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JASA ARTICLE

Effect of inharmonicity on pitch perception and subjective tuning of piano tones

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ABSTRACT:
The consensus in piano tuning philosophy explains the stretched tuning scale by the inharmonicity of piano strings. This study aimed to examine how variable inharmonicity influences the result of the piano tuning process, compare the tuning curves of aurally tuned pianos with the curves derived from subjective octave enlargement experiments, and evaluate whether the pitches of inharmonic or harmonic versions of the same tone are perceived differently. In addition, the influence of strings of other piano keys on the measured inharmonicity of a single piano string was investigated. The inharmonicity of all individual strings was measured on a Steinway D grand piano. Variable inharmonicity was implemented by additive synthesis with frequency-adjusted sinusoidal partials. Fifteen piano tuners and 18 orchestra musicians participated in the experiments. The results indicate that the inharmonic piano tones produced a keyboard tuning curve similar to the Railsback curve and differed significantly from the harmonic counterpart. The inharmonic tuning curve was reminiscent of the subjective octave enlargement curve. Inharmonic tone pitches were perceived to be higher than harmonic tones up to *C*♯7. The covibrating strings of the other keys did not exhibit any meaningful effect on the measured inharmonicity of a single string of the played key.
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I. INTRODUCTION

In well-tuned upward and grand pianos, the tuning is stretched without exception. This means that intervals, including octaves, are not the same size, and they enlarge as one moves away from the tuning reference key. In previous studies, the inharmonicity of a single piano string has been reported to be an explanation for such a stretched tuning (Fletcher, 1964; Giordano, 2015; Lattard, 1993; Martin and Ward, 1961; Rasch and Heetvelt, 1985; Rigaud *et al.*, 2013; Schuck and Young, 1943; Young, 1952). However, a similar type of stretched tuning has also been observed in stringed and wind instruments with strictly harmonic overtone series (Jaatinen *et al.*, 2019; Sundberg and Lindqvist, 1973) (see also Fig. 1). In wind and string instruments, stretched tuning is a natural consequence of the octave enlargement phenomenon. When one adds stretched octaves to the other, the result is a stretched tuning. In earlier psychoacoustic experiments, the octave enlargement phenomenon has been observed to be the largest in the high register for sinusoidal and complex tones (Dobbins and Cuddy, 1982; Hartmann, 1993; Jaatinen *et al.*, 2019; Sundberg and Lindqvist, 1973; Terhardt, 1971; Walliser, 1969; Ward, 1954).

Although a piano is called an equal-tempered instrument, the truth is something else (see Fig. 1). Actually, in a normally tuned grand piano, there are hardly any equally tempered semitones present. A tuning center or a reference tone, e.g., A4 or C4, may be the only tone that is located on the mathematically equally tempered scale. All other tones are tuned so that deviation from the mathematical scale increases with distance from the tuning center. However, in the tempering octave (see Sec. II B), the scale may be close to an equal-tempered scale. In upright and grand pianos, the shape of the stretching curve is slightly different in the low register, whereas in the high register they resemble each other more closely (see Fig. 1).

The harmonic structure of piano tones and tuning has been under interest for many decades. In the 1930s, Railsback invented a chromatic stroboscope (Railsback, 1937), which allowed one to properly measure the fundamental frequency and harmonic structure of a presented complex tone. With this machine, he measured the tunings of 16 upright pianos (Railsback, 1938b). In Fig. 1 his data are visualized as a curve, concurrently with a Steinway D (Steinway & Sons, New York, NY) grand piano template of the Tunelab tuning software (Tunelab, Real-Time Specialties Inc., Hopkins, MN). In the lower register, the Railsback curve stretches more downward, whereas in the upper register it is similar to the Tunelab curve.

The frequently cited studies of Railsback about stretched tuning have probably disappeared,[1] and only short reviews have survived (Railsback, 1938a,b). However, Schuck and Young (1943) obviously had the original Railsback tuning data available when they wrote their paper on the vibrations

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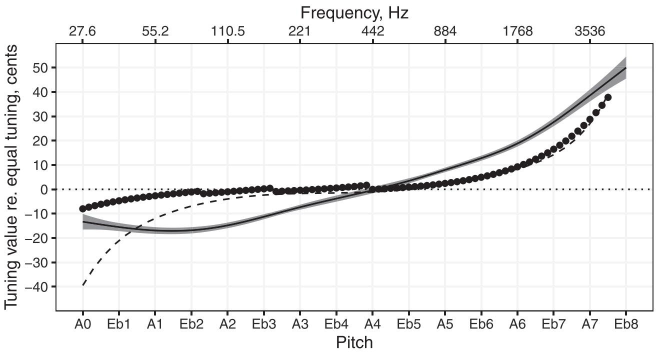

FIG. 1. Black dots = Tunelab tuning software's (Real-Time Specialties, Inc., Hopkins, MN) Steinway D grand piano curve; Solid line, an average stretching curve of orchestra instruments (Jaatinen *et al.*, 2019) (shaded area = 95% confidential interval); dashed line, the Railsback curve for upright piano, average reproduced from Schuck and Young (1943); small dots, equal tempered tuning (*x* axis).

of the piano string. In that paper, the existence of an inharmonicity of a piano string was reported for the first time. They also suggested that inharmonicity is a reason for the stretched tuning of a piano.

