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FIELDS, PARTICLES, AND NUCLEI

Particle Creation: Schwinger + Unruh + Hawking

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We discuss the interconnection between the Schwinger pair creation in electric field, Hawking radiation and particle creation in the Unruh effect. All three processes can be described in terms of the entropy and temperature. These thermodynamic like processes can be combined. We consider the combined process of creation of charged and electrically neutral particles in the electric field, which combines the Schwinger and Unruh effects. We also consider the creation of the charged black and white holes in electric field, which combines the Schwinger effect and the black hole entropy. The combined processes obey the sum rules for the entropy and for the inverse temperature. Some contributions to the entropy and to the temperature are negative, which reflects the quantum entanglement between the created objects.

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1. INTRODUCTION

In this paper we discuss the connection of Schwinger particle creation in the constant electric field [1, 2] with the particle production in the Unruh [3] and Hawking [4] effects. For that we consider the combined effects, which involve simultaneously the Schwinger particle production and the other effects. These combined effects demonstrate that the entropy and temperature can be associated not only with the event horizons, as it was suggested by Gary Gibbons and Stephen Hawking [5], but also can be extended to the Schwinger effect.

The plan of the paper is the following. In Section 2, we recall the derivation of the Schwinger pair production.

In Section 3, the combined Schwinger and Unruh process is considered. It is the correlated process of creation of particle with mass $M + m$ in electric field, in which the charged particle with mass M is created in the Schwinger process, and chargeless particle with mass $m \ll M$ is created in the Unruh process, which accompanies the Schwinger creation. This correlated Schwinger + Unruh process suggests that the Schwinger pair creation can be also characterized by the temperature, and thus by the corresponding entropy, which is discussed in Section 4. For the Schwinger pair creation, the entropy appears to be negative, which reflects the quantum entanglement of the created particles.

In Section 5, the combined Schwinger + Hawking process is considered, in which the pair of the charged black holes is created in external electric field. It is shown that this process obeys the sum rule for the entropy; i.e., the total entropy is the sum of the Schwinger negative entropy and positive entropies of the created black holes. The same sum rule is applicable to creation of the pair of charged white holes, and to creation of black hole–white hole pair. The similar sum rule takes place for the inverse temperature.

Here we do not consider the Schwinger pair production in curved spacetime (see, e.g., [6] and references therein, and also the analog system, in which both gravity and electromagnetic field are emergent effective fields [7]). Although this also combines Schwinger, Unruh and Hawking processes, they do not reflect the proper thermodynamics.

2. SCHWINGER PROCESS IN THE SEMICLASSICAL APPROXIMATION

2.1. Quantum Tunneling Approximation

The spectrum of a charged particle in a constant electric field:

$$E(\mathbf{p},\mathbf{r}) = \pm\sqrt{M^2 + p^2} + q\mathcal{E}\cdot\mathbf{r}, \qquad (1)$$

where q is electric charge. Introducing the coordinate z along the electric field, one obtains for Schwinger case:

$$E(\mathbf{p},z) = \pm\sqrt{M^2 + p_\perp^2 + p_z^2} + q\mathcal{E}z. \qquad (2)$$

In the semiclassical approximation the probability of the particle creation is given by the tunneling exponent $2\mathrm{Im}\int dz p_z(z)$:

$$W = \sum_{\mathbf{p}} w_{\mathbf{p}} = \sum_{\mathbf{p}} \exp\left(-2\mathrm{Im}\int dz p_z(z)\right), \quad (3)$$

where the tunneling trajectories along z are given by equation $E(\mathbf{p}, z) = E$. In case of Schwinger pair production, the tunneling exponent depends only on the transverse momentum $\mathbf{p}_\perp$:

$$w_{\mathbf{p}}^{\mathrm{Schw}} = \exp\left(-\frac{\pi\tilde{M}^2}{q\mathscr{E}}\right), \quad \tilde{M}^2 = M^2 + p_\perp^2. \quad (4)$$

The semiclassical approximation is valid for $M^2 \gg q\mathscr{E}$.

