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Non-Hermitian skin effect and point-gap topology in photonic crystals

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Abstract: We show that two-dimensional non-Hermitian photonic crystals made of lossy material can exhibit nontrivial point gap topology and non-Hermitian skin effect. Our work shows that these effects can be exhibited in systems beyond tight-binding models. **OCIS codes:** (160.5298) Photonic crystals (350.4238) Nanophotonics and photonic crystals

1. Introduction

In recent years, there has been growing interest in exploring the topological band theory of non-Hermitian topological photonics systems [1]. Photonic platforms are of interest as non-Hermitian features such as gain and loss are quite common and useful for applications. In this work, we focus on point-gap topology. An energy band is said to have a point gap at a reference energy if the reference energy does not lie on the energy band. For non-Hermitian systems, the complex energy bands can form loops which can give rise to a non-zero winding number with respect to a reference energy. The presence of nontrivial point-gap topology can give rise to the non-Hermitian skin effect when the medium is truncated. States become localized on the edge of the finite structure and the energies of these localised states lie inside the loop formed by the complex band structure. Previous studies have mainly been limited to tight-binding models. In this work, we show that the non-Hermitian skin effect can be exhibited in 2D reciprocal and nonreciprocal photonic crystals [2].

2. Results and discussion

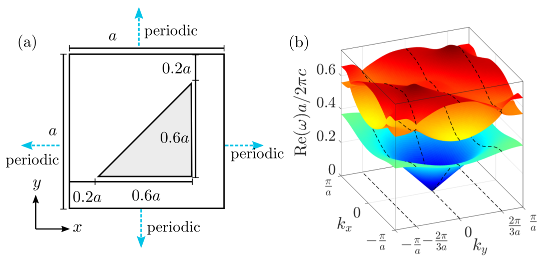

Fig. 1. (a) Unit cell of photonic crystal where the lattice periodicity is a. Here, the background is air and the triangle has refractive index $n_{\text{tri}} = 2 - 1i$. (b) The lowest three bands of the real part of the eigenfrequences in the first Brillouin zone for the photonic crystal geometry in (a). The dashed black lines are at fixed $k_y = \frac{-2\pi}{3a}$, 0 and $\frac{2\pi}{3a}$.

2.1. *Reciprocity and point-gap topology*

The non-Hermitian point-gap topology for a single energy band in a 1D system corresponds to a closed loop traced by the band structure $\omega(k)$ in the complex frequency plane. The point-gap topology is characterised by the winding number W. W gives the number of times the complex eigenfrequency winds around a reference energy $\omega_0 \in \mathbb{C}$ as k varies across the first Brillouin zone. In order to achieve a nontrivial winding in a 1D system, a necessary condition is to ensure that $\omega(k) \neq \omega(-k)$. If $\omega(k) = \omega(-k)$ for all k, the band structure always has trivial winding (a loop with no enclosed area), as the band necessarily retraces itself in the complex plane as k varies from one end of the Brillouin zone to the other. Moreover, the system must break reciprocity, since in a reciprocal system $\omega(k) = \omega(-k)$. For a 2D system with a band structure defined by $\omega(k_x, k_y)$, we can assign a similar winding number for any loop inside the first Brillouin zone. In this work, for simplicity, we only consider a square lattice with a lattice constant a. At any given k_y, the straight path from $\left(k_x = -\frac{\pi}{a}, k_y\right)$ to $(k_x = \frac{\pi}{a}, k_y)$ forms a closed loop due to the periodic property of the first Brillouin zone. Therefore, one can define a k_y -dependent winding number

$$W\left(k_y\right) = \int_{-\pi/a}^{\pi/a} \frac{dk_x}{2\pi} \partial_{k_x} \arg\left(\omega(k_x, k_y) - \omega_0\right). \tag{1}$$

For a fixed k_y away from $k_y = 0$ or $k_y = \pi/a$, it is possible for $W(k_y)$ to be nonzero even for a reciprocal system so long as there is no symmetry that maps $k_x \to -k_x$. This means that it is possible to achieve nontrivial winding in a reciprocal 2D system.

