Lopez Ramirez, Alejandra; Löfman, Monica; Korkiala-Tanttu, Leena

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Safety margin from characteristic values in settlement calculations of embankments on clay

Alejandra López Ramírez MSc
Doctoral Candidate, Department of Civil Engineering, Aalto University, Espoo, Finland (Orcid:0000-0003-0086-1989) (corresponding author: alejandra.lopezramirez@aalto.fi)

Monica Susanne Löfman MSc
Doctoral Candidate, Department of Civil Engineering, Aalto University, Espoo, Finland (Orcid:0000-0003-3086-1031)

Leena Korkiala-Tanttu PhD
Associate Professor, Department of Civil Engineering, Aalto University, Espoo, Finland (Orcid:0000-0003-0894-7218)

Settlement calculations in serviceability limit state (SLS) design of embankments require the estimation of characteristic values of soil properties to account for different sources of uncertainty. The upcoming revised version of EN 1997-1:2004 (Eurocode 7) presents a statistically based equation to determine characteristic values of soil properties with the aim of providing a more consistent safety margin. The aim of this study is to quantify the safety margin in settlement calculations using this equation and performing probabilistic analyses. Four different cases of embankments on soft soil with different soil conditions were considered, including significant non-linear stress–strain behaviour. This allowed evaluation of the consistency of the safety margin from statistically derived characteristic values for different soil conditions. Probabilistic analyses were performed using Monte Carlo simulations to determine the statistics of total settlements. The safety margin was then defined based on the distance between the settlement from characteristic values and the mean settlement value of the simulations. The quantified safety margin was not consistent among the cases, which may be related to varying levels of non-linear compressibility and degree of consolidation. Moreover, for some cases, the safety margin might be too conservative for SLS design of embankments.

Notation

C_c	compression index
C_s	swelling index
e_0	void ratio
G	performance function
K	coefficient for defining the 5% fractile of student's t distribution
k_n	coefficient for defining the 5%, the 50%, or 95% fractile of normal dristibution with Eurocode 7
m_1	modulus number (Janbu, 1963)
N	number of simulation runs
n	number of samples
P	cumulative probability
p_e	probability of exceeding total settlement
p_f	probability of unsatisfactory performance
S_{best}	settlement calculated using best estimate values
S_{Xk}	settlement calculated using characteristic values
V_x	coefficient of variation
X_k	characteristic value
X_{mean}	sample mean
β	reliability index
β_1	stress exponent (Janbu, 1963)
β_{SLS}	target reliability index for serviceability limit state
Γ^2	variance reduction factor
$\Delta\sigma_v$	increment in vertical stress
ε_{NC}	vertical strain in normal compression state
μ_S	probabilistic mean value of total settlement
ξ^2_{inh}	inherent variability variance
ξ^2_{meas}	measurement uncertainty variance
ξ_S	standard deviation
ξ^2_{stat}	statistical uncertainty variance
ξ^2_{tot}	total variance
ξ^2_{trans}	transformation uncertainty variance
σ'_p	preconsolidation pressure
σ_{ref}	reference stress
σ'_{vo}	effective vertical in situ stress
σ_x	standard deviation of sample

1. Introduction

Uncertainty in the estimation of soil properties continues to receive considerable attention as it affects the prediction of soil behaviour and the subsequent limit state verification. EN 1997-1:2004 (Eurocode 7) (CEN, 2004) – henceforth referred to as EC7 in this paper – acknowledges the different sources of uncertainty affecting geotechnical design: inherent soil variability, measurement error, transformation uncertainty and statistical uncertainty (Phoon and Kulhawy, 1999). Accounting for these sources of uncertainty, the partial-factor method is one of the approaches applied in geotechnical design to achieve the required level of safety. The partial-factor method requires the estimation of characteristic values and the application of partial factors. According to EC7 (CEN, 2004: p. 27), 'The characteristic value of a geotechnical parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state'. This 'cautious estimate' can be selected subjectively or by using statistical methods.

In EC7 (CEN, 2004), estimation of the total and differential settlements for serviceability limit state (SLS) verification is performed using characteristic values of soil parameters. This provides a safety margin, accounting for certain sources of uncertainty. However, the partial factor is unity for SLS verification. This means that, unlike in ultimate limit state (ULS) estimates, there is no safety margin beyond the characteristic values in SLS estimates. The non-life-threatening nature of the conditions verified through SLS design explains EC7's decision on the partial factor for SLS. However, as the characteristic values can be subjectively chosen cautious estimates, the safety margin can be insufficient or unnecessarily large, depending on the chosen level of cautiousness. Furthermore, defining the characteristic values using statistical methods may provide a more consistent safety margin in SLS, although the possibility of having an unnecessarily large safety margin remains. In statistical methods, a coefficient of variation (CoV) V_x (defined as the ratio of the standard deviation to the mean) has to be selected for the soil property. When site-specific data are not available, literature values of V_x are commonly used as prior knowledge. According to Phoon and Kulhawy (1999), these literature values offer a certain level of confidence. However, the range can sometimes be considerably wide for some properties and without site-specific data, selection becomes challenging.

Moreover, it is unclear how the uncertainties accounted for during determination of characteristic values can influence the system's response to non-linear soil behaviour, especially for soft, sensitive clays that are characterised by non-linear behaviour along the compression range. This non-linearity has led geotechnical engineers to use the tangent modulus method (TMM) for a better approximation of the stress–strain behaviour of some Fennoscandian clays. The aim of this study was thus to quantify the safety margin provided by statistically based characteristic values for settlement calculations. To this end, both probabilistic and deterministic estimations of total settlements were made. Total settlements, representing the response of the system, were calculated for four different cases of embankments built on soft Finnish clays. The uncertainty in input parameters was quantified through three different values of the CoV, which fall within the range reported in the literature for the relevant soil parameters.

In this paper, the results of deterministic analyses are presented first, with the characteristic values of soil parameters estimated using the equation presented in the October 2020 draft of the second-generation EC7 Part 1 (prEN 1997 1:202x, 2020). In this paper, this document is henceforth referred to as the 'October draft'. An alternative equation, proposed by Schneider (1997) for determining the characteristic values of soil parameters, was also used. Probabilistic assessments of total settlements were carried out using Monte Carlo simulations (MCSs). The probabilistic mean value of total settlement (μ_S) was used as a reference value for quantitatively assessing the safety margin from the characteristic values. This quantification was done through computation of the probability of exceeding the total settlement (p_e) from the characteristic values (S_{Xk}). Likewise, the distance between the standard deviations of μ_S and S_{Xk} was also defined as an alternative quantification of the safety margin.

The selected cases represent different soil conditions, including normally and overconsolidated clays with slight to strong non-linear behaviours. The TMM was preferred for estimating total settlements, but the compression index method (CIM) was also utilised. Even though CIM might be less accurate for soft Finnish clays, its use allowed for a better assessment of how the safety margin is affected by the varying the uncertainty in soil properties when non-linear behaviour is considered.

2. Statistical methods for defining the characteristic value

2.1 Uncertainties in soil property determination

Clause 2.4.5.2(7) in EC7 (CEN, 2004) states that the value affecting the occurrence of the limit state is 'often the mean of a range of values covering a large surface or volume of the ground' (p. 27). This value is referred to as the population mean. The estimate of the population mean from a limited dataset is the sample mean. The characteristic value then accounts for the uncertainty arising from the difference between the sample mean and the population mean. Equation 1 shows the total uncertainty associated with the estimate of the mean of a soil property within a relevant soil layer. According to Equation 1, the total uncertainty can be quantified by way of the sum of variances.

