_i}$ is constant and $\lim_{t\to t_1}A_0^i(t) = A_0$, where t_1 is a finite settling time defined in (5).

On the other hand, substituting the designed input (15) into the transformed delay-free system (12) generates

$$z_i(t) = e^{A_i\tau_i}\tilde{x}_i(t) + \int_{t-\tau_i}^{t} e^{A_i(t-s)}B_iK_iz_i(s)ds, \tag{16}$$

where $\tilde{x}_i(t), i \in \mathbf{I}_1^N$ is defined in (13). As $e^{A_i\tau_i}$ is reversible and $\lim_{t\to\infty}z_i(t) = 0$, then $\lim_{t\to\infty}\tilde{x}_i(t) = 0, i \in \mathbf{I}_1^N$.

Theorem 1: Consider Assumptions 1-3, Problem 1 is solved by the FDC (see Fig. 2) consisting of the input (15), state predictor (16), with the Luenberger-like observer (6), FO (3) in Lemma 1 and AO (7) in Lemma 2, if K_i, L_i are respectively chosen such that $A_i + B_iK_i, A_i - L_iC_i, i \in$

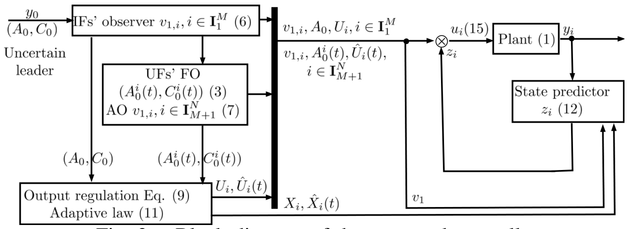

Fig. 2. Block diagram of the proposed controller.

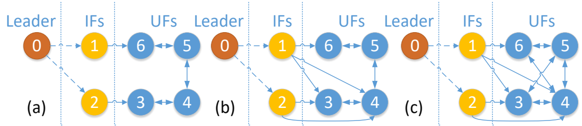

Fig. 3. Communication graph $\mathcal{G}$ with more and more links from (a) to (c).

$\mathbb{IF} \cup \mathbb{UF}$ are Hurwitz, $(X_i, U_i), i \in \mathbb{IF}$ are the solution of output regulation equation (9) and $(\hat{X}_i(t), \hat{U}_i(t)), i \in \mathbb{UF}$ are the solution of adaptive law (11) in Lemma 3.

Proof: It is based on Lemma 4 and $\lim_{t\to\infty} \tilde{x}_i(t) = 0$ in (16). ∎

Remark 4: The input delay for heterogeneous MASs is also studied in [12], [20]. In [20] the constant input delay is required to satisfy two constraints. In [12] the controller is not fully distributed. In addition, all the followers in both [20] and [12] are assumed to know the leader's dynamics A_0. In this paper, there is no constraint on delay, the controller is fully distributed and, only IFs need to know A_0.

IV. Simulations

A. FDC Simulation

To solve Problem 1, denote $M = 2, N = 6$. The dynamics of MAS (1) are

$$A_0 = \begin{bmatrix} 0_{2\times 2} & I_2 \\ \mathrm{diag}(-\vartheta^2, -\vartheta^2) & 0_{2\times 2} \end{bmatrix}, A_i^{'} = \begin{bmatrix} 0 & 1 \\ a(i) & b(i) \end{bmatrix},$$

$$A_i^{''} = \begin{bmatrix} 0_{2\times 1} & & I_2 \\ 0 & a(i) & d(i) \end{bmatrix}, A_i^{'''} = \begin{bmatrix} 0_{3\times 1} & & I_3 & \\ 0 & a(i) & b(i) & d(i) \end{bmatrix},$$

$$B_i^{'} = \mathrm{diag}(d(i), d(i+1)), B_i^{''} = \mathrm{diag}(b(i), b(i+1), b(i)),$$

$$B_i^{'''} = \mathrm{diag}(b(i), b(i+1), c(i), c(i+1)),$$

$$C_0 = C_i^{'''} = \begin{bmatrix} I_2 & 0_{2\times 2} \end{bmatrix}, C_i^{'} = I_2, C_i^{''} = \begin{bmatrix} I_2 & 0_{2\times 1} \end{bmatrix}, \tag{17}$$

where $a = [1, 2, 3, 4, 5, 6]^T$, $b = [1, 2, 2, 4, 4, 8, 10]^T, c = [2, 4, 5, 6, 8, 10, 1]^T, d = [2, 2, 4, 5, 1, 8, 6]^T$ and $\vartheta = 2$. The corresponding states are $x_i^{'} = (x_{i1}^{'}, x_{i2}^{'})^T \in \mathbb{R}^2$, $x_i^{''} = (x_{i1}^{''}, \ldots, x_{i3}^{''})^T \in \mathbb{R}^3$ and $x_i^{'''} = (x_{i1}^{'''}, \ldots, x_{i4}^{'''})^T \in \mathbb{R}^4$.