In this work, we approach inharmonicity by four different methods: (a) by investigating how the presence of inharmonicity influences piano tuning in practice (Sec. III C 1), (b) by comparing the stretching curves of aurally tuned pianos with the curves derived from subjective octave enlargement experiments (Sec. III C 2), (c) by comparing whether the pitch of inharmonic or harmonic (harmonized) versions of the same tone is perceived differently (Sec. III C 3), and (d) by elaborating what is the influence of other strings in the spectral harmonicity of a single piano string (Sec. III C 4).

The purpose of our study was to investigate whether professional tuners tune pianos differently if the strings (piano tones) are inharmonic or harmonic. If there is no difference or if a stretched scale also occurs with strictly harmonic strings, inharmonicity cannot be the only reason for stretched tuning in the piano.

This article contributes to the existing literature by studying the effect of varied inharmonicity on tuning curves. To the authors' knowledge, similar types of tuning experiments with different inharmonicity factors or the results from a controlled full-keyboard tuning experiment, which enables a free comparison of tones like with real pianos, have not been published earlier.

II. BACKGROUND

In this section, we review the principles of inharmonicity, musical perceptual effects, and practical methodology in piano tuning.

A. Piano string inharmonicity and musical perceptual effects

The structure of a piano string varies in different registers. The lower strings are wound and thick (max. diameter of a wound string is about 8.5 mm), while the highest strings are plain and thin (min. diameter about 0.85 mm). Therefore, the stiffness and then, by implication, the assumed degree of inharmonicity should be different in every string. Jorgensen (1991) explains the upward movement of harmonics using the theory of "nodes." According to this theory, the strings of a piano are unusually thick compared to their length and the nodes shorten the length of a vibrating string. That is, for example, in a string whose width is 1 mm, a node is 1 mm long, and every finite node shortens a vibrating part of a string by the respective 1 mm. In the second harmonic (half-length), the vibrating part is shortened by one node (the width of the string), and the frequency of the harmonic moves upward. This phenomenon replicates in the upper harmonics and accumulates with every added node. In the literature, the degree of inharmonicity is typically described by the inharmonicity coefficient B, where values above zero indicate the general enlargement of harmonics from integer multiples of the fundamental frequency (Fletcher, 1964; Rauhala *et al.*, 2007).

As mentioned earlier, the inharmonicity of strings influences the tuning of pianos, achieving stretching on a tuned scale. However, the tuning of a stretched scale is not linear. Closer to the tuning reference, the deviation is smaller, and in the highest and lowest registers it increases more exponentially (see Fig. 1). This can be explained by the commensurately smaller inharmonicity of the middle register's plain strings, the position of the tempering octave in the aural tuning method (see Sec. II B) or by musical context. According to Rasch and Heetvelt (1985), the wound strings are less inharmonic than the plain strings. Among these string type groups, the inharmonicity is not constant. In wound strings, the inharmonicity increases when going downward. In the plain strings, the inharmonicity is lowest in the middle register and increases towards the highest register.

In addition to the physical features of the piano strings, the conventions of writing music may also play a role. In the low register, classical composers avoid using thirds or fourths due to perceived roughness. Instead, octave couplings and fifths are preferable, and such intervals tend to be stretched more (Rosner, 1999). In the middle register, close to the tuning reference, the chords and the melodic elements dominate (Adler, 2002). According to Terhardt and Zick (1975), in a musical context, when chords and melodics are close together (typically in the middle register), equal-tempered or even contracted intonation may be preferred. In the high register, octave couplings and melodic figures are commonly used musical elements. In such cases, in conjunction with chords with a large ambitus, stretched tuning is the most agreeable.

B. The phases of the aural piano tuning method

A modern grand piano usually has 88 keys and, on average, 220 strings. Obligatory accessories in aural piano tuning are a good quality tuning hammer, a tuning felt, and tuning wedges. In addition, an electronic tuning machine or a tuning fork is necessary to tune a reference tone (typically C4 or A4 by convention).

At the beginning of a tuning, a chosen tempering octave (usually F3-F4 or A3-A4) in the middle range will be tempered so that each half-step has the size of the selected tempering system (i.e., equal size in the equal-tempered tuning). In tuning of the tempering octave, a tuner listens to the first common harmonic of the tones in a musical interval, the so-called harmonic unison. This unison should beat in different ways depending on the type of interval, range, and inharmonicity of the strings. A professional piano tuner is experienced in listening to beats between observed harmonics and is concurrently capable of ignoring irrelevant beats. For example, in a major third F3-A3, the first common harmonic is A5. In the case of a major third F3-A3, the theoretical beating frequency is 6.95 Hz. In the example interval F3-A3 with tuning reference A4 of 440 Hz, this can be calculated by subtracting the frequency of the fourth harmonic (A5) of the A3 note (880 Hz) from the frequency of the fifth harmonic (A5) of the mathematically equal tempered F3 note (873.05 Hz). Since the inharmonicity of particular strings has not been considered in calculating the theoretical beating frequency, instrument-specific charts are necessary. There are several tempering schemes in use, but the result of beating all intervals is equivalent. At present, the most commonly used tuning intervals are major thirds and minor sixths, while fourths and fifths serve as control intervals.

After tempering, notes of the tempered octave will be transferred to the lower and upper range using octaves and control intervals, e.g., 15ths, 16ths, 17ths, and 18ths plus octaves, if wider intervals are needed.