1. $$\xi^2_{\text{tot}} = \xi^2_{\text{inh}}\Gamma^2 + \xi^2_{\text{meas}} + \xi^2_{\text{trans}} + \xi^2_{\text{stat}}$$

In this equation, ξ^2_{tot} denotes the total variance for the soil parameter in question and ξ^2_{inh}, ξ^2_{meas}, ξ^2_{trans} and ξ^2_{stat} are, respectively, the variances accounting for the inherent variability, measurement uncertainty, transformation uncertainty and statistical uncertainty. This sum of variances is often replaced with the sum of squared CoVs (V_x) (Phoon and Kulhawy, 1999). Γ^2 is a variance reduction factor, which takes into account the effect of spatial averaging (Vanmarcke, 1977). If the soil layer is sufficiently thick, the soil properties can be fully averaged. In such a case, $\Gamma^2 = 0$, and the effect of the weak local zones in the soil is eradicated. In the case of the non-existing averaging effect, $\Gamma^2 = 1$.

The statistical uncertainty ξ^2_{stat} related to estimating the mean soil property from limited measurements is defined through the inherent variability ξ^2_{inh} of the spatially averaged soil property (Lo and Li, 2007). If ξ^2_{meas} and ξ^2_{trans} are assumed to be zero, the variance accounting for the total uncertainty is reduced to:

2. $$\xi^2_{\text{tot}} = \xi^2_{\text{inh}}\Gamma^2 + \xi^2_{\text{stat}} = \xi^2_{\text{inh}}\Gamma^2 + \xi^2_{\text{inh}}\frac{1}{n} = \left(\Gamma^2 + \frac{1}{n}\right)\xi^2_{\text{inh}}$$

where n indicates the number of samples used to obtain the sample mean. Hence, in the case of the full-averaging effect ($\Gamma^2 = 0$), the only source of uncertainty accounted for is the statistical uncertainty. This corresponds to the statistical definition of the characteristic value (e.g. Prästings *et al.*, 2019). If a normal distribution is assumed, the total variance ξ^2_{tot} related to the uncertainty in the estimation of the population mean is defined as:

$$3. \quad \xi^2_{\text{tot}} \approx \xi^2_{\text{inh}} \frac{1}{n} = (V_{\text{x}} X_{\text{mean}})^2 \frac{1}{n}$$

where V_{x} is the CoV and X_{mean} is the sample mean. For a normal distribution, the total variance in Equation 2 allows the characteristic value to be defined as: (Lo and Li, 2007)

$$4. \quad X_{\text{k}} = X_{\text{mean}} \left[1 - K \frac{V_{\text{x}}}{\sqrt{n}} \right]$$

where K is a coefficient that depends on n and represents the probability of occurrence of the limit state. When the population mean is estimated from an unlimited number of test results, $K = 1.645$. This value corresponds to the Student's t correlation factor evaluated for a 95% confidence level in a probability value not greater than 5% of a worse value governing the occurrence of the limit state. This is the prescribed probability defined in clause 2.4.5.2(11) of EC7 (CEN, 2004). For small values of n, Student's t can be used to estimate K.

2.2 Equation in the October draft

The October draft provides a statistically based equation (Equation 5) for the assessment of the characteristic value of ground properties (X_{k}) with a 95% confidence level. Equation 5 is a general form of Equation 4 as it allows for the computation of X_{k} when full averaging is not considered (local failure, $\Gamma^2 = 1$). Depending on the design situation, the equation can be used to obtain an estimate of the mean value or of the 5% or 95% fractile of a ground property.

$$5. \quad X_{\text{k}} = X_{\text{mean}} \left[1 \mp \frac{k_n \sigma_x}{X_{\text{mean}}} \right] = X_{\text{mean}} [1 \mp k_n V_{\text{x}}]$$

where σ_x is the standard deviation of the sample and k_n is a coefficient that depends on the number of derived values of the ground property (n) and the fractile for which X_{k} is estimated. A positive sign is used if X_{k} is an upper-bound estimate of the value governing the limit state. When V_{x} is known or assumed, Equation 6 is used to calculate X_{k} as the estimate of the mean value and Equation 7 is used to calculate X_{k} as an estimate of the inferior or superior value (5% or 95% fractile). Substituting Equation 7 into Equation 5 implies that the failure domain is small with respect to the scale of fluctuation, based on the equations for characteristic values presented by Lo and Li (2007).

$$6. \quad k_n = N_{95} \sqrt{\frac{1}{n}}$$

$$7. \quad k_n = N_{95} \sqrt{1 + \frac{1}{n}}$$

where N_{95} is a value (1.64) that represents the normal distribution. The values of V_{x} can also be calculated as the ratio between the standard deviation and the mean of the derived values of the ground property. For this latter option, the value of k_n is calculated using the values of Student's t evaluated for $n - 1$ degrees of freedom ($t_{95,n-1}$), which can be found in the thesis of López Ramirez (2020). These values of $t_{95,n-1}$ replace the values of N_{95} in Equations 6 and 7 for this case.

2.3 Schneider's method for determining the characteristic value

Schneider (1997) demonstrated that a reasonable estimate of X_{k} can be obtained when $k_n = 0.5$, which corresponds to $n = 13$ assuming a t-distribution and $n = 10$ for a normal distribution. For less than ten samples, the statistically defined characteristic values are too conservative. Thus, X_{k} as an estimate of the mean value with a 95% confidence level according to Schneider's method is given by:

$$8. \quad X_{\text{k}} = X_{\text{mean}} (1 \mp 0.5 \, V_{\text{x}})$$

According to Schneider (1997), Equation 8 can be applied even in cases where there are no test results at all and for any kind of distribution followed by the data. In this work, this equation was especially useful for one of the cases (the Kujala embankment, described in Section 3.2) where the limited amount of data did not allow the use of a statistical approach.

3. Study cases

3.1 Haarajoki test embankment

The Haarajoki test embankment was built and instrumented by the Finnish Road Administration near the city of Järvenpää in southern Finland in 1997. The embankment is 2.9 m high and 100 m long. The crest is 8 m wide and the slopes have a gradient of 1:2. The material of the embankment consists of sandy gravel and gravel, with an estimated density of 21 kN/m^3. The groundwater table is at ground level. The embankment is founded on a soft soil deposit about 20 m thick, consisting of overconsolidated soft clay overlain by a 2 m thick dry crust. The clay deposits on the site are characterised by high compressibility and sensitivity. Beneath the soft soil deposit, there are 2–3 m thick silt and till layers. The soils below half the length of the embankment contain vertical drains to accelerate the consolidation process. The other half of the deposit is composed of virgin soil layers (Lojander and Vepsäläinen, 2001).

Figure 1 shows the distribution of some of the soil compressibility parameters derived for the Haarajoki case. The mean value in each profile in Figure 1 corresponds to the mean of the test results of a specific soil sublayer. The subscript 1 in parameters m_1 and β_1 indicates that the deformations calculated from those values belong to the normal consolidated range. For deformations belonging to the overconsolidated part, subscript 2 is used. Figure 1 shows, in addition to the in situ stress, how the preconsolidation pressure σ'_p was defined for each soil sublayer. The values of σ'_p to be used in the settlement estimations were defined in two ways: (*a*) by calculating the mean value of σ'_p from all the oedometer results of the sublayer or (*b*) by calculating the mean value of all the pre-overburden pressures (POPs) within a sublayer. POPs were obtained as the difference between the values of σ'_p from oedometer tests and the in situ vertical stress at the sample depth. The mean of all the POPs was then calculated for a sublayer, which is the value shown in Figure 1. Further details of the site properties and general information can be found elsewhere (López Ramirez, 2020; Vepsäläinen *et al.*, 1997; Yildiz *et al.*, 2009).