Choose the dynamics of IFs as $A_1 = A_1^{''}, A_2 = A_2^{'}$ and the dynamics of UFs as $A_3 = A_3^{'}, A_4 = A_4^{'}, A_5 = A_5^{'''}, A_6 = A_6^{'''}$. The corresponding $B_i, C_i, i \in \mathbf{I}_1^6$ are chosen similarly. We can see that agents have different dynamics with different state dimensions. Assumptions 1 and 3 are thus satisfied. The solution to output regulation equation (9) is

$$X_1 = \begin{bmatrix} I_2 & & 0_{2\times 2} \\ -\vartheta^2 & 0 & 0_{1\times 2} \end{bmatrix}, X_i = \begin{bmatrix} I_2 & & 0_{2\times 2} & \\ -\vartheta^2 & 0 & 0 & 0 \\ 0 & 0 & -\vartheta^2 & 0 \end{bmatrix}, i = 5, 6;$$

$$X_i = \begin{bmatrix} I_2 & 0_{2\times 2} \end{bmatrix}, i = 2, 3, 4;\ U_i = B_i^{-1}(X_i A_0 - A_i X_i).$$

Remark 5: In this example, for IF $i, i \in \mathbf{I}_1^2$, X_i is firstly calculated and then U_i. The reason of designing A_0 and $A_i, i \in \mathbf{I}_1^2$ in the format of (17) is for the convenience of calculating the corresponding X_i. Then, designing B_i as the diagonal matrix is for the easiness of calculating U_i. Even if B_i is not a square matrix or has no inverse matrix B_i^{-1}, we can still using adaptive law (11) to calculate X_i and U_i at the same time simply by replacing $(A_0^i(t), C_0^i(t))$ to (A_0, C_0).

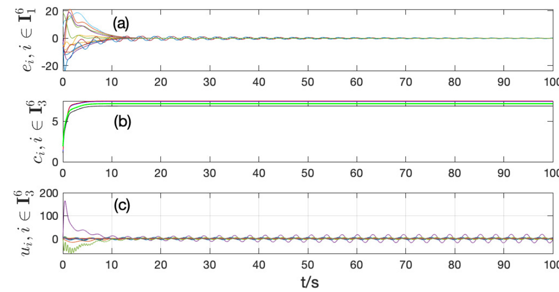

Fig. 4. (a) OS error; (b) adaptive parameters in AO (7); (c) inputs of UFs.

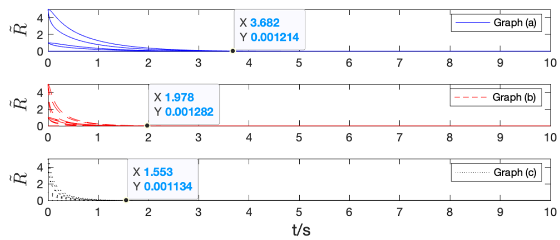

Fig. 5. FO error (4) comparisons from graph (a) to (c).

For UF $i, i \in \mathbf{I}_3^6$, $\hat{X}_i(t)$ and $\hat{U}_i(t)$ are calculated by the adaptive law (11). According to $A_0^i(t)$ in FO (3) and ARE (8), P_i can be calculated accordingly. Using pole placement method to have $A_0 - F_i C_0, i \in \mathbf{I}_1^2, A_i + B_i K_i, A_i - L_i C_i, i \in \mathbf{I}_1^6$ be Hurwitz. Set FDC parameters $\chi_1 = 2, \chi_2 = 2, \phi = 2, \kappa = 0.8, \epsilon_i = 1, i \in \mathbf{I}_1^6$. Set input delays are $\tau_i = 0.1s, i \in \mathbf{I}_1^6$. All initial conditions are chosen randomly. The graph $\mathcal{G}$ is shown in Fig. 3 (a) satisfying Assumption 2. Then, we can calculate the settling time t_1 for FO satisfying $t_1 \leq 158.4430s$ based on (5).

The OS errors shown in Fig. 4 (a) converge to zero asymptotically, meaning that Problem 1 is indeed solved. In detail, the performance of FO in Fig. 5 (a) presents clearly that the uncertain leaders dynamics (A_0, C_0) is estimated by UFs in finite-time around $t_1 = 3.682s$ which satisfies the calculated result $t_1 \leq 158.4430s$. In Fig. 4 (b) and 6 (a), one can see that AO works well.

B. FDC Convergence Discussion

From Lemma 4, one can see the convergence rate of OS error e_i is related to the rate of $\tilde{x}_i(t) \to 0, \tilde{v}_{1,i}(t) \to 0$ and $\tilde{X}_i(t) \to 0$.

Take the UF i for analyzing. From (26): $\dot{V}_4 \leq -\varrho^T[G(\hat{c} + \hat{\rho}) \otimes I]\varrho$ in the proof of Lemma 2, one can see the convergence rate of AO is related to the value of G which is defined in Lemma 8 as $G = \mathrm{diag}(\mathcal{L}_1^{-1}\mathbf{1}_{N-M})^{-1}$. Then we have the following relation about the convergence rate of AO as

$$\tilde{v}_{1,i}\ (22) \begin{cases} \tilde{v}_{1,j}\ (6) \leftarrow \text{poles of } A_0 - F_j C_0, j \in \mathbf{I}_1^M, \\ \varrho_i \leftarrow G\ (26) \leftarrow \mathcal{L}_1^{-1}\mathbf{1}_{N-M} \text{in Lemma 8.} \end{cases}$$

Specifically, the smaller $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ is, the larger G is, the smaller $\dot{V}_2$ is, the faster V_2 decrease, and the quicker $\tilde{v}_{1,i}$ convergences.