The sizes of the octaves vary in different ranges depending on the inharmonicity of the instrument. Typically, in the middle or high range, the listened-to harmonic unison is 2:1 or 4:2. In the low range, 4:2, 6:3, 8:4, or even 10:5 are used. For example, 6:3 means that the harmonic beatless unison is aimed to achieve between the sixth harmonic of the lower tone and the third harmonic of the upper tone.

Finally, tones on several parallel strings (bi- and trichords) will be tuned in unison.

Traditionally, piano tuning has been a subjective task performed exclusively manually. However, mechanical approaches and algorithms have recently been proposed to automate the process (e.g., Tuovinen *et al.*, 2019; Zhou *et al.*, 2021).

III. METHODS

The applied audio stimuli used in the tuning and listening experiments were based on measurements on a Steinway D model grand piano and were analyzed and processed with specific algorithms to obtain parametric synthesis of piano tones with the desired amount of inharmonicity. The following sections present the steps for each part of the method of the participants, piano recording, stimulus processing, experimental design, and data analysis.

We also considered, when recording and measuring a single piano string installed on a grand piano, it may be demanding to eliminate the influence of other strings in the recorded spectrum. Although other strings are normally attenuated by dampers, they may still resonate to some extent with measured (played) strings. Therefore, it may be difficult to determine which of the upper harmonics belongs to the played string and which may come from the other strings. In the case of a normally tuned piano, other strings are already stretch-tuned. That is, if other strings vibrate with a measured string, their fundamental frequencies or upward-stretched upper partials may produce an inharmonicity to the spectrum of the measured string. In that case, if other strings are not completely damped in a recording session, ostensible inharmonicity may be observed even in harmonically pure strings. For further exploration of this hypothesis, an additional experiment was conducted with the analysis of recorded A notes from a Steinway C model grand piano, where we carefully damped all non-measured strings as well as all string bridges with thick felt. This experiment is described in Sec. III C 4.

A. Participants

Because the piano tuning experiment was similar to a normal piano tuning procedure, it was necessary to involve only trained piano tuners as participants. Fifteen professional tuners with different backgrounds (instrument, experience, age) completed the tuning experiments, as well as pairwise and octave listening tests [12 males, 3 females, mean age 41.9 years, standard deviation (SD) 11.0]. As a control group, 18 professional musicians participated in the pairwise and octave experiments (11 males, 7 females, mean age 44.3 years, SD 9.1). They all represent the highest professional level and are employed in major symphony orchestras in Helsinki area, Finland.

B. Stimuli

1. Piano tone recording

For the stimuli used in our tuning and listening experiments, we recorded a Steinway & Sons model D concert grand piano with high precision: two AKG C414 B-ULS mics (AKG Acoustics GmbH, Vienna, Austria) and a Zoom H6 recorder (Zoom Inc., Tokyo, Japan), sample rate $f_s = 96$ kHz, 24-bit stereo. The microphones were placed under the open lid at 0.3 m distance from the strings. The recorded instrument is one of the soloist grand pianos of the Helsinki Music Center, Finland. The instruments are stored in a dedicated storage room with an approximately 1.3 s reverberation time, and they are tuned and serviced at least once a week. Other grand pianos in the same space had closed lids and dust covers.

The 88 keys were recorded in a middle-range dynamic (mezzoforte) played by a trained pianist (author, J.P.). If there was more than one string per key, the best sounding string was subjectively chosen in accordance with a professional piano tuner, while the others were carefully damped. That is, only one string (*una corda*) in each key was present in the recorded signals. Additionally, keys without dampers in the upper register were muted with felt cloth and wedges to ensure that only one string vibrated freely. The recorded tones were kept depressed as long as they naturally vibrated (4–20 s) without a sustain pedal. All recorded tones (88 keys with three repetitions) were subjectively evaluated by the authors, and the best representation of each key was selected for subsequent analysis and synthesis.

We applied a time-frequency analysis and synthesis of the recorded piano tones through the estimation of partial frequencies and their respective amplitude decay after tone onset.

2. Analysis and synthesis of piano tones

The stimulus type for the tuning experiment was modeled as an additive sum of sinusoidal partials, each of which has time-variant amplitudes according to the recorded tones. In addition, the attack transient from the onset of the recorded tone was added to the synthesized tone. This process is described in the following sections.

3. Estimation of partial frequencies

To construct realistic piano tones with desired harmonic properties, the partial overtone series was estimated on the basis of the recorded tones. This process is illustrated in Fig. 2. First, the spectral peak frequencies for each sampled tone $x[n]$ were estimated from a quasi-stationary part of the tone sustain. This region was defined from 0.1 s from the tone onset (sample index n_1) until the instant of the absolute normalized amplitude falling below 0.05 (-26 dB) (sample index n_2). Furthermore, for long-sustain bass register tones, the end of the analysis window was limited to 8 s [see Fig. 2(a)]. The selected segment $x[n_1 \ldots n_2]$ was windowed with Tukey window w with a tapering width of 0.5 and transformed into the frequency domain $X[k]$ with K-point Fourier transform, where K is the next power of two from $n_2 - n_1$. The fundamental frequency $\hat{f}_0$ of the tone was estimated from the magnitude spectrum as the maximum amplitude within half-semitone from the nominal fundamental frequency for the respective tone, *re.* $A4 = 441.5$ Hz. Frequencies of overtones $m = 0 \ldots M$ were calculated iteratively as a series where the next spectral peak $\hat{f}_{m+1}$ lies within the interval $[\hat{f}_m + 0.5\hat{f}_0; \hat{f}_m + 1.5\hat{f}_0]$ [see Fig. 2(b)]. This approach takes into account the different degrees of inharmonicity while preserving the order of partials. For low-register tones, the inharmonicity was distinctly observed, as shown in Fig. 2(c).