The instrumentation for the Haarajoki embankment consists of inclinometers, piezometers, pressure cells and settlement plates (Yildiz *et al.*, 2009). The observed settlements 2 years after construction were 372 mm and 633 mm for the virgin soil layers and the section with vertical drains, respectively (Lojander and Vepsäläinen, 2001). Additional observed values were reported by Länsivaara (2001) at 3.5 years: a settlement of approximately 400 mm was reached in the virgin soil layers and around 700 mm in the section with vertical drains. Based on these 3.5 year observations, primary consolidation was still ongoing for both halves of the soil deposit.

3.2 Kujala test embankment

Two test embankments were built in 2017 for the Kujala interchange of highway 12, which is located near the city of Lahti, Finland. Only one of the embankments was selected for analysis in this work. The selected embankment is, on average, 4 m high and around 20 m long. The crest is 18 m wide. The slopes were designed at 1:1.5. The embankment material is composed of crushed rock with a unit weight of approximately 20 kN/m^3. The embankment is underlain by a 2 m thick dry crust layer. The dry crust is followed by a 12 m clay/silt layer, which is mostly overconsolidated. The depth of the groundwater level is around 2 m.

The Kujala test embankments were monitored over a period of 1.5 years, during which the settlements were 65–115 mm.

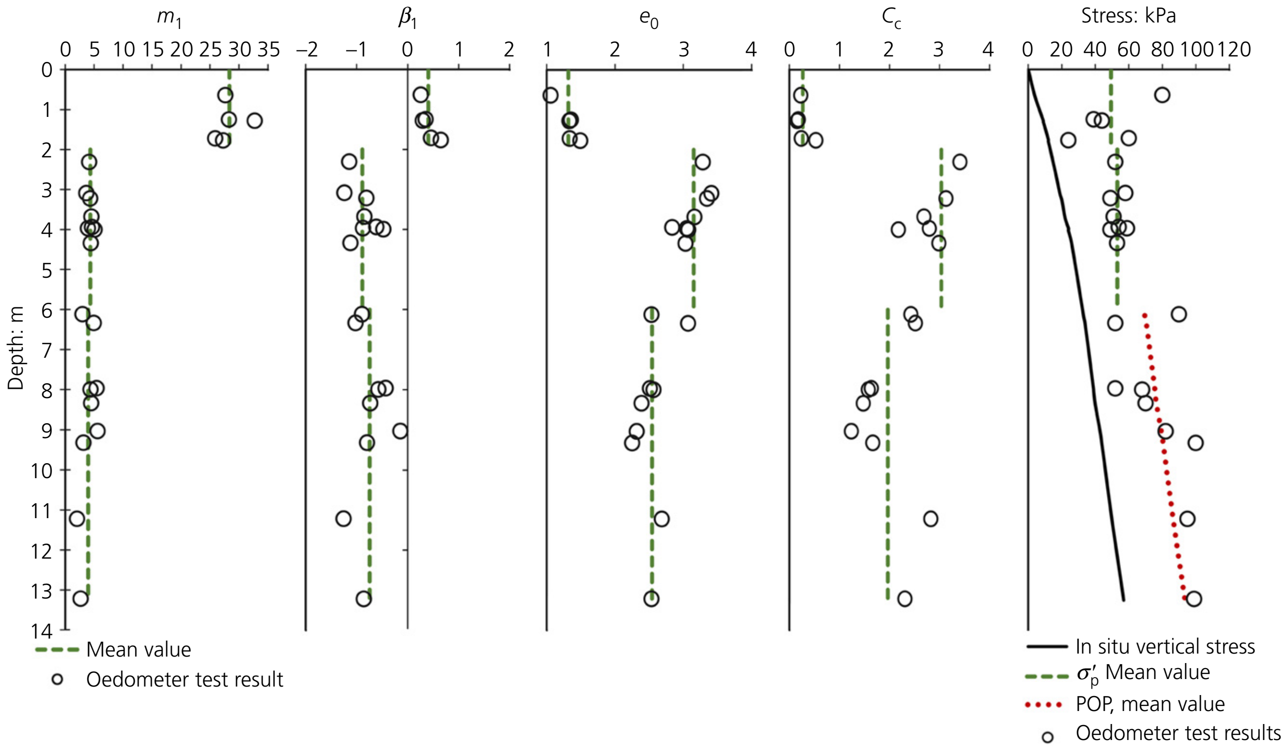

Figure 1. Compressibility properties of the Haarajoki test embankment

Much of the primary consolidation was attained during this observation time (Löfman and Korkiala-Tanttu, 2021a). Some of the apparent preconsolidation pressure values (σ'_p) determined from oedometer tests on silty clay samples were estimated to be affected by sampling disturbance. For those tests, the POP was increased to better correspond to the σ'_p profile estimated from field vane tests. After this modification, the calculated settlements agreed well with the measured settlements. The dry crust is considered non-compressible and there are two silt layers at depths of 3.2 m and 6 m, whose TMM parameters were obtained from the literature. The distribution of the main compressibility properties and the modified σ'_p are shown in Figure 2.

3.3 Murro test embankment

The Murro test embankment was built by the Finnish Road Administration in 1993 near Seinäjoki, Finland. The embankment is 2 m high and 20 m long, with a 10 m crest and 1:2 slopes. The fill material of the embankment is crushed rock with a unit weight of 20 kN/m^3. Underneath the embankment is a soft, silty clay deposit about 23 m deep. The soft soil deposit is normally consolidated. The dry crust is around 1.6 m thick and is heavily overconsolidated. The groundwater level is estimated to be at 0.8 m below ground level. Figure 3 shows the distribution with depth of the main compressibility parameters for each layer.

The Murro test embankment was monitored over a period of 8 years after construction. During that time, a total settlement of 798 mm was measured, with most of the settlements occurring at a depth of 1.6–6.7 m (Karstunen and Yin, 2010).

3.4 Östersundom test embankment

The Östersundom test embankment was built in two stages, the first beginning in March 2014 and the second in December 2014. The embankment is located in the Östersundom district in eastern Helsinki, Finland. In the first stage, a 42 m long embankment was built with a crest width of 19.2 m and a height of 0.4 m. In the second stage, an upper section was added to the centre of the existing embankment. This section is 0.8 m high, 21 m long and has a 10 m wide crest with 3.2 m shoulders on both sides. The total height of the embankment after the second stage was 1.2 m. The slopes in both stages were constructed at a 1:2 gradient. The filling material has an estimated unit weight of 21 kN/m^3. The embankment is founded on a 5 m thick soft clay deposit overlain by a dry crust layer about 0.8 m deep. The groundwater level is at a depth 0.6 m below ground. The soil profiles for the main soil compressibility parameters are shown in Figure 4. The last sublayer, consisting of silt, is 16.4 m deep. For this layer, literature values of the modulus number (m_1) and stress exponent (β_1) were used.

According to the values reported by Köylijärvi (2015), the observed settlements for the Östersundom test embankment over 1.5 years were about 125–300 mm. The end of the primary consolidation had not been attained at the time of the observations in this study.