Similarly, the effectors influencing the convergence rate of $\tilde{x}_i(t)$ are $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ and $\lambda_{\min}(\mathcal{L}_1)$. To be specific, in addition to the same influencing effect of $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ above, the larger $\lambda_{\min}(\mathcal{L}_1)$ is, the faster FO error convergences, and the quicker $\tilde{x}_i$ convergences.

For the above two factors, which one plays a more important role? Three simulations are made from graph (a) to (c) shown in Fig. 3. From Table I, $\lambda_{\min}(\mathcal{L}_1)$ becomes larger and larger from graph (a) to (c). The value of $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ becomes smaller from (a) to (b), but there is no obvious rule from (b) to (c). The OS average error in Fig. 7 demonstrates that increasing the directed paths from IFs to UFs can improve the convergence rate greatly. The detailed comparisons about the components of control input (15) from graph (a) to (c) are presented in Fig. 5, 6, respectively, which can demonstrate our findings in the following.

Remark 6: Both $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ and $\lambda_{\min}(\mathcal{L}_1)$ can influence the convergence rate of OS, and $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ plays a more weighted role. Increasing the directed paths from IFs to UFs can improve the convergence rate greatly.

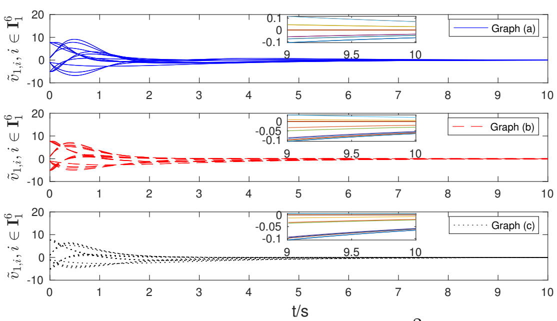

Fig. 6. Luenberger-like observer error $\tilde{v}_{1,i}, i \in \mathbf{I}_1^2$ in (6) and AO error $\tilde{v}_{1,i}, i \in \mathbf{I}_3^6$ in (7) from graph (a) to (c).

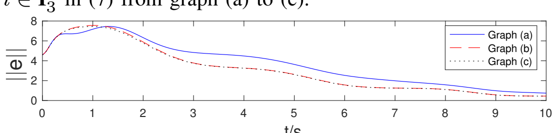

Fig. 7. OS average error $\|e\| = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\|e_i\|^2}$ from graph (a) to (c).

TABLE I

EIGENVALUES OF $\mathcal{L}_1$ AND $\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$ IN LEMMA 8.

	$\lambda(\mathcal{L}_1)$				$\mathcal{L}_1^{-1}\mathbf{1}_{N-M}$			
Fig. 3 (a)	**0.38**	1.38	2.62	3.62	2.00	2.00	3.00	3.00
Fig. 3 (b)	**0.80**	2.29	3.00	4.91	0.59	0.78	1.26	1.52
Fig. 3 (c)	**1.05**	3.43	4.63	5.88	0.73	0.78	0.98	1.16

V. CONCLUSION

The output synchronization problem of general linear heterogeneous multi-agent systems considering different state dimensions and heterogeneous constant input delays is addressed based on output measurements. The leader's dynamics is not known to all the followers. The proposed controller for followers, which consists of the finite-time observer, adaptive observer and state predictor, is fully distributed and due to its distributed nature and scalability, it can be applied to large-scale systems. Factors affecting the performance of the controller are further discussed. More specifically, increasing the communication links from the informed followers (having leader as neighbor) to uninformed followers (do not have leader as neighbor) can improve the synchronizing convergence rate greatly.

Since the input delays are constant, future work will focus on dealing with time-varying unknown input delays by designing the corresponding observer. Another possible direction is how to design finite-time observers for directed graphs in order to generalize our results to uninformed followers forming a strongly connected directed graph.

APPENDIX

A. Proof of Lemma 1

To facilitate the presentation of proof, the following three lemmas are firstly needed.

Lemma 5: [21] Let $\varsigma_1, \varsigma_1, \ldots, \varsigma_N \geq 0$ and $\phi > 1$. Then, $\sum_{i=1}^{N}\varsigma_i^{\phi} = N^{1-\phi}(\sum_{i=1}^{N}\varsigma_i)^{\phi}$.

Lemma 6: [22] Consider the system $\dot{x} = f(t,x), x(0) = x_0$ where $x \in \mathbb{R}^n$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function. If there exists a continuous radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that $V(x) = 0 \Rightarrow x \in E$ with $E \subset \mathbb{R}^n$ and any solution $x(t)$ satisfies the inequality: $\dot{V}(x(t)) \leq -(\nu_1 V^{\phi}(x(t)) + \nu_2 V^{\kappa}(x(t)))^{\eta}$ for some $\nu_1, \nu_2, \phi, \kappa, \eta > 0$ and $\phi\eta > 1, \kappa\eta < 1$, then the set E is globally finite-time attractive and the settling time is

$$t(x_0) \leq \frac{1}{\nu_1^{\eta}(\phi\eta - 1)} + \frac{1}{\nu_2^{\eta}(1-\kappa\eta)}, \forall x_0 \in \mathbb{R}^n. \tag{18}$$