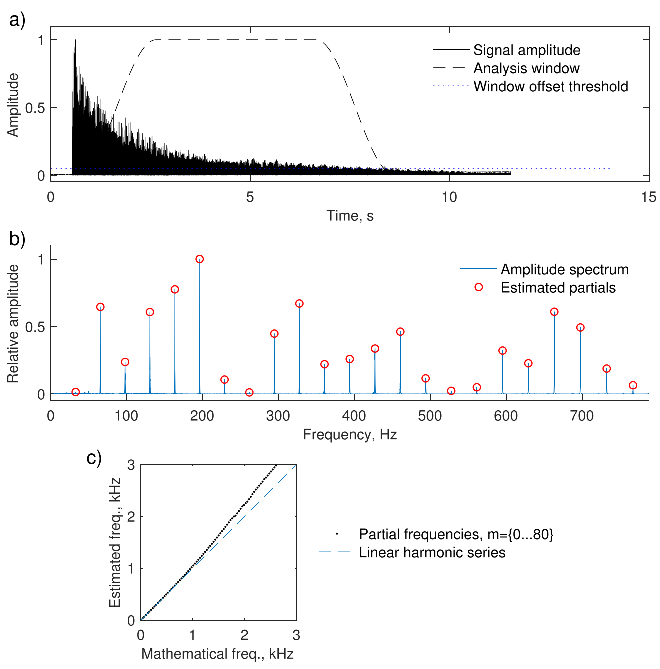

FIG. 2. (Color online) Example of quasi-stationary amplitude spectrum analysis for estimating partial frequencies for tone C1 ($f_0 = 32.8$ Hz). (a) Relative absolute time-amplitude curve with respective windowing function. (b) Amplitude spectrum and estimated partial frequencies. (c) Comparison of overtone series with respect to pure harmonic series.

The analysis of partials provided a flexible starting point for the synthesis of piano notes. With accurate data on partial frequencies and their respective order, the entire overtone series was corrected for equal tempering f_0^{ref} *re.* A4 = 441.5 Hz. Furthermore, the degree of inharmonicity could be parametrically adjusted with respect to the series of harmonic overtones as a harmonization factor b. Thus, the subsequent analysis uses partial frequencies f_m:

$$f_m = \frac{\hat{f}_m - b(\hat{f}_m - m\hat{f}_0)}{\hat{f}_0 / f_0^{ref}}, \tag{1}$$

where $\hat{f}_m$ denotes the estimated partial frequency of the original recording. Two levels of harmonization factor $b = 0$ (original inharmonic) and $b = 1$ (harmonic) were used to synthesize the stimuli for the experiments.

4. Time-varying partial amplitude estimation and synthesis

The varying amplitudes for the partials were calculated from the short-time Fourier transform (STFT) of the recorded tone,

$$X[l, \omega] = \sum_{n=-\infty}^{\infty} x[n] w^{\{p\}}[l - n] e^{-j\omega l}, \tag{2}$$

where $w^{\{p\}}$ was a Hanning window of length $p = 2^{14}$, $2^{13}, 2^{12}, 2^{11}$, or 2^{10} for tones in octaves 0, 1–2, 3–4, 5–6, and 7–8, respectively, and 75% overlap. The index of the STFT frame is denoted by l. STFT frequency bins are denoted by ω as opposed to the natural frequency values (f). Different lengths of the analysis window were chosen to attain an improved frequency resolution to separate individual partials in low-register tones while having an adequate temporal resolution in high-register tones.

The synthesized complex tone was constructed in the time domain using the overlap-add method for each frame and a single partial:

$$x_m^{\{l\}}[n] = A[m, l] \sin(2\pi f_m (n + pl/4)/f_s + \phi_m), \tag{3}$$

where $n = [pl/4 \ldots p(l+1)/4]$, and $A[m, l]$ denotes the amplitude of the partial number m in frame l. f_s stands for the signal sampling rate, and the term $pl/4$ represents the phase increment reciprocal to 75% analysis window overlap. The initial phase angles ϕ_m were randomized as in Anderson and Strong (2005).

The frequency resolution in STFT analysis and synthesis suffers inherently from a limited window length, which in turn is required for acceptable temporal resolution. For example, the length of the analysis window of $w^{\{p\}} = 4096$ samples for tone D3 ($f_0 = 147.3$ Hz) produces a relatively sparse frequency resolution of 23.4 Hz. For determining the matrix $A[m, l]$ for each partial m, the energy from the discrete frequency domain bins must be assigned to the respective partial. Due to the limited frequency resolution, some partials coincide accurately with specific frequency-domain bins, whereas other partial frequencies aligned between bins yield broader spectral peaks. Together with the inharmonicity of partial frequencies, these effects suggest against a constant, equidistant grouping of frequency-domain bins.