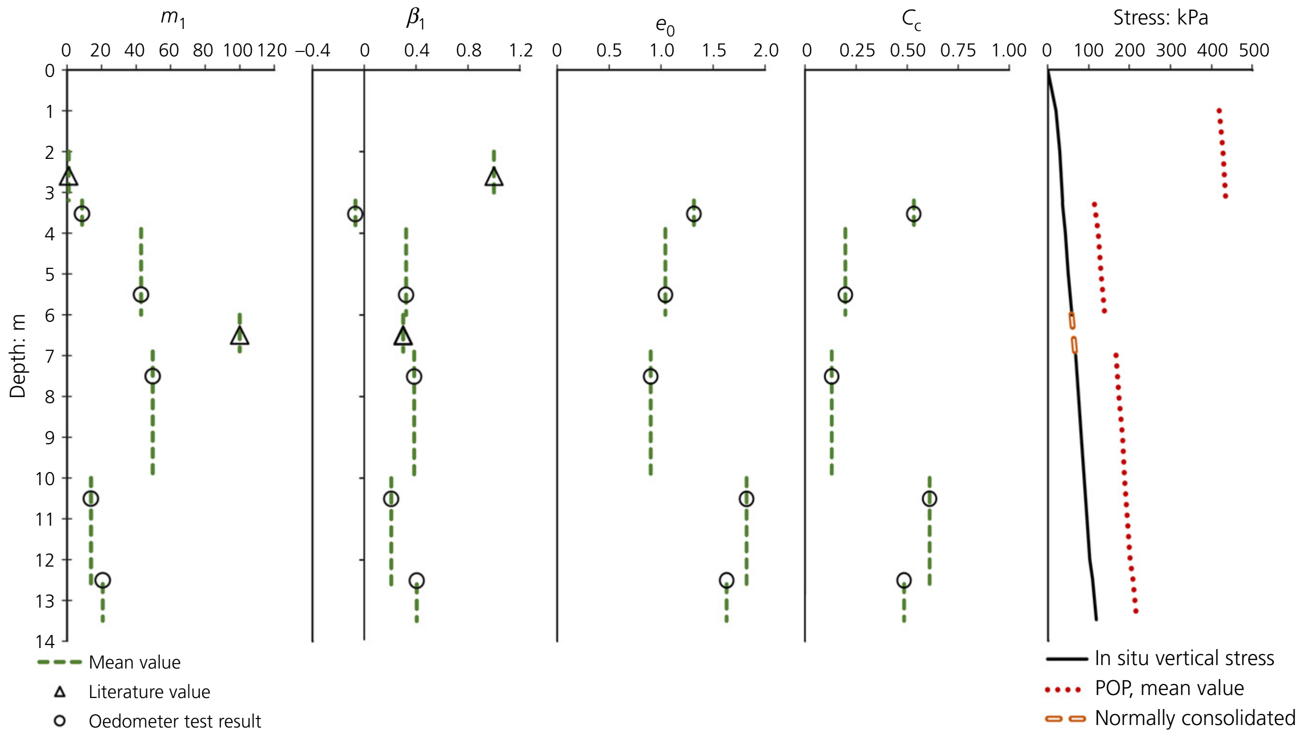

Figure 2. Compressibility properties of the Kujala test embankment

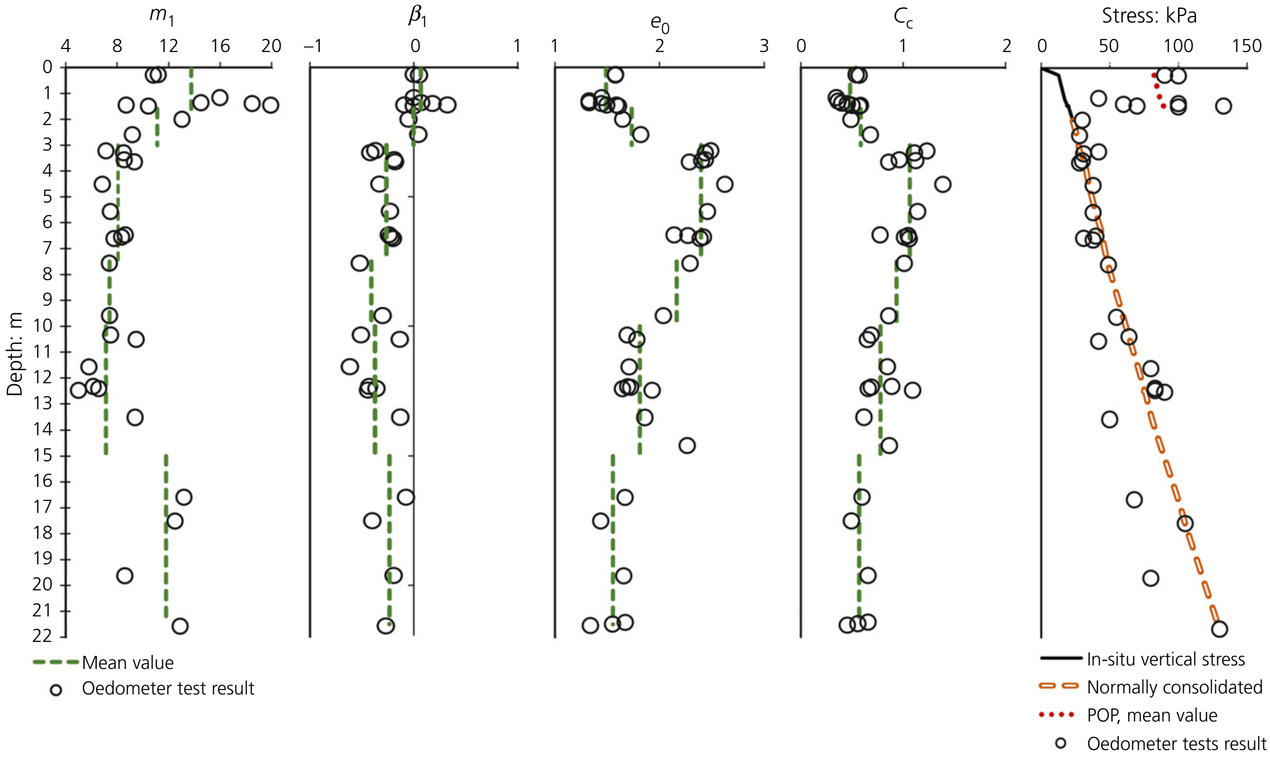

Figure 3. Compressibility parameters of the Murro test embankment

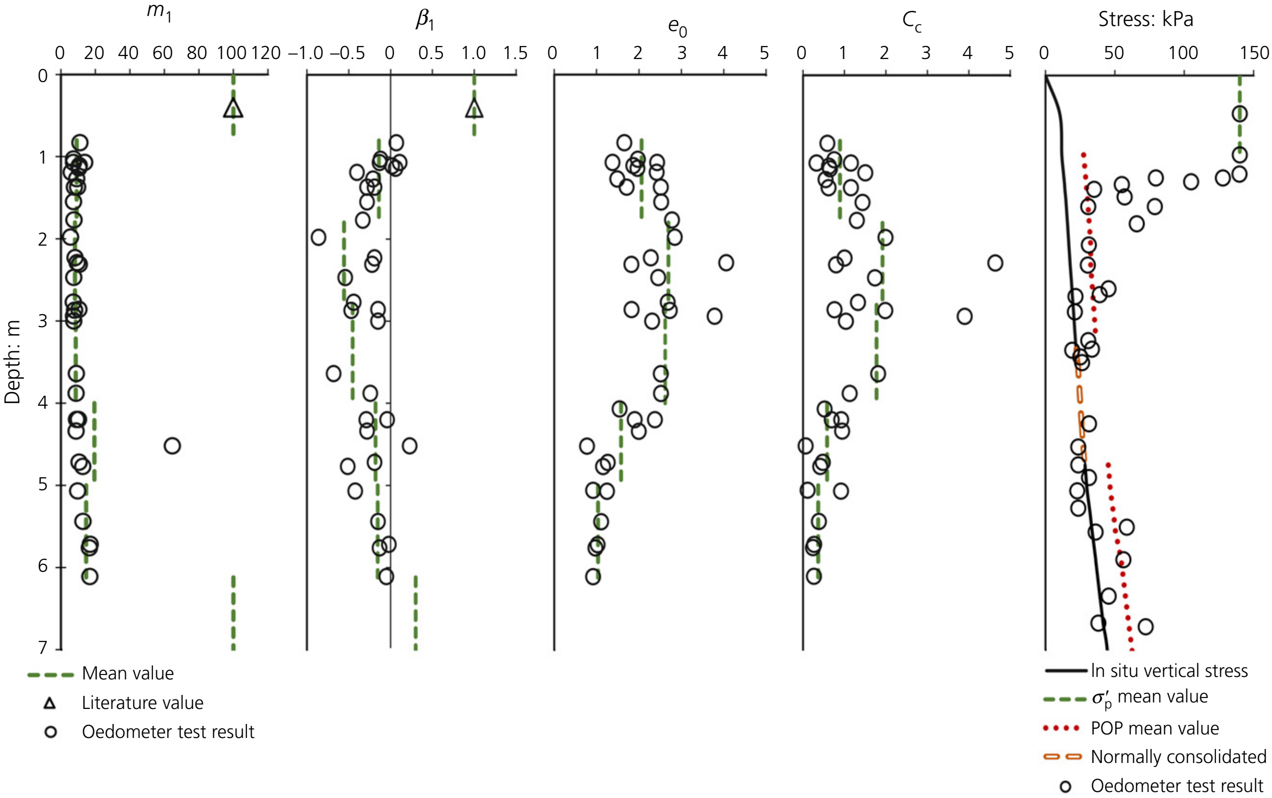

Figure 4. Compressibility properties of the Östersundom test embankment

4. Settlement analyses and selection of characteristic values

4.1 Input parameters for the TMM

The CIM is the conventional method used for the computation of the settlements of fine-grained soils. This method assumes a linear dependence of the vertical strains on the vertical effective stress on a logarithmic scale. However, a non-linear stress–strain relationship on a semi-logarithmic scale has been observed for soft Finnish clays of post-glacial origin. This has led to the use of more suitable methods to allow a better representation of the stress–strain behaviours of sensitive Finnish clays. One of these methods is the TMM (Janbu, 1963), which has been widely used in Finland. In this method, the vertical strain in the normal compression state ε_{NC} is defined as shown in Equation 9 (Helenelund, 1974; Janbu, 1963). The general equation for estimating settlements according to the CIM can be found elsewhere (Terzaghi *et al.*, 1996).

9a. $$\varepsilon_{NC} = \frac{1}{m_1\beta_1}\left[\left(\frac{\sigma'_{vo} + \Delta\sigma_v}{\sigma_{ref}}\right)^{\beta_1} - \left(\frac{\sigma'_p}{\sigma_{ref}}\right)^{\beta_1}\right] \qquad \beta_1 \neq 0$$

9b. $$\varepsilon_{NC} = \frac{1}{m_1}\ln\left(\frac{\sigma'_{vo} + \Delta\sigma_v}{\sigma_{ref}}\right) \qquad \beta_1 = 0$$

where m_1 is the modulus number, β_1 is the stress exponent, σ'_{vo} is the effective vertical in situ stress, σ_{ref} is the reference stress (=100 kPa) and $\Delta\sigma_v$ is the increment in vertical stress. The parameters m_1 and β_1 can be derived from oedometer tests by using curve fitting. The characteristic values were calculated for σ'_p and fitting parameters m_1 and β_1. As m_1 and β_1 are not physical compressibility parameters, it is difficult to find the indicative values of V_x describing the variability of such parameters. Therefore, a 20–70% V_x range was used for m_1 and β_1, which was considered wide enough to include their actual variability. The selected range for m_1 and β_1 corresponds to an indicative value for the compressibility modulus (Schneider, 1997). On the other hand, V_x within the range 10–35% was assigned for input parameter σ'_p, based on the value reported by Uzielli *et al.* (2006). The characteristic values of these input parameters (m_1, β_1 and σ'_p) were chosen as the 95% confidence levels in the mean value. The unit weight was not treated as an uncertain parameter because it normally exhibits negligible variability within homogeneous Finnish clay layers (Löfman and Korkiala-Tanttu, 2019). The load of the embankment was also left unmodified.

As the characteristic values are conservative estimates of the real soil properties, these uncertain parameters were decreased or increased with respect to the best estimate, depending on which alternative yielded a more conservative settlement. Thus, the σ'_p values were reduced with respect to their mean value using a negative sign in Equations 5 and 8 to obtain settlements with higher and more conservative values than the results of the best-estimate analyses. Likewise, decreasing values of m_1 and β_1 led to higher values of the settlement and, as such, these parameters were modified accordingly to estimate their characteristic values.