Lemma 7: [23] Denote vector $\delta = [\delta_1^T, \ldots, \delta_N^T]^T \in \mathbb{R}^{nN}$ with $\delta_i = [\delta_{i1}, \ldots, \delta_{in}]^T \in \mathbb{R}^n, i \in \mathbf{I}_1^N$, then

$$\delta^T\delta^{[\phi]} \geq n^{-\phi}N^{\frac{1-\phi}{2}}(\delta^T\delta)^{\frac{1+\phi}{2}}, \ \phi > 1, \tag{19}$$

$$\delta^T\delta^{[\kappa]} \geq (\delta^T\delta)^{\frac{1+\kappa}{2}}, \ \kappa \in (0,1). \tag{20}$$

Now we come to the convergence of FO (3). Based on the compact form of FO error dynamics (4), denote $\tilde{R}^j \in \mathbb{R}^{(N-M)(n+q)}, j \in \mathbf{I}_1^n$ as the j-th column of matrix $\tilde{R}$. Then, $\dot{\tilde{R}}^j = -\chi_1[(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j]^{[\phi]} - \chi_2[(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j]^{[\kappa]}$.

Let $V_1(\tilde{R}^j) = \tilde{R}^{jT}(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j$.

Denote $\delta_j = (\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j \in \mathbb{R}^{(N-M)(n+q)}$, then

$$\begin{aligned}\dot{V}_1(\tilde{R}^j) =& -2\chi_1\tilde{R}^{jT}(\mathcal{L}_1 \otimes I_{n+q})[(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j]^{[\phi]} \\ & -2\chi_2\tilde{R}^{jT}(\mathcal{L}_1 \otimes I_{n+q})[(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j]^{[\kappa]} \\ =& -2\chi_1\delta_j^T\delta_j^{[\phi]} - 2\chi_2\delta_j^T\delta_j^{[\kappa]},\end{aligned} \tag{21}$$

where the symmetric property of $\mathcal{L}_1$ from Assumption 2 is used. As $\phi > 1$ and $\kappa \in (0,1)$, based on Lemma 7, we have $\dot{V}_1(\tilde{R}^j) \leq -2\chi_1(n+q)^{-\phi}(N-M)^{\frac{1-\phi}{2}}(\delta_j^T\delta_j)^{\frac{1+\phi}{2}} - 2\chi_2(\delta_j^T\delta_j)^{\frac{1+\kappa}{2}}$. The calculation of $\delta_j^T\delta_j$ is as follows: $\delta_j^T\delta_j = \tilde{R}^{jT}(\mathcal{L}_1 \otimes I_{n+g})(\mathcal{L}_1 \otimes I_{n+q})\tilde{R}^j \geq \lambda_{\min}(\mathcal{L}_1 \otimes I_{n+q})V_1(\tilde{R}^j) = \lambda_{\min}(\mathcal{L}_1)V_1(\tilde{R}^j)$, where the last equality comes from the property that if the eigenvalues of $S \in \mathbb{R}^{n\times n}$ and $T \in \mathbb{R}^{m\times m}$ are $\lambda_1, \ldots, \lambda_n$ and $\mu_1, \ldots, \mu_m$, respectively, then the eigenvalues of $S \otimes T$ are $\lambda_i\mu_j, i \in \mathbf{I}_1^n, j \in \mathbf{I}_1^m$.

Finally, we obtain $\dot{V}_1(\tilde{R}^j) \leq -2\chi_1(n+q)^{-\phi}(N-M)^{\frac{1-\phi}{2}}\lambda_{\min}^{\frac{1+\phi}{2}}(\mathcal{L}_1)V_1(\tilde{R}^j)^{\frac{1+\phi}{2}} - 2\chi_2\lambda_{\min}^{\frac{1+\kappa}{2}}(\mathcal{L}_1)V_1(\tilde{R}^j)^{\frac{1+\kappa}{2}}$.

Based on Lemma 6, we can conclude that $\lim_{t\to t_1}\tilde{R}^j(t) = 0, j \in \mathbf{I}_1^n$ are globally finite-time stable with the settling time t_1 satisfying (5).

B. Proof of Lemma 2

Lemma 8 ([24], Theorem 4.25): For the nonsingular M-matrix $\mathcal{L}_1$, there exists a matrix $G \triangleq \operatorname{diag}(g)^{-1} > 0$ such that $G\mathcal{L}_1 + \mathcal{L}_1^T G > 0$, where $g = [g_{M+1}, \dots, g_N]^T = \mathcal{L}_1^{-1}\mathbf{1}_{N-M}$.

Now, from (7) we have $\varrho_i = \sum_{j=1,j\neq i}^N a_{ij}[(v_{1,i} - x_0) - (v_{1,j} - x_0)] = \sum_{j=1}^N l_{ij}\tilde{v}_{1,j}$.

Rewrite the leader's state observer error $\tilde{v}_1 = [\tilde{v}_{1,IF}^T, \tilde{v}_{1,UF}^T]^T$, where $\tilde{v}_{1,IF} = [\tilde{v}_{1,1}^T, \dots, \tilde{v}_{1,M}^T]^T$ for all IFs and $\tilde{v}_{1,UF} = [\tilde{v}_{1,M+1}^T, \dots, \tilde{v}_{1,N}^T]^T$ for all UFs.