To this end, we used a mapping approach in which the spectrum was grouped into peak regions by first calculating a p-point Fourier transform of the quasi-stationary signal window [see Fig. 2(a)]. Then, the entire amplitude spectrum was divided into regions based on local maxima nearest to the original partial frequencies $\hat{f}_m$. Finally, the frequency domain bin groups were assigned the nearest partial m based on the distance of the mean frequency of each group. Therefore, each partial refers to a set of frequency bins Ω_m, whose amplitudes are accumulated for the corresponding sinusoidal as

$$A[m, l] = \sum_{\omega \in \Omega_m} |X[l, \omega]|. \tag{4}$$

An example of this procedure is given in Fig. 3, where the local peak regions are first identified and then assigned to the nearest original partial frequency.

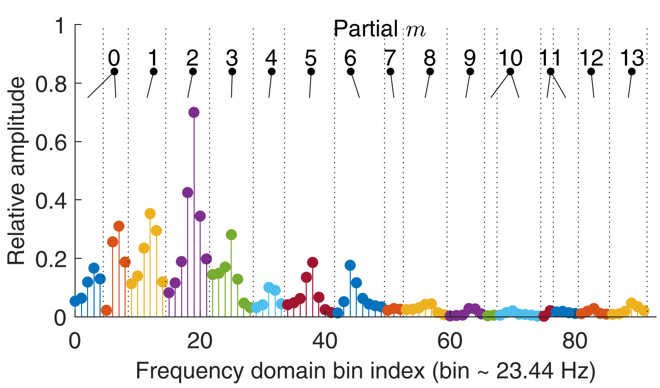

FIG. 3. (Color online) Example of the local spectral peak region mapping for calculating the time-frequency amplitude matrix $A[m, l]$ in the synthesis for tone D3 ($f_0 = 147.3$ Hz). Regions separated with vertical lines and by colors are formed around local spectral peaks. The regions nearest to each partial frequency are finally mapped to the synthesized partial frequencies. In the example, energy from two nearest peak regions is assigned to partials 0, 10, and 11. The varying number of frequency-domain bins in each region comes from the approach of finding local maxima.

The partials were synthesized on the list of frequencies and frames with a schema corresponding to the STFT analysis and synthesis. As any spectral energy in the local peak regions will manifest itself as energy in the corresponding sinusoidal partial, the synthesis amplitudes were treated with noise thresholding in the silent part of the recording to prevent the random noise spectrum to be synthesized as tonal artifacts.

The method described above produces high spectral fidelity through time-domain sinusoidal synthesis at the expense of temporal resolution due to relatively long STFT analysis windows. Therefore, tone onset transients lose the sharp attack quality, and the spectrum of the hammer impact sound cannot be accurately reproduced by harmonic partial sinusoids. To overcome this issue, the attack transient was sampled from the original recording with a smooth window and combined with the optimized phase of the synthesized part as follows: The recording was first resampled by the ratio of detected and adjusted f_0 for the correct fundamental frequency. The onset instant was determined with a –6 dB threshold of the absolute signal maximum, and the window extended 0.1 s before and p samples after the onset. The optimal delay for the transient part was chosen samplewise by the maximum of cross correlation between the transient window and the synthesized part within one fundamental period from the original STFT frame index of tone onset. In essence, this provides a good estimate for overlapping, considering the randomized phase in synthesis. The end of the onset window was tapered to half of the analysis window $w^{\{p\}}$ and joined to the synthesized signal with the respective complement window.

To summarize, the generation of piano tones consisted of an additive synthesis with frequency-adjusted sinusoid partials, each modulated by the respective local spectral energy, and finally combined with the original onset transient for a more realistic note attack.

5. Evaluation of harmonization method

The result of the synthesis of the stimulus was objectively evaluated by the overall effect of the inharmonicity of the piano tone on the two levels of harmonicity parameters. The inharmonicity coefficient B of the final stimulus signals was estimated using the algorithm proposed by Rauhala *et al.* (2007). The result is shown in Fig. 4 for each piano tone: Below A4, the difference in inharmonicity is on the order of magnitude of 102–103. Above A4, the results follow a similar trend. However, the reliability of inharmonicity estimation suffers from the decreasing number of overtones present in the signal. It should be noted that the inharmonicity could be better estimated for harmonized tones in the high register. Estimates of the inharmonicity coefficient for missing data points exceeded the upper limit of visualization. Thus, they were considered false estimates by the method. Rauhala *et al.* have provided an evaluation and comparison of their algorithm only for tones up to G3. Below this, the current synthesized inharmonic tones closely

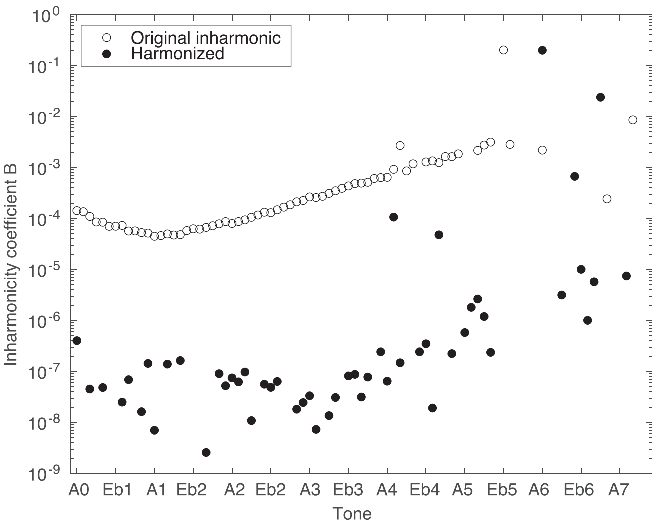

FIG. 4. Comparison of inharmonicity coefficient B, estimated for final synthesized piano tone stimuli with original inharmonic (harmonization factor $b = 0$) and fully harmonized ($b = 1$) spectra. The reliability of inharmonicity coefficient estimation (Rauhala *et al.*, 2007) becomes degraded above A5.