When the characteristic values of σ'_p are calculated for the TMM, m_1 should be adjusted according to the real preconsolidation pressure that was obtained through the oedometer tests. This is due to the stress dependency of m_1 and the uniqueness of the stress–strain behaviour of the soil layer. A further explanation of the stress dependency of m_1 and the equation for the modulus number adjustment that was used in this paper can be found elsewhere (Länsivaara, 2003).

The total settlement analyses were grouped into two main analyses: (*a*) best-estimate analyses using the sample mean of each layer and (*b*) conservative analyses with the characteristic values of the selected input parameters based on Equations 5, 6 and 8. For the conservative analyses, several sub-analyses were carried out because three different values within the indicative ranges of V_x were used. The results of analyses (*a*) and (*b*) were compared by means of a ratio between the settlements calculated using characteristic values (S_{Xk}) and the best-estimate settlements (S_{best}). This ratio provides an indication of the safety margin within the total settlement measurements. S_{best} is analogous to the probabilistic mean value of total settlement (μ_S), whereas S_{Xk} should correspond to an upper percentile of the total settlement distribution.

4.2 Input parameters for the CIM

For the settlement analyses carried out using the CIM, the compression index (C_c), swelling index (C_s) and preconsolidation pressure (σ'_p) were treated as uncertain parameters, whose characteristic values were calculated. The characteristic values of C_c and C_s were obtained as values higher than the mean from the test results, yielding more conservative settlements. On the contrary, σ'_p was modified to a value below the mean value from the test results, as described in Section 4.1.

Indicative values of V_x for the compressibility indices were used based on the ranges reported by Uzielli *et al.* (2006), where a V_x range of 10–37% was reported for C_c. The same range was applied when modifying the mean value of C_s. The initial void ratio (e_0) was not modified. The results of the analyses of the total settlements calculated using the CIM were grouped in the same way as the results of the analyses for the TMM, using Equations 5 and 8 and varying the values of V_x.

5. Reliability analyses

5.1 Target safety margin

The settlement ratios (S_{Xk}/S_{best}) from the deterministic analyses provide a measure of the 'safety margin' from the characteristic values with respect to the best estimates. The safety margin should increase when the probability of unsatisfactory performance decreases (p_f). On the contrary, p_f usually increases with increasing uncertainty in the soil properties, resulting in a narrower safety margin. However, high V_x values of the input parameters do not imply high V_x values in response (Phoon, 2017). Hence, probabilistic analyses were performed to estimate the uncertainty in the total settlement. Reliability analyses will provide a more robust measure of the safety margin in the case of the non-linear stress–strain relationship.