Denote $\varrho = [\varrho_{M+1}^T, \dots, \varrho_N^T]^T$, then,

$$\varrho = (\mathcal{L}_1 \otimes I_n)\tilde{v}_{1,UF} + (\mathcal{L}_2 \otimes I_n)\tilde{v}_{1,IF}. \tag{22}$$

Based on Assumption 2, it can be easily shown that all eigenvalues of $\mathcal{L}_1$ have positive real parts. Thanks to this non-singularity of $\mathcal{L}_1$ and $\lim_{t\to\infty}\tilde{v}_{1,IF}(t) = 0$ for IFs in (6), it is obvious that for UFs, $\lim_{t\to\infty}\tilde{v}_{1,UF}(t) = 0$ if and only if $\lim_{t\to\infty}\varrho(t) = 0$.

Denote $\rho_i(t) = \varrho_i^T(t)P_i\varrho_i(t), K_i = -P_i, \Gamma_i = P_i^2$ and $\tilde{A}_0^i(t) = A_0^i(t) - A_0$, then the derivative of $\varrho_i(t)$ is: $\dot{\varrho}_i = \sum_{j\in\mathbb{UF}} l_{ij}[A_0^j v_{1,j} + (c_j + \rho_j)K_j\varrho_j - A_0x_0 + A_0v_{1,j} - A_0v_{1,j}] + \sum_{j\in\mathbb{IF}} l_{ij}[A_0v_{1,j} + F_j(y_0 - C_0v_{1,j}) - A_0x_0] = A_0\varrho_i + \sum_{j\in\mathbb{UF}}[l_{ij}(c_j + \rho_j)K_j\varrho_j] + \sum_{j\in\mathbb{UF}} l_{ij}\tilde{A}_0^j v_{1,j} - \sum_{j\in\mathbb{IF}} l_{ij}F_jC_0\tilde{v}_{1,j}, i \in \mathbb{UF}$. Its compact format is

$$\begin{aligned}\dot{\varrho} =&(I_{N-M} \otimes A_0)\varrho + [\mathcal{L}_1(\hat{c} + \hat{\rho}) \otimes I_n]\operatorname{diag}(K_i)\varrho \\ &+ (\mathcal{L}_1 \otimes I_n)\operatorname{diag}(\tilde{A}_0^i)v_{1,UF} - (\mathcal{L}_2 \otimes I_n)\bar{F}\tilde{v}_{1,IF},\end{aligned} \tag{23}$$

where $\hat{c} = \operatorname{diag}(c_{M+1}, \dots, c_N), \hat{\rho} = \operatorname{diag}(\rho_{M+1}, \dots, \rho_N)$ and $\bar{F} = \operatorname{diag}(F_1C_0, \dots, F_MC_0)$. $\operatorname{diag}(\tilde{A}_0^i)$ is an abbreviated expression of $\operatorname{diag}(\tilde{A}_0^{M+1}, \dots, \tilde{A}_0^N)$ (similar abbreviations are also adopted in the rest of this proof). Let

$$V_2 = \sum_{i\in\mathbb{UF}}(2c_i + \rho_i)\rho_i/(2g_i) + \sum_{i\in\mathbb{UF}}(c_i - \alpha)^2/(2g_i), \tag{24}$$

where $G = \operatorname{diag}(\frac{1}{g_{M+1}}, \dots, \frac{1}{g_N}) > 0$ is set as in Lemma 8. Since $\mathcal{L}_1$ is a nonsingular M-matrix, $G\mathcal{L}_1 + \mathcal{L}_1^TG \geq \lambda_0 I$ is guaranteed with $\lambda_0 > 0$ being the smallest eigenvalue of $G\mathcal{L}_1 + \mathcal{L}_1^TG$. From $c_i(0) > 0, \dot{c}_i(t) \geq 0$ in (7), we have $c_i(t) > 0, \forall t > 0$. $\alpha > 0$ is a constant to be decided. With $\rho_i = \varrho_i^TP_i\varrho_i \geq 0$, V_2 is positive definite. Then,

$$\begin{aligned}\dot{V}_2 =& \sum_{i\in\mathbb{UF}}[(c_i + \rho_i)\dot{\rho}_i/g_i + \rho_i\dot{c}_i/g_i + (c_i - \alpha)\dot{c}_i/g_i] \\ =&\varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(P_iA_0 + A_0^TP_i + A_0^{iT}P_i + P_iA_0^i \\ &- A_0^{iT}P_i - P_iA_0^i) + [G(\hat{c} + \hat{\rho} - \alpha I) \otimes I]\operatorname{diag}(\Gamma_i) \\ &+ [((\hat{c} + \hat{\rho})(G\mathcal{L}_1 + \mathcal{L}_1^TG)(\hat{c} + \hat{\rho})) \otimes I]\operatorname{diag}(P_iK_i)\}\varrho \\ &+ \Theta_1 \\ \leq&\varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(A_0^{iT}P_i + P_iA_0^i - \tilde{A}_0^{iT}P_i \\ &- P_i\tilde{A}_0^i) - \lambda_0[(\hat{c} + \hat{\rho})^2 \otimes I]\operatorname{diag}(P_i^2) + [G(\hat{c} + \hat{\rho} \\ &- \alpha I) \otimes I]\operatorname{diag}(P_i^2)\}\varrho + \Theta_1,\end{aligned} \tag{25}$$

where $\Theta_1 = 2\varrho^T[G(\hat{c} + \hat{\rho})\mathcal{L}_1 \otimes I]\operatorname{diag}(P_i\tilde{A}_0^i)v_{1,UF} - 2\varrho^T\{[G(\hat{c} + \hat{\rho})\mathcal{L}_2] \otimes I\}\operatorname{diag}(P_i)\bar{F}\tilde{v}_{1,IF}$.