follow the typical inharmonicity reported by the referenced authors. In summary, the evaluation of the inharmonicity shows that the harmonized tones have substantially lower inharmonicity than those in the original spectra. Furthermore, the synthesized tones with the original spectrum retain a natural degree of inharmonic behavior.

C. Experimental setups

All experiments were conducted with piano tones synthesized with the above methods, and they were presented to the participants with a Macbook Pro computer (Apple Inc., Cupertino, CA), a Zoom H6 (Zoom Corporation, Tokyo, Japan) as an external audio card, AKG K550 headphones, magnitude response 12 Hz–28 kHz (AKG Acoustics, Vienna, Austria) and Max 8 software (Cycling '74, San Francisco, CA). The sampling frequency was 48 kHz and the bit depth was 24 bits. All tones were initially tuned according to the reference tone $A4 = 441.5$ Hz.

1. Tuning experiment setup

Data on piano tuning were gathered by an experiment in which professional piano tuners could conduct a complete tuning of a virtual piano by adjusting each key separately. The test included two complete tunings: one for inharmonic piano tones and the other for harmonized piano tones. Bi- or tri-chords were not used, so a single sampled tone represented a single key (una corda).

The experiment setup consisted of a Nektar Impact LX88+ MIDI keyboard (Nektar Technology, Inc., San Francisco, CA) and a Shuttle Pro V2 (Contour Design A/S, Ballerup, Denmark) rotary controller connected to a Macbook Pro computer that runs the experiment logic implemented in the Max 8 environment. The subject could select one key at a time on the MIDI keyboard to be tuned using the continuous rotary knob, while all other keys could be played in their last adjusted tuning. Sample playback was arranged with three identical monophonic samplers, which contained all 88 synthesized piano tones with the chosen harmonization factor. Note commands for the currently adjustable key were routed to the first sampler, whereas the two other samplers were dedicated for all other keys. The tuning was controlled in one-cent resolution using the pitch-bend algorithm in MaxMSP 8. Adjusted tuning values were stored in memory and recalled upon the respective MIDI keyboard press.

The initial tuning was randomized between ± 10 cents for each key for each subject. Tone A4 was selected as the tuning reference at 0 cents and was not adjustable at any stage. Subjects could complete the tuning in any preferred order, and their progress was shown on the screen by a virtual keyboard where the keys with accepted tuning were highlighted, as well as with a numeric indicator of the remaining untuned keys. The average sound pressure level (SPL) was 75 dB.

2. Subjective octave experiment setup

In the subjective octave experiment, participants listened to pairs of sound stimuli that alternated between lower- and higher-octave versions of the same signal type (inharmonic or harmonized piano tones). The task was to adjust all the $A-$ and $E\flat-$ based octaves and double-octaves to their preferable sizes. The reference tone $A4$ was the scale divider. In pairs $A4-A5, A5-A6, A6-A7, A3-A5, A4-A6, A5-A7$ and $E\flat4-E\flat5, E\flat5-E\flat6, E\flat6-E\flat7, E\flat3-E\flat5, E\flat4-E\flat6, E\flat5-E\flat7$, the lower tone was a reference tone and the upper was adjustable. In pairs $A4-A3, A3-A2, A2-A1, A1-A0, A5-A3, A4-A2, A3-A1, A2-A0$ and $E\flat4-E\flat3, E\flat3-E\flat2, E\flat2-E\flat1, E\flat5-E\flat3, E\flat4-E\flat2, E\flat3-E\flat1$, the upper tone was a reference and the lower one adjustable. That is, the pairs that crossed the reference tone were measured in both directions. The total number of pairs was 52. In pairs (with a total length of 3.0 s), 1.5 s long tones with the same stimulus type (inharmonic or harmonized piano tones) were presented successively and repeated until the participant was satisfied with the tuning. The amplitudes were equalized according to the C-weighted SPL. The average SPL at presentation was 75 dB. Initial tuning of the octave pairs was randomized between ± 20 cents. Pairs were presented in randomized order.

3. Pairwise comparison experiment setup

In the pairwise comparison experiment, participants compared inharmonic and harmonized versions of the same note and indicated which one's perceived pitch was higher. If the difference between pitches was indistinguishable, participants could indicate a guessed choice separately. Together, 44 successively presented piano tone pairs (harmonized - inharmonic) were included in this experiment from A0 to B7 by whole-tone steps $(A-B-C\sharp-E\flat-F-G-A)$. The tones in pairs were 1.5 s long and the order of the tones, as well as the order of the pairs, was randomized. The other presentation attributes were equal to the subjective octave experiment (Sec. III C 2).