The reliability index β is an alternative measure of the safety margin. This index describes the distance (in standard deviation) between the mean value and a critical value of the performance function G. According to EN 1990:2002 (CEN, 2002), structures designed according to this code should aim to achieve a minimum β for both ULS and SLS. However, target β values for the SLS verification of geotechnical structures have not been defined in the code. Phoon *et al.* (2003) argued that for the SLS design of shallow foundations subjected to uplift, the suitable target β would be 2.6 (corresponding to an annual probability ($p_{f,1y}$) of 5×10^{-3} and a 50-year probability ($p_{f,50y}$) of 0.20). Naghibi *et al.* (2014) concluded that, for a single pile, a typical maximum lifetime probability ($p_{f,lifetime}$) of excessive settlement would lie somewhere between 10^{-1} and 10^{-4} ($\beta \approx 1.3$–3.7). For SLS verification of ground-supported embankments, to the best of the authors' knowledge, a suitable target β has not been estimated. Generally, lower target β values are acceptable if the associated consequences are less severe (e.g. Phoon *et al.*, 2003). Regarding the SLS design of road embankments on fine-grained soil, the risks are mostly economical, such as related to additional repair works. Furthermore, settlements accumulate slowly in clays, meaning that the target $p_{f,1y}$ cannot be used when assessing the reliability in the total settlement estimate. All in all, the range of available target β_{SLS} values is wide and, as such, no single value can be selected for the study cases.

Lastly, it should be noted that this paper does not define the performance function G in a traditional manner (i.e. $G = 0$ being the limit state and $G < 0$ meaning unsatisfactory performance). Instead, G is defined as the total settlement because this study investigated the safety margin related to S_{Xk} rather than the reliability of the geotechnical design.

5.2 MCSs

The MCS method was used as a probabilistic approach to generate a histogram of the total settlement for each case study. 50 000 simulation runs were performed for each obtained distribution. The simulation was performed using the Latin hypercube sampling incorporated in @Risk software (Palisade, 2016). Perfect spatial correlation within a soil layer was assumed; in other words, one soil property per soil layer was simulated in each iteration. The total settlement was then calculated by summing through 100 mm thick calculation layers. The correlation between the soil properties was not taken into account for better replication of the deterministic analyses. The stress dependency of the modulus number was considered, as described in Section 4.1.

The variance reduction factor Γ^2 was set to 0 because the applied definition of the characteristic value corresponds to a case of full averaging. Thus, the total uncertainty for the probabilistic analyses was defined as in Equation 2.

6. Results

6.1 Safety margin from the characteristic values of the input parameters

Figures 5(a) and 5(b) show the settlement ratios (S_{Xk}/S_{best}) from the deterministic analyses for the October draft and the Schneider equation, respectively. The calculated settlement values (S_{best} and S_{Xk}) can be found elsewhere (López Ramirez, 2020).

It can be seen in Figures 5(a) and 5(b) that, for a higher CoV (V_x), the ratios increased, indicating a larger safety margin. The ratios for the Haarajoki case differ from those in the rest of the cases, with the difference becoming more significant with higher values of the CoV. This difference may be due to the strong non-linearities of the compression range of the clay layers at the Haarajoki site. The large negative values of β_1 obtained for this case reflect these non-linearities and the strong de-structuration of the clay after the yield point. The average value of β_1 throughout the Haarajoki soft clay deposit was −0.82; for the other cases, the average of the deposits varied from −0.20 to −0.32. Cautious assessment of the negative values of β_1 implies that the negative values of this parameter become larger, making the non-linearities and de-structuration more substantial.

In the case of the Murro test embankment, in Figure 5(a) the high ratio obtained for the highest value of V_x when using Equation 5 and the TMM was caused by the large statistical uncertainty in two of the five subsoil layers for which only two samples were available. Apart from the large statistical uncertainty, the safety margin seems to have been further amplified by the non-linear stress–strain behaviour of the soil because the settlement ratio showed a minor change in the highest values of V_x when the CIM was used. On the other hand, the ratio for the Murro case was lower and more consistent with the other cases for all the values of V_x when Schneider's formula was used. This is because this formula always subtracts 0.5 times the standard deviation from the mean, regardless of the number of samples.

6.2 Safety margin according to reliability analyses

An example of the MCS result for the Haarajoki case (CIM; $V_x = 24\%$; $\Gamma^2 = 0$) is shown in Figure 6. The statistics of the total settlement (mean μ_S, standard deviation ξ_S and the percentiles)

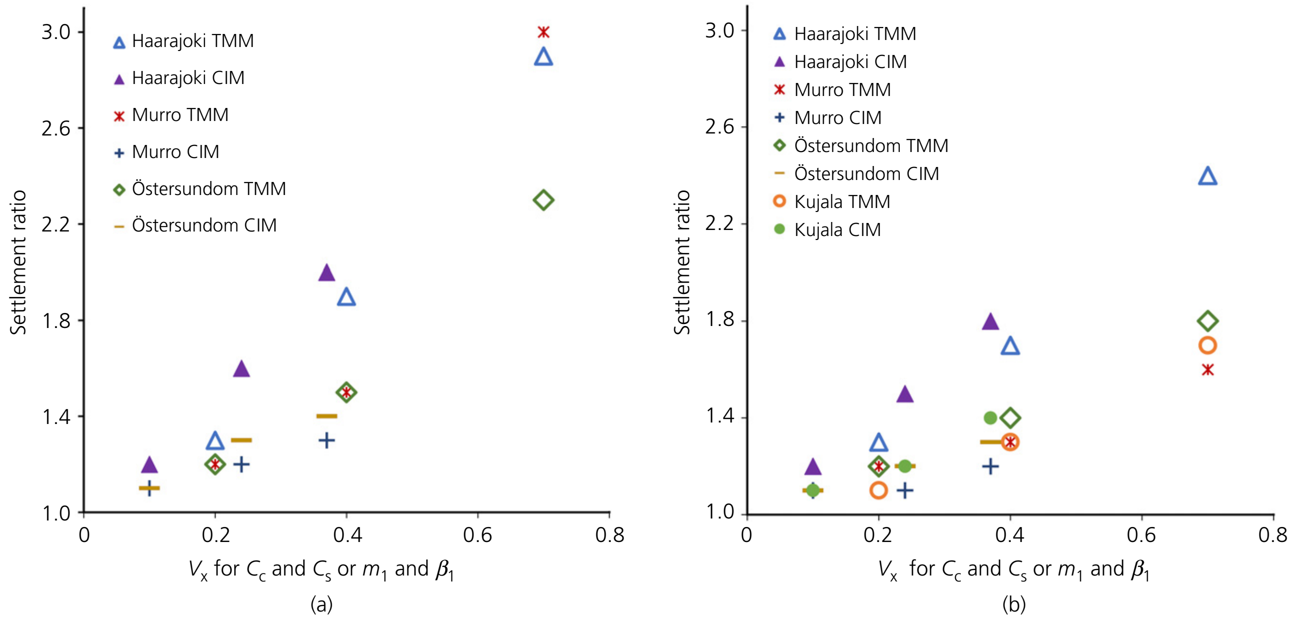

Figure 5. Safety margin for settlement calculations using deterministic analyses exceedance for (a) October draft equation and (b) Schneider's equation

were defined from the sample of 50 000 simulations. Besides the MCS results, the corresponding deterministic best estimate (S_{best}) and the conservative settlement calculated with the characteristic values ($S_{Xk;\text{October}}$) are also shown in the figure.

The safety margin was estimated by computing the probability of the total settlement being larger than the conservative value calculated with the characteristic values. This probability of exceedance $p_e = P(S > S_{Xk})$ can be treated as analogous to the probability of unsatisfactory performance of failure. In other words, $1 - p_e$ is the probability of settlements remaining smaller than $S_{Xk;\text{October}}$. In the MCSs, p_e was estimated from the simulated values by means of the cumulative probability $P(S > S_{Xk})$. More specifically, the probability was calculated from the MCS sample's empirical cumulative distribution (instead of from a fitted distribution). Hence, if $p_e = 0$, not a single simulated value of settlement was higher than S_{Xk}. To simplify, when $N = 50\ 000$, the smallest p_e that can be calculated is $1/50\ 000 = 2 \times 10^{-5}$. This probability is much smaller than the lower limit of p_e in the SLS suggested by Naghibi *et al.* (2014) (i.e. 1×10^{-4}) for the lifetime of a single pile. Hence, considering the suggested range of p_e in the SLS, the selected number of simulation runs ($N = 50\ 000$) in the Latin hypercube MCS can be considered large enough to evaluate the safety margins. However, it should be noted that the required N is also affected by the desired confidence level and the selected sampling technique (e.g. Baecher and Christian, 2003).