Integration over transverse momenta gives

$$\int \frac{d^2 p_\perp}{(2\pi)^2} \exp\left(-\frac{\pi\tilde{M}^2}{q\mathscr{E}}\right) = \frac{q\mathscr{E}}{(2\pi)^2} \exp\left(-\frac{\pi M^2}{q\mathscr{E}}\right). \quad (5)$$

Integral over $dp_z/2\pi$ diverges because the exponent does not depend on p_z. Due to the motion equation $dp_z = q\mathscr{E}dt$, one obtains the known Schwinger pair creation per unit volume per unit time (the integer factors for polarization and for spin of particles are ignored):

$$\Gamma^{\mathrm{Schw}}(M) = \frac{dW^{\mathrm{Schw}}}{dt} = \frac{q^2\mathscr{E}^2}{(2\pi)^3} \exp\left(-\frac{\pi M^2}{q\mathscr{E}}\right). \quad (6)$$

2.2. Comparison with Unruh Radiation and the Factor 2 Problem

The straightforward comparison of Eq. (6) with the Unruh radiation in the accelerated frame reveals the factor of 2 problem. The Unruh temperature is $T_{\mathrm{U}} = a/2\pi$, where a is acceleration. Since in the Schwinger case the acceleration in the electric field is $a = q\mathscr{E}/M$, one obtains:

$$\exp\left(-\frac{\pi M^2}{qE}\right) = \exp\left(-\frac{\pi M}{a}\right) = \exp\left(-\frac{M}{2T_{\mathrm{U}}}\right). \quad (7)$$

In this naive approach the temperature of thermal radiation is twice the Unruh temperature. The factor of 2 problem arises also in the consideration of the Hawking radiation in the de Sitter expansion, see [8] and references therein. Subtleties in the tunneling approach to Hawking and Unruh radiation see in [9–14]. Different scenarios of resolving the above discrepancy between the Schwinger and Unruh mechanisms see in [15–17] and references therein. We consider the scenario somewhat similar to that in [16].

3. CORRELATED SCHWINGER + UNRUH PROCESS

According to Eq. (6), the probability of creation of particle with mass $M + m$ and charge q in the limit $m \ll M$ can be expressed in terms of the probability of creation of particle with mass M and the extra term:

$$\Gamma^{\mathrm{Schw}}(M + m) = \Gamma^{\mathrm{Schw}}(M) \exp\left(-\frac{2\pi Mm}{q\mathscr{E}}\right). \quad (8)$$

The extra term can be described in terms of the temperature of the Unruh radiation in the accelerated frame:

$$\exp\left(-\frac{2\pi Mm}{q\mathscr{E}}\right) = \exp\left(-\frac{m}{T_{\mathrm{U}}}\right) = \Gamma^{\mathrm{Unruh}}(m), \quad (9)$$

$$T_{\mathrm{U}} = \frac{a}{2\pi}, \quad a = \frac{qE}{M}, \quad (10)$$

where a as before is the acceleration of charged particle with mass M in electric field.

Altogether this gives:

$$\Gamma^{\mathrm{Schw}}(M + m) = \Gamma^{\mathrm{Schw}}(M)\Gamma^{\mathrm{Unruh}}(m). \quad (11)$$

This relation can be interpreted as follows: the process on the left-hand side of Eq. (11) can be considered as the combination of two correlated (entangled) processes on the right-hand side of Eq. (11). The first process is the Schwinger creation of two particles with large mass M and the charges $\pm q$. The second term describes the creation of two neutral particles, $q = 0$, with masses $m \ll M$. These electrically neutral particles do not feel the electric field. But each neutral particle is entangled with its charged partner, moves along the same trajectory and thus feels the same acceleration $a = \pm q\mathscr{E}/M$. As a result, the creation of the neutral particles is fully described by the Unruh process.

This coherent combined process can be also interpreted in the following way. The charged particle with heavy mass M is created by the Schwinger process and moves with the acceleration $a = q\mathscr{E}/M$. In the accelerated reference frame (in the Rindler spacetime), the moving massive particle plays the role of the detector in the Unruh vacuum. This detector experiences the emission of neutral particles providing the Unruh radiation with the Unruh temperature $T_{\mathrm{U}} = a/2\pi$.

The combination of the several processes, which gives rise to the product of probabilities, is similar to the phenomenon of cotunneling in the electronic systems. In these condensed matter systems, the electron experiences the coherent sequence of tunneling events: from an initial to the virtual intermediate states and then to the final state [18, 19]. In our case the virtual intermediate state is the state of the created charged particle with mass M. Its motion with acceleration a triggers the creation of the neutral particle by the Unruh mechanism.