4. Varyingly damped grand piano inharmonicity analysis

This experiment differed from the three subjective listening tests and piano tones were objectively analyzed using the inharmonicity estimation algorithm as in the evaluation of piano tone synthesis (Rauhala *et al.*, 2007). A similar set of recording equipment was used as in Sec. III B 1. However, only all A notes were recorded in three different damping setups. Only one string of each tone was recorded (una corda). Notes in all three conditions were played six times by the author, J.P., in mezzoforte dynamics. Then the author, J.J., chose aurally the most noiseless and natural-sounding alternatives for further analyses.

In the fully damped setup, all other strings except the played one were damped at least two points (between the bridges), on top of which all strings behind the bridges were also fully damped at both ends. We used 3 mm thick felt as damping material. In the half-damped setup, the difference from the first setup was that the strings behind the bridges were free. In the normal setup, we did not use any external damping material.

Two microphone positions, near-field and far-field, were experimented with for the most stable inharmonicity estimation. Recordings with the near-field microphone, about 10 cm distance from the midpoint of the measured string, were observed to provide more distinct frequency-domain peaks and thus stable analysis and synthesis results compared to an alternative far-field microphone just outside the open grand piano lid, about 1.5 m above the floor level. Therefore, near-field recordings were applied.

D. Statistical analysis methods

The primary interest in subjective tuning experiments lies in the resulting tuning curves and their differences. Since the shape of the curves is not limited to specific linear functions or polynomial orders, conventional statistical methods are not optimal for modeling the results.

Instead, we modeled the tuning curves using the Generalized Additive model (GAM) (Wood, 2017). In principle, GAM models the data as a combination of spline lines. The key advantage of GAM in the current context is its ability to model arbitrary shapes that would not be achieved by straightforward polynomial fits. GAM analysis does not require *a priori* information or the assumption of the polynomial degree but instead aims to minimize the degrees of freedom while retaining an accurate fit.

Similar to conventional linear models, GAM is sensitive to violations of homoscedasticity. For this reason, the data were weighted by the inverse of local variance per tone $1/(1 + var(\vec{x}))$, where $\vec{x}$ is the subset of the data for the respective combination of factors. Since the variance of the data were higher at the frequency extrema, the weighting of variance further widens the confidence intervals that are eventually associated with estimating the statistical differences between compared inharmonicity condition, subject grouping, or experiment type factors.

GAM analysis was performed in the R environment with the `mgcv` package. Among several options for spline types, current data with multiple simultaneous variables and interactions within them advocate for selecting thin plate splines, which is also the default smooth term in the `mgcv` package. The same principle of GAM analysis was applied in a previous study on orchestra instrument tuning curves (Jaatinen *et al.*, 2019).

1. Tuning experiment analysis

Data from the piano keyboard tuning experiment apply directly to GAM analysis. The fundamental frequency of the tone as an independent variable is used as the octave number relative to the reference tone A4. The tuning value for each piano key in cents *re.* equal tuning is the dependent variable. Here, the comparison of the analyzed experiment factors is achieved through the differences between the respective tuning curves and their confidence intervals. This approach has been used by Rose *et al.* (2012) and was suggested by the last author of the referred article to identify areas where the curves differ from each other at a significant level. In essence, in the curve regions where the confidence intervals do not overlap, the results are statistically significant at the chosen confidence level of 95%. Note that GAM would also enable an analysis of variance (ANOVA) for the smooth terms fitted to the tuning curves. However, in the case of comparing distinctly linear and strongly nonlinear curves, ANOVA was observed to yield misleading values if interpreted in the traditional manner, with low p-values referring to statistically significant differences. Therefore, the results are analyzed using the above approach.

2. Subjective octave analysis

Data from the subjective octave experiment included enlargement values for individual single- and double-octaves. Therefore, these data do not initially describe a tuning curve over the presented tones. To obtain results comparable to holistic whole-keyboard tuning, raw data requires preprocessing. This step was carried out with the same procedure as described earlier in Jaatinen *et al.* (2019).

In essence, the stretching curve is calculated on a reference point at tone A4. The single-octave interval tuning values above and below the center were stacked according to the additivity assumption. The process started near the tuning center and proceeded iteratively toward the low and high octave extrema. By repeating the procedure separately for the data for each combination of participant, A or E♭ tones, and two conditions of inharmonicity, the process produced distinct tuning curves for these independent variables. Separate tuning curves for the A and E♭ tones were combined into a single curve by adjusting the E♭ curve to have its reference point E♭4 at the average of the A3 and A4 tuning values (Jaatinen *et al.*, 2019).

The corresponding tuning curves for double-octave tone pairs were obtained with the same procedure added to the decision steps by splitting the cumulative stretching contribution over the reference point A4. This process is explained in detail in Jaatinen *et al*. (2019).

The formed tuning curves were analyzed and compared using GAM analysis. GAM also allows for a comparison between keyboard tuning and subjective octave listening experiment results.

3. Pairwise comparison analysis

The pairwise comparison yielded choice data between subjective judgments with inharmonic or harmonized tones perceived as higher. An additional option to indicate uncertain answers formed a three-alternative paradigm. Statistically, the data were analyzed using the subset with uncertain answers omitted. Therefore, it was obtained whether a specific tone was judged to be significantly higher in either harmonic condition with a two-sided proportional z-test. That is, the proportion of inharmonic or harmonized answers is compared to the no-difference level of 0.5.