Another measure of the safety margin can be acquired by assessing the position of S_{Xk} relative to the simulated values. For example, if $S_{Xk;\text{October}}$ is more than three standard deviations away from the mean settlement, it can be considered rather rare (i.e. conservative) because 99.7% of all the settlement values are contained within ±3 standard deviations in the case of normal distribution. This distance, referred to herein as Distance β, was selected as another measure to represent the safety margin provided by the characteristic values. Hence, Distance β is the distance of S_{Xk} from mean μ_S expressed as number of standard deviations ξ_S. Therefore, Distance β is analogous to the reliability index β. A higher β indicates a lower probability of settlement exceedance p_e.

It should be noted that the best estimate S_{best} was slightly smaller than mean settlement value of simulations μ_S. Such a difference is to be expected when comparing the results of deterministic and probabilistic analyses, especially if the distribution of the performance function is non-symmetric (e.g. non-normal) and marked with a large deviation about the mean. Figure S1 in the online supplementary material shows that the difference between S_{best} and μ_S was greatest for high settlement values. It was also found that the difference was greatest when V_x was 70%.

Figures 7(a) and 7(b) show that the probability $P(S > S_{Xk})$ was in the scale of $p_e = 0–1.3 \times 10^{-2}$ when full averaging was assumed ($\Gamma^2 = 0$). The CIM seems to be more robust to varying V_x; that is, the probability remains consistent with increasing V_x. The highest scatter in probability was observed with V_x = 70%, using the TMM. In such cases, the shape of the settlement histogram is marked with a considerable positive

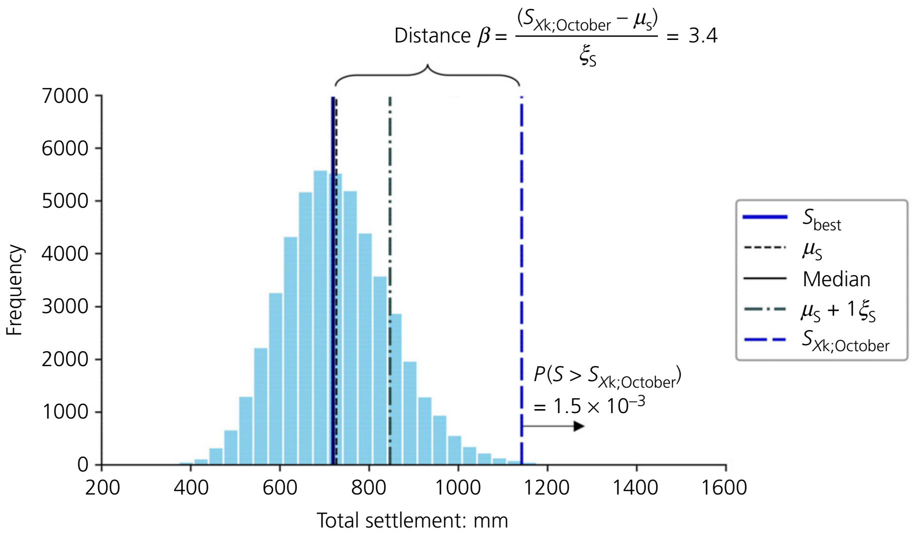

Figure 6. Total settlement for the Haarajoki case (CIM, medium uncertainty)

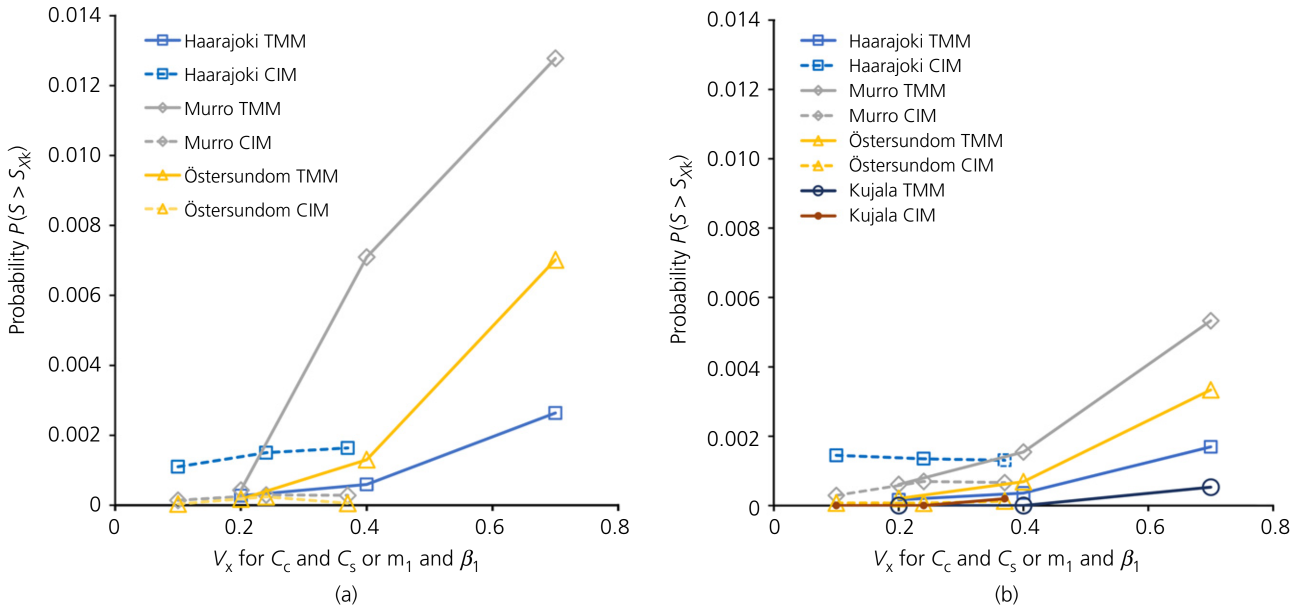

Figure 7. Probability of settlement exceedance for (a) October draft equation and (b) and Schneider's equation; $\Gamma^{-2} = 0$

skew, meaning that very high settlement values were more probable compared to the more normally distributed settlement cases (i.e. the cases corresponding to a low V_x and the CIM). Consequently, the apparent safety margin induced by S_{Xk} was also smaller for these cases with a positive skew (i.e. the TMM with a high V_x). Figure 8 shows that, in terms of Distance β, the highest conservatism (safety margin) is related to the Kujala case (Distance $\beta > 5$). The patterns of Distance β with respect to V_x are not fully identical to the patterns of probability p_e because Distance β does not depend on the occurrence of very high settlements as much as probability p_e does in the case of settlement distributions with positive skews.