The combination of two processes in Eq. (11)—the Schwinger creation of mass M and the Unruh creation of mass $m \ll M$—is similar to the combination of two processes in the creation of pairs of Reissner–Nordström (RN) monopole black holes in magnetic field [20]:

$$\Gamma^{\mathrm{BH,Monopole}} = \Gamma^{\mathrm{Monopole}}\Gamma^{\mathrm{BH}}, \tag{12}$$

$$\Gamma^{\mathrm{BH}} = \exp(S_{\mathrm{BH}}). \tag{13}$$

According to [20], the instanton amplitude contains an explicit factor corresponding to the black hole entropy, $S_{\mathrm{BH}} = A/4$ in Eq. (13), where A is the area of the event horizon. The Schwinger pair creation is modified by the black hole entropy and thus by the Hawking temperature.

We consider such combination in Section 5.1.

4. QUANTUM TUNNELING AND ENTROPY

In principle, the black hole thermodynamics can be obtained by consideration of the tunneling processes. The Hawking temperature can be derived by comparing the tunneling rate with the Boltzmann factor [21–23]. The black hole entropy can be found [24, 25] by comparing the tunneling rate with the thermodynamic fluctuations [26]. In this approach the tunneling process is considered as fluctuation, with the probability of fluctuation being determined by the entropy difference between the initial and final states, $w \propto \exp(S_{\mathrm{final}} - S_{\mathrm{initial}})$. Let us consider the process of Schwinger pair creation as the thermodynamic fluctuations, which can be expressed via the corresponding entropy $S_{\mathrm{Schw}}(M)$.

4.1. First Guess

At first glance, Eq. (6) can be written in the form

$$\Gamma^{\mathrm{Schw}}(M) = \frac{q^2\mathcal{E}^2}{(2\pi)^3}\exp(-S_{\mathrm{Schw}}(M)). \tag{14}$$

This kind of entropy may give the Unruh temperature:

$$T^{-1}_{\mathrm{Schw}} = \frac{dS_{\mathrm{Schw}}}{dM} = \frac{2\pi M}{q\mathcal{E}} = \frac{2\pi}{a} = T^{-1}_{\mathrm{U}}. \tag{15}$$

However, this guess is not correct. The situation is different and is more interesting, see Section 4.2 below.

4.2. Negative Entropy

In the entropy-fluctuation relation [26, 24] the probability of fluctuation is $w \propto \exp(S_{\mathrm{final}} - S_{\mathrm{initial}})$. The initial state is the vacuum, and its entropy is zero, $S_{\mathrm{initial}} = S_{\mathrm{vac}} = 0$. Thus, the Schwinger entropy is negative, $S_{\mathrm{final}} \equiv S_{\mathrm{Schw}}(M) = -\frac{\pi M^2}{q\mathcal{E}} < 0$, and

$$\Gamma^{\mathrm{Schw}}(M) = \frac{q^2\mathcal{E}^2}{(2\pi)^3}\exp(S_{\mathrm{Schw}}(M)). \tag{16}$$

The negative entropy comes from the quantum entanglement between the created particles and also between the particles and quantum fields in the vacuum, see discussions of correlations between the radiated particles in [27, 28].

Then the corresponding Schwinger temperature is also negative and is opposite to the Unruh temperature, $T_{\mathrm{Schw}} = -T_{\mathrm{Unruh}}$:

$$T^{-1}_{\mathrm{Schw}} = \frac{dS_{\mathrm{Schw}}}{dM} = -\frac{2\pi M}{q\mathcal{E}} = -\frac{2\pi}{a} = -T^{-1}_{\mathrm{U}}. \tag{17}$$

The negative temperature is a well-defined quantity, especially in condensed matter systems, see review [29].

5. COMBINED SCHWINGER + HAWKING

5.1. Sum Rule for Entropy

The description of the quantum tunneling in terms of the thermodynamic fluctuation [26] can be also applied to the creation of two charged RN black holes discussed in [20]. This corresponds to the triple cotunneling—the coherent sequence of three tunneling events. Adding the entropy of two created black holes one obtains for the full entropy, which determines the probability of creation:

$$\begin{aligned} &S_{\mathrm{final}} - S_{\mathrm{initial}} \\ &= S_{\mathrm{Schw}}(M,q) + S_{\mathrm{RN\,BH}}(M,q) + S_{\mathrm{RN\,BH}}(M,q) \end{aligned} \tag{18}$$

$$= -\frac{\pi M^2}{q\mathcal{E}} + 4\pi G M^2 + 4\pi G M^2. \tag{19}$$

Here it is assumed that $q\mathcal{E} \ll G^{-1} \sim E^2_{\mathrm{Planck}}$; i.e., the force between the charges is much smaller than the Planck "maximum force" limit conjectured by Gibbons [30] (see also recent papers [31, 32] and references therein, and the criticism in [33]). As a result, the total contribution to Eq. (19) is negative. We also take into account that the entropy of the RN black hole does not depend on charge q, which is due to the correlations between the inner and outer horizons [25].