By using the Young's inequality, we have: 1) $\varrho^T[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(P_i^2)\varrho \leq \varrho^T[(\frac{\lambda_0}{4}(\hat{c} + \hat{\rho})^2 + \frac{G^2}{\lambda_0}) \otimes I]\operatorname{diag}(P_i^2)\varrho$; 2) $2\varrho^T[G(\hat{c} + \hat{\rho})\mathcal{L}_1 \otimes I]\operatorname{diag}(P_i\tilde{A}_0^i)v_{1,UF} \leq \varrho^T[\frac{\lambda_0}{4}(\hat{c} + \hat{\rho})^2 \otimes I]\operatorname{diag}(P_i^2)\varrho + \frac{4}{\lambda_0}\|(G\mathcal{L}_1 \otimes I)\operatorname{diag}(\tilde{A}_0^i)\|^2\|v_{1,UF}\|^2$; 3)$-2\varrho^T[G(\hat{c}+\hat{\rho})\mathcal{L}_2 \otimes I]\operatorname{diag}(P_i)\bar{F}\tilde{v}_{1,IF} \leq \varrho^T[\frac{\lambda_0}{4}(\hat{c}+\hat{\rho})^2 \otimes I]\operatorname{diag}(P_i^2)\varrho + \frac{4}{\lambda_0}\|(G\mathcal{L}_2 \otimes I_n)\bar{F}\|^2\|\tilde{v}_{1,IF}\|^2$.

Substituting the above three inequalities into (25) yields

$$\begin{aligned}\dot{V}_2 \leq&\varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(A_0^{iT}P_i + P_iA_0^i - \tilde{A}_0^{iT}P_i - P_i\tilde{A}_0^i) \\ &- [(\frac{\lambda_0}{4}(\hat{c} + \hat{\rho})^2 + \alpha G - \frac{G^2}{\lambda_0}) \otimes I]\operatorname{diag}(P_i^2)\}\varrho + \Theta_2 \\ \leq&\varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(A_0^{iT}P_i + P_iA_0^i - \tilde{A}_0^{iT}P_i - P_i\tilde{A}_0^i) \\ &- [(\frac{\lambda_0}{4}(\hat{c} + \hat{\rho})^2 + \frac{G^2}{\lambda_0}) \otimes I]\operatorname{diag}(P_i^2)\}\varrho + \Theta_2 \\ \leq&\varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(A_0^{iT}P_i + P_iA_0^i - P_i^2 \\ &- \tilde{A}_0^{iT}P_i - P_i\tilde{A}_0^i)\}\varrho + \Theta_2 \\ =&\varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(-I_n - \tilde{A}_0^{iT}P_i - P_i\tilde{A}_0^i)\}\varrho + \Theta_2,\end{aligned}$$

where $\Theta_2 = \frac{4}{\lambda_0}\|(G\mathcal{L}_1 \otimes I)\operatorname{diag}(\tilde{A}_0^i)\|^2\|v_{1,UF}\|^2 + \frac{4}{\lambda_0}\|(G\mathcal{L}_2 \otimes I_n)\bar{F}\|^2\|\tilde{v}_{1,IF}\|^2$. From the fact of $a + b \geq 2\sqrt{ab}, \forall a, b \in \mathbb{R}^+$, choose $\alpha \geq \max 2/(g_i\lambda_0), i \in \mathbb{UF}$ to obtain the last two inequalities, and the equality comes from ARE (8).

Define $V_3 = \sum_{i=1}^M \tilde{v}_{1,i}^TQ_i\tilde{v}_{1,i}$, where $Q_i > 0$ satisfies $(A_0 - F_iC_0)^TQ_i + Q_i(A_0 - F_iC_0) = -I_n$.

Combined with $\dot{\tilde{v}}_{1,i} = (A_0 - F_iC_0)\tilde{v}_{1,i}$, we obtain $\dot{V}_3 = -\tilde{v}_{1,IF}^T\tilde{v}_{1,IF} = -\|\tilde{v}_{1,IF}\|^2$.

Based on the above analysis, consider the following Lyapunov function candidate $V_4 = V_2 + \gamma V_3$.

Then, $\dot{V}_4 \leq \varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}(-I_n - \tilde{A}_0^{iT}P_i - P_i\tilde{A}_0^i)\}\varrho + \frac{4}{\lambda_0}\|(G\mathcal{L}_1 \otimes I)\operatorname{diag}(\tilde{A}_0^i)\|^2\|v_{1,UF}\|^2$, where the inequality comes from the choice of $\gamma \geq \frac{4}{\lambda_0}\|(G\mathcal{L}_2 \otimes I_n)\bar{F}\|^2$.