The aim of this study was to derive a method for measurement of the safety margin provided by characteristic values using reliability analysis. Based on the suitable target β reported in the literature, the safety margins obtained in this paper should be large enough. It was also observed that the safety margin, defined as settlement ratio (Figures 5(a) and 5(b)), increases almost linearly with increasing uncertainty in the settlement

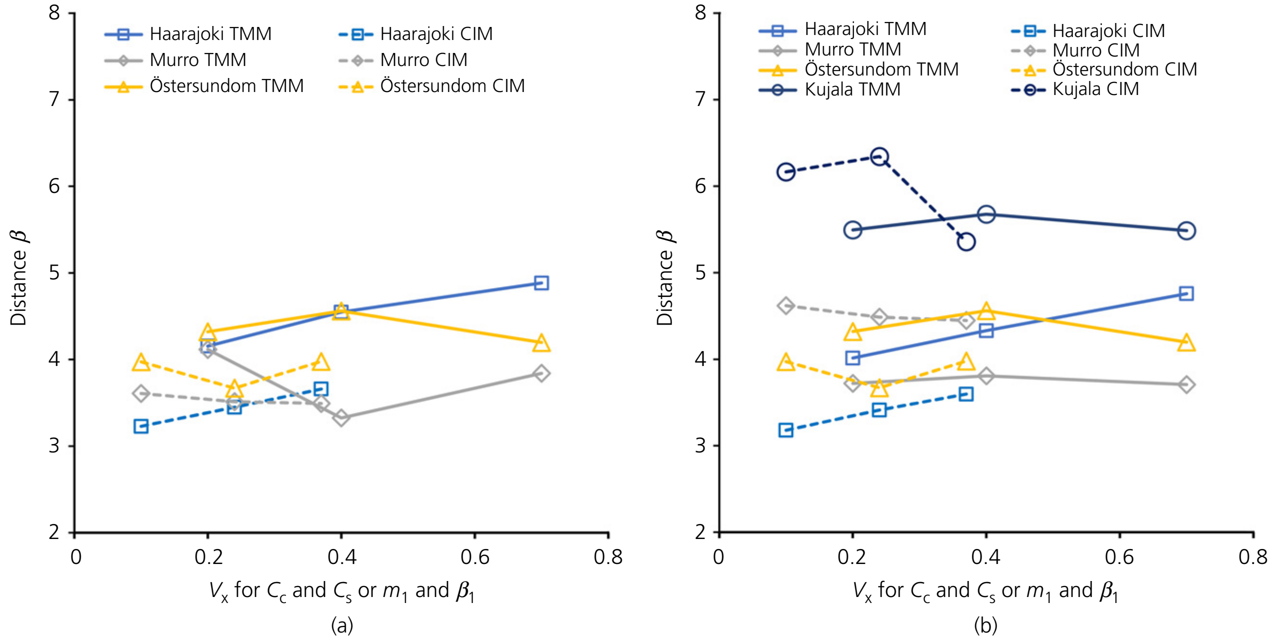

Figure 8. Distance β for (a) October draft equation and (b) and Schneider's equation; $\Gamma^2 = 0$

(i.e. the CoV of the simulated settlement; see Figure S2 in the online supplementary material). In other words, using a statistically defined characteristic value provides a safety margin that is dependent on the degree of uncertainty in the settlement.

On the other hand, the provided safety margin may be too conservative. In order to assess the adequacy of such a safety margin, the allowed probability of unsatisfactory performance p_f in the SLS verification of embankments should be established. The main aspect to consider when establishing a target value for p_f is the period over which SLS verification is to be carried out. In this study, the probability of settlement exceedance p_e was estimated for the ultimate total settlements accumulated during a period that depended on the consolidation properties and drainage conditions. However, in a design situation involving SLS verification, settlement estimates are usually calculated for specific operational periods of the structure. These periods may or may not coincide with the period over which total settlement is attained. This means that, in order to establish whether the safety margin provided by S_{Xk} is adequate for an actual design, the relevant reference period has to be considered.

The results shown in this paper are especially relevant for soft clays exhibiting non-linear stress–strain behaviour. The results show that the system' response is sensitive to varying values of V_x when the TMM is used. This increases the possibility of obtaining highly conservative values of settlements, particularly if there is no previous information on the distributions of input parameters. Such is the case for the input parameters of the TMM. Moreover, m_1 and β_1, the preconsolidation pressure parameters, are correlated to some degree, and the current version of EC7 (CEN, 2004) does not offer any guidance on how characteristic values should be applied in this case. In this paper, the cross-correlation between m_1 and the preconsolidation pressure was taken into account by means of modulus number adjustment, as discussed in Section 4.1.

The characteristic values, as described in EC7 (CEN, 2004), were applied to the soil parameters that exhibited inherent variability. Therefore, the embankment load (which depends on the unit weight of the fill material) was left unmodified for both the deterministic and probabilistic analyses. However, the embankment load may be modelled as an uncertain variable in probabilistic SLS design. For instance, Spross and Larsson (2021) used a CoV of 5% for the embankment load in their illustrative example related to a design framework for surcharges on vertical drains.

It should be noted that since the stress exponent β_1 is closer to a curve shape parameter than a stiffness parameter, keeping it constant is a viable alternative that should be investigated further. In this paper, β_1 was treated as an uncertain soil property due to its notable variability within soil layers. However, MCSs could be combined with a sensitivity study to investigate how much the variability in β_1 affects the uncertainty in the settlement. If the system response is insensitive to changes in a given parameter, the parameter can be treated as a deterministic constant.

Another relevant consideration was the size of the failure domain within a layered soil. The soil deposit beneath each embankment was divided into several layers, which were approximated as homogeneous. This assumption led to consideration of the full-averaging effect, whose failure domain was the entire soil layer. Thus, a mean value of each variable ground property was estimated as the governing parameter of the soil layer. This is the most common interpretation of E7's provision regarding the selection of the zone governing a limit state. It is noteworthy that if no averaging effect is considered in probabilistic analyses, the CoV calculated from total variance defined in Equation 2 will be greater. As a result, the standard deviation of the settlement will be larger and hence the safety margin related to $S_{X\mathrm{k}}$ will be smaller (i.e. greater probability p_e and smaller Distance β). However, this will no longer be valid if the non-existing averaging effect case ($\Gamma^2 = 1$) is considered simultaneously in both probabilistic and deterministic analyses.

Lastly, a normal distribution was assumed in both the probabilistic and deterministic results. If the total CoV is large (e.g. due to the assumption of no averaging effect), one may need to truncate the normal distributions to avoid reversion of their signs. Alternatively, the results can be replicated using a log-normal distribution to avoid the occurrence of negative values. For instance, Schneider and Schneider (2013) suggested assuming a log-normal distribution if the total CoV is equal to or greater than 30%. However, one should then apply the alternative form of Equation 5 when the ground property is considered following a log-normal distribution. According to Lacasse and Nadim (1996) and Löfman and Korkiala-Tanttu (2021b), both normal and log-normal distributions may be suitable for compressibility properties such as the overconsolidation ratio and the compression index.

7. Conclusions

The analyses and comparisons made in this study allowed a quantification of the safety margin obtained by statistically defined characteristic values using the probabilistic approach. The main objective was to assess the safety margin related to settlement calculations for embankments on clay. The results show that a sufficient safety margin is provided by the equation included in the draft version of the next-generation EC7, given that the full-averaging effect is assumed. The settlement value obtained from the characteristic values ($S_{X\mathrm{k}}$) was in the scale of 3–5 standard deviations apart from the mean settlement, indicating a rather large safety margin. For the SLS design of ground-supported infrastructures, such safety margins may be overly conservative.

Moreover, even though the safety margin is sufficient, the wide range of observed safety margins implies that a consistent safety margin cannot be attained by means of the characteristic value. The observed inconsistency may be related to the varying levels of non-linear compressibility and the degree of consolidation. Hence, a probabilistic approach may offer better robustness for the SLS design of embankments on clays. However, it is evident that designers need to assess suitable values of the CoV for the soil parameters, regardless of the design approach used (statistically defined characteristic value or probabilistic approach).

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