2.2. *Numerical results*

We consider a lossy reciprocal photonic crystal with unit cell as shown in Fig. 1(a). When we consider an infinite photonic crystal which is periodic in both x and y, we can calculate the full 2D band structure in the first Brillouin zone as shown in Fig. 1(b). We can trace along the real and imaginary eigenfrequencies along fixed k_y as indicated by the black dashed lines in Fig. 1(b). This complex band structure for $k_y = \frac{-2\pi}{3a}$,0 and $k_y = \frac{2\pi}{3a}$ is plotted as solid black lines in Fig. 2 in the lower panels. The complex band structure forms loops. We now consider a finite stripe geometry which is periodic in the y direction and has finite 20 unit cells along the x direction. We can plot the eigenvalues at $k_y = \frac{-2\pi}{3a}$,0 and $k_y = \frac{2\pi}{3a}$. We do this in the lower panels using red-blue dots in the lower panels of Fig. 2, where the color indicates the mean position of the electric field distribution along the finite stripe geometry. We see that these eigenvalues lie within non-trivial point gap loops. We plot the normalized electric field eigenstates corresponding to a few of these points in panels A-I. We see that the eigenstates from the finite stripe geometry that lie within the band loops are localized, which shows that this system exhibits the non-Hermitian skin effect. One can also find the non-Hermitian skin effect in a lossy nonreciprocal 2D photonic crystal structure [2].

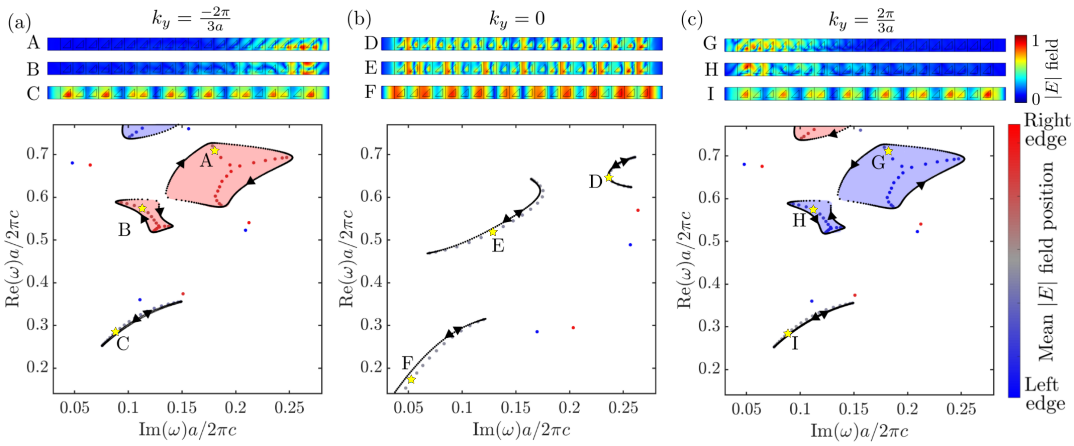

Fig. 2. The lower part of each panel shows the complex band structure of the infinite 2D photonic crystal in solid black, where arrows give the direction of winding. The red and blue dots are the eigenfrequencies of the finite stripe geometry. (a), (b) and (c) correspond to $k_y = \frac{-2\pi}{3a}$,0 and $k_y = \frac{2\pi}{3a}$ respectively and A-I are the normalized electric field eigenstates which show the skin effect.

3. Conclusion

In conclusion, we have shown that system in 1D must break reciprocity in order to achieve non-trivial point-gap topology. This condition is not true for 2D systems, if one considers a winding number traced along k_x in the first Brillouin zone at a fixed k_y. We numerically demonstrate this effect in a lossy 2D reciprocal and nonrecriprocal photonic crystal and show that we can find non-trivial point-gap topology and non-Hermitian skin effect in a continuous media. Our work shows that the non-Hermitian skin effect goes beyond tight-binding models which is more promising for future applications.

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