Thanks to $\lim_{t\to t_1}\tilde{A}_0^i(t) = 0$ in Lemma 1, $\exists t_2 > t_1, \forall t > t_2$, we have $-\tilde{A}_0^{iT}P_i < \varepsilon_1 I_n$ where $\varepsilon_1 > 0$ is an arbitrary small parameter. Similarly, $\exists t_3 > t_1, \forall t > t_3$, we have $\operatorname{diag}(\tilde{A}_0^i) < \varepsilon_2 I_{(N-M)n}$ where $\varepsilon_2 > 0$ is an arbitrary small parameter. So, when $t > \max\{t_2, t_3\}$, we have $\dot{V}_4 \leq \varrho^T\{[G(\hat{c} + \hat{\rho}) \otimes I]\operatorname{diag}((-1 + 2\varepsilon_1)I_n)\}\varrho + \frac{4\varepsilon_2^2}{\lambda_0}\|(G\mathcal{L}_1 \otimes I)\|^2\|v_{1,UF}\|^2$.

By choosing $\varepsilon_1 < \frac{1}{2}$ with ε_2 being arbitrarily small, as $\lambda_0, G, \mathcal{L}_1$ and $v_{1,UF}$ are bounded in reality, we have

$$\dot{V}_4 \leq -\varrho^T[G(\hat{c} + \hat{\rho}) \otimes I]\varrho. \tag{26}$$

Since $g_i > 0, c_i > 0, \rho_i \geq 0$, we get $\dot{V}_4(t) \leq 0$. Therefore, $V_4(t)$ is bounded and the same as ρ_i, c_i. Given $\dot{c}_i(t) \geq 0$ in (7), we can conclude that each coupling weight $c_i(t)$ increases monotonically and converges to the finite value finally. Note that $\dot{V}_4 \equiv 0$ means $\varrho = 0$. Thus, by LaSalle's Invariance principle, it follows $\lim_{t\to\infty}\varrho(t) = 0$. So $\lim_{t\to\infty}\tilde{v}_{1,UF}(t) = 0$ for UFs in (22).

C. Calculation of derivative of state predictor $z_i(t)$

1) Derivative for IF $i, i \in \boldsymbol{I}_1^M$: From (1), (6), (9), (12) and $\tilde{x}_i(t) = x_i(t) + \bar{x}_i(t) - X_iv_{1,i}(t)$ in (13), we have

$$\begin{aligned}\dot{z}_i =&e^{A_i\tau_i}\{A_ix_i + B_iu_i(t - \tau_i) + \dot{\bar{x}}_i - X_i[A_0v_{1,i} \\ &+ F_i(y_0 - C_0v_{1,i})]\} + B_iu_i - B_iU_ie^{A_0\tau_i}v_{1,i} \\ &- e^{A_i\tau_i}[B_iu_i(t - \tau_i) - B_iU_ie^{A_0\tau_i}v_{1,i}(t - \tau_i)] \\ &+ A_i\int_{t-\tau_i}^t e^{A_i(t-s)}B_i(u_i(s) - U_ie^{A_0\tau_i}v_{1,i}(s))ds.\end{aligned}$$

Thanks to Eq. (9a), then

$$\begin{aligned}\dot{z}_i =& A_i z_i + B_i u_i - B_i U_i e^{A_0\tau_i} v_{1,i} + e^{A_i\tau_i} X_i F_i C_0 \tilde{v}_{1,i} \\ & - e^{A_i\tau_i} B_i U_i [v_{1,i} - e^{A_0\tau_i} v_{1,i}(t-\tau_i)] + e^{A_i\tau_i}\dot{\bar{x}}_i. \quad (27)\end{aligned}$$

The prediction of observer $v_{1,i}(t+\tau_i)$ in (6) at time t is $v_{1,i}(t+\tau_i) = e^{A_0\tau_i} v_{1,i}(t) - \int_{t-\tau_i}^{t} e^{A_0(t-s)} F_i C_0 \tilde{v}_{1,i}(s+\tau_i) ds$, which is also expressed as

$$v_{1,i}(t) = e^{A_0\tau_i} v_{1,i}(t-\tau_i) - \int_{t-\tau_i}^{t} e^{A_0(t-s)} F_i C_0 \tilde{v}_{1,i}(s) ds. \quad (28)$$

Substituting (28) into (27) yields

$$\dot{z}_i = A_i z_i + B_i u_i - B_i U_i e^{A_0\tau_i} v_{1,i} + \Omega_i(t), \quad (29)$$

where $\Omega_i(t) = e^{A_i\tau_i} B_i U_i \int_{t-\tau_i}^{t} e^{A_0(t-s)} F_i C_0 \tilde{v}_{1,i}(s) ds + e^{A_i\tau_i} X_i F_i C_0 \tilde{v}_{1,i}(t) + e^{A_i\tau_i}\dot{\bar{x}}_i, i \in \mathbb{IF}$. From $\lim_{t\to\infty} \bar{x}_i(t) = 0$, it is obvious that $\lim_{t\to\infty} \dot{\bar{x}}_i(t) = 0$. As it has been proven that the observer error $\lim_{t\to\infty} \tilde{v}_{1,i}(t) = 0$ for IFs in (6), it is easily deduced that $\lim_{t\to\infty} \Omega_i(t) = 0$.