4. Damped grand piano inharmonicity analysis

In the analysis, we compared the inharmonicity coefficients (B) of the piano tones. The same algorithm (Rauhala *et al*., 2007) was applied here as in the evaluation of the synthesis method.

The inharmonicity coefficients were analyzed with one-way ANOVA to compare the degree of the effect of damping within the same tones with the overall variance of inharmonicity between different tones.

IV. RESULTS

The following sections describe the results obtained from the experiments. The conditions compared, the types of experiments, and the resulting differences in the tuning curves are collected in Table I.

A. Results of the piano tuning experiment

The tuning curves with contrasting inharmonicity deviated significantly from each other. Due to the aural piano tuning method (Sec. II B), beats play an essential role when tuning simultaneously sounding tone pairs. That is, the inharmonicity of the strings must have a very significant influence on the outcome. As seen in Fig. 5, strictly harmonic (harmonized) tones have been tuned completely differently due to beats. It should also be mentioned that in the case of harmonized tones, the less experienced tuners (<6 years) had a more upward-bending curve in the highest register. In addition, the tuning curves of the harmonized tones of all the tuners deviated significantly from the null hypothesis (equal-tempered scale). This also indicates that the subjective octave-based method overrides the beat-based method in the high register.

Although the resulting curve from the inharmonic piano tuning experiment resembles the Railsback curve in the literature (Railsback, 1938a; Schuck and Young, 1943), the GAM analysis between these curves showed a prominent difference, particularly below A1. Since a specific confidence interval for the Railsback curve is not available in the literature for curve difference comparison (see Sec. III D 1), a provisional statistic using ANOVA produces a statistically significant difference [$F(1,16) = 16.05$ $p < 0.01$).

Although Shah and Välimäki (2020) suggested that 2:1 octaves were ranked as the most preferable in the highest register inharmonic piano tones, our data indicate a larger enlargement in the highest register.

B. Results of the subjective octave experiment

In the case of inharmonic tones (by the tuners), it also coincided accurately with the piano tuning curve (Fig. 6).

TABLE I. Comparison of subjective octave and tuning experiments and respective results.

Experiment[a]	Group 1[b]	N(G1)	Group 2[b]	N(G2)	Stimulus type[c]	Sig. different registers[d]	Figure
PT vs PT	T	15	-	-	H vs I	A0-C8	5
PT vs PT	T < 6	4	T > 6	11	H vs H	-	5
PT vs PT	T < 6	4	T > 6	11	I vs I	A0-Eb1	5
PT vs NH	T < 6	4	-	-	H	A0-A2, A6-C8	5
PT vs NH	T < 6	4	-	-	I	A0-C8	5
PT vs NH	T > 6	11	-	-	H	A0-A4, Eb5-C8	5
PT vs NH	T > 6	11	-	-	I	A0-C8	5
PT vs SO	T	15	-	-	H vs H	A0-Eb3, A4-C6, G6-C8	6
PT vs SO	T	15	-	-	I vs I	A4-A5, A6-C8	6
SO vs SO	M	18	T	15	combined H and I	B4-F5	7
PT vs SO	T	15	OE	36	I vs O(1)	Eb1-C8	8
SO vs SO	A	33	OE	36	I vs O(1-2)	A0-Eb7	9

[a]SO, subjective octave experiment; PT, piano tuning experiment; NH, null hypothesis (mathematically equal-tempered tuning).
[b]M, musicians; A, all participants in this study; T, all tuners; T < 6 = tuners with a tuning experience of 6 years or less; T > 6 = tuners with a tuning experience over 6 years; OE, all participants in the orchestra instrument experiment (Jaatinen *et al*., 2019).
[c]I, inharmonic piano tone; H, harmonized piano tone; O(1), grand average of orchestra instruments on 1-octave pairs; O(1-2), grand average of orchestra instruments on 1- and 2-octave pairs.
[d]Registers with significant difference, no overlap in confidence intervals.

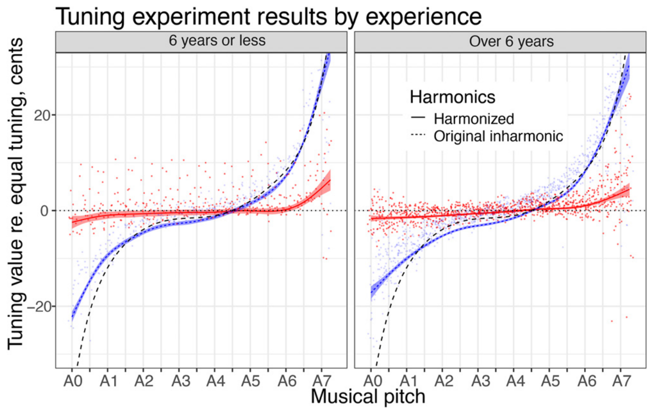

FIG. 5. (Color online) Results of tuning experiments. On the left side, tuners with 6 years or less experience in tuning. On the right side, corresponding results of experienced tuners. Red solid line/opaque dots, harmonized tones; blue dotted line, original inharmonic tones; dashed line, the Railsback curve. Dots indicate individual measured values, shown with a small horizontal jitter for readability.

Sundberg (1991, p. 104) suggested earlier that the stretched tuning of a piano is fortuitously similar to the general octave enlargement phenomenon. Our result verifies his observation.

For harmonized tones, the curves differed significantly. The keyboard tuning yielded a nearly linear result, whereas the