As in [20] (see also [34] and references in [35]), the positive terms in Eq. (19) demonstrate that due to the positive entropy of the black holes, the creation of pairs of RN black holes is enhanced compared to pair production of particles or magnetic monopoles with the same masses and charges.

On the other hand, the creation of the pairs of the RN white holes should be suppressed due to their negative entropy, $S_{WH} = -S_{BH}$ [25]:

$$S_{final} - S_{initial} \\ = S_{Schw}(M,q) + S_{RNWH}(M,q) + S_{RNWH}(M,q) \tag{20}$$

$$= -\frac{\pi M^2}{q\mathcal{E}} - 8\pi G M^2. \tag{21}$$

There can be also the process of the creation of the pair: black hole + white hole. In this process the conventional Schwinger production is restored due to mutual cancellation of entropies of black and white holes:

$$S_{final} - S_{initial} \\ = S_{Schw}(M,q) + S_{RNWH}(M,q) + S_{RNBH}(M,q) \tag{22}$$

$$= S_{Schw}(M,q) = -\frac{\pi M^2}{q\mathcal{E}}. \tag{23}$$

5.2. Sum Rule for Temperature

The sum rule for the entropy in Eqs. (18), (20), and (22) is also valid for the inverse temperatures. The corresponding temperature $1/T = dS_{final}/dM$ describing the above three processes of creation is:

$$\frac{1}{T} = \frac{1}{T_{Schw}} + \frac{1}{T_{BH}} + \frac{1}{T_{BH}}, \quad T_{BH} = \frac{1}{8\pi G M}, \tag{24}$$

$$\frac{1}{T} = \frac{1}{T_{Schw}} + \frac{1}{T_{WH}} + \frac{1}{T_{WH}}, \quad T_{WH} = -\frac{1}{8\pi G M}, \tag{25}$$

$$\frac{1}{T} = \frac{1}{T_{Schw}} + \frac{1}{T_{BH}} + \frac{1}{T_{WH}} = \frac{1}{T_{Schw}}. \tag{26}$$

The summation law for inverse temperature is similar to that, which takes place for the black hole with several horizons [25], where the temperature is expressed in terms of the Hawking temperatures on different horizons. For the black hole with two horizons, such as RN black hole, one has $1/T_{RNBH} = 1/T_+ + 1/T_-$, where T_+ and T_- are the temperatures of outer and inner horizons correspondingly. The temperature T_{RNBH} determines the Hawking radiation rate, which corresponds to cotunneling – the coherent sequence of tunneling events at different horizons: $\exp(-E/T) = \exp(-E/T_-)\exp(-E/T_+)$.

The same was found for the Schwarzschild–de Sitter black hole [36, 37], with T_+ and T_- being the temperatures of the cosmological horizon and the black hole horizon correspondingly. On the other hand, the total entropy of the RN black hole is not necessarily the sum of the entropies of the inner and outer horizons due to correlations between the two horizons, see, e.g., [25, 38–43]. In addition, the consideration of the black holes in the de Sitter environment requires the use of special modifications of the Painleve–Gullstrand coordinates [44], which may modify the thermodynamics.

6. DISCUSSION

We considered the extension of the phenomenon of the black hole entropy to the general processes of quantum tunneling, including the macroscopic quantum tunneling. It appears that the Schwinger pair creation obeys the same thermodynamic laws as in the black hole thermodynamics. The thermodynamics naturally connects the Schwinger pair creation in electric field with the Hawking radiation and with the particle creation in the Unruh effect. All three processes can be described in terms of the entropy and temperature. Moreover, these 3 processes can be combined. For example, the process of creation of charged and electrically neutral particles in the electric field combines the Schwinger and Unruh effects. The same takes place in the process of the creation of the charged black and white holes in electric field. This process combines the Schwinger effect and the black hole entropy. The combined processes obey the sum rules for the entropy and for the inverse temperature. Some contributions to the entropy and to the inverse temperature are negative, which reflects the quantum entanglement between the created objects: charged particles, magnetic monopoles and black and white holes.

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CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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