Remark 7: The integration calculation (28) of the observer $v_{1,i}(t)$ is the key technique to deduce $\dot{z}_i(t)$.

2) Derivative for UF $i, i \in \boldsymbol{I}_{M+1}^N$: Denote $\tilde{X}_i(t) = \hat{X}_i(t) - X_i$. Similar to derivative calculations of state predictor for IFs, from (1), (7), (10), (12) and (13), we get

$$\begin{aligned}\dot{z}_i =& e^{A_i\tau_i}\{A_i x_i + B_i u_i(t-\tau_i) + \Delta_{1,i}(t) - X_i[A_0 v_{1,i} \quad (30) \\ & - (c_i + \varrho_i^T P_i \varrho_i) P_i \varrho_i]\} + B_i u_i - B_i \hat{U}_i(t) e^{A_0^i(t)\tau_i} v_{1,i} \\ & - e^{A_i\tau_i}[B_i u_i(t-\tau_i) - B_i \hat{U}_i(t) e^{A_0^i(t)\tau_i} v_{1,i}(t-\tau_i)] \\ & + A_i \int_{t-\tau_i}^{t} e^{A_i(t-s)} B_i(u_i(s) - \hat{U}_i(t) e^{A_0^i(t)\tau_i} v_{1,i}(s)) ds,\end{aligned}$$

where $\Delta_{1,i}(t) = -\int_{t-\tau_i}^{t} e^{A_i(t-s)} B_i[\dot{\hat{U}}_i(t) e^{A_0^i(t)\tau_i} v_{1,i}(s) + \hat{U}_i(t) e^{A_0^i(t)\tau_i} \dot{A}_0^i(t)\tau_i v_{1,i}(s)] ds - e^{A_i\tau_i}(\dot{\tilde{X}}_i(t) v_{1,i}(t) + \tilde{X}_i(t)\dot{v}_{1,i}(t) + X_i \tilde{A}_0^i v_{1,i}(t) + \dot{\bar{x}}_i(t))$. Since $\tilde{A}_0^i(t) \to 0$, $\tilde{X}_i(t) \to 0$, $\dot{\tilde{X}}_i(t) \to 0$, $\dot{\hat{U}}_i(t) \to 0$, $\dot{A}_0^i(t) \to 0$ and $\dot{\bar{x}}_i(t) \to 0$ as $t \to \infty$, we can have $\lim_{t\to\infty} \Delta_{1,i}(t) = 0$.

Thanks to Eq. (9a), the calculation of term $X_i A_0 v_{1,i}$ in (30) is as follows: $X_i A_0 v_{1,i} = A_i(\hat{X}_i(t) - \tilde{X}_i(t)) v_{1,i} + B_i U_i v_{1,i}$. Now (30) changes to

$$\begin{aligned}\dot{z}_i =& A_i z_i + B_i u_i - B_i \hat{U}_i(t) e^{A_0^i(t)\tau_i} v_{1,i} + \Delta_{2,i}(t) \quad (31) \\ & - e^{A_i\tau_i}[B_i U_i v_{1,i} - B_i \hat{U}_i(t) e^{A_0^i(t)\tau_i} v_{1,i}(t-\tau_i)],\end{aligned}$$

where $\Delta_{2,i}(t) = \Delta_{1,i}(t) + e^{A_i\tau_i}[X_i(c_i + \varrho_i^T P_i \varrho_i) P_i \varrho_i + A_i \tilde{X}_i(t) v_{1,i}(t)]$. It is easy to have that $\lim_{t\to\infty} \Delta_{2,i}(t) = 0$.

Based on FO (3) in Lemma 1, we have $\lim_{t\to t_1} A_0^i(t) = A_0$ with t_1 in (5), i.e., we could regard $A_0^i(t)$ as unchanged to be A_0 as $t > t_1$. Then, similar as calculating (28), the prediction $v_{1,i}(t+\tau_i)$ in (7) at time t is $v_{1,i}(t) = e^{A_0^i(t)\tau_i} v_{1,i}(t-\tau_i) - \int_{t-\tau_i}^{t} e^{A_0^i(t-s)} P_i(c_i(s) + \rho_i(s))\varrho_i(s) ds$, $t > t_1 + \tau_i$. Substitute the above equation into (31), then we obtain

$$\dot{z}_i = A_i z_i + B_i u_i - B_i \hat{U}_i(t) e^{A_0^i(t)\tau_i} v_{1,i} + \Omega_i(t), \quad (32)$$

where $\Omega_i(t) = e^{A_i\tau_i} B_i(\hat{U}_i(t) - U_i) v_{1,i}(t) + \Delta_{2,i}(t) + e^{A_i\tau_i} B_i U_i \int_{t-\tau_i}^{t} e^{A_0^i(t-s)} P_i(c_i(s) + \rho_i(s))\varrho_i(s) ds, i \in \mathbb{UF}$. Since $\lim_{t\to\infty} \varrho_i(t) = 0$ is proved in Lemma 2 and $\lim_{t\to\infty} \hat{U}_i(t) = U_i$, we get $\lim_{t\to\infty} \Omega_i(t) = 0, i \in \mathbb{UF}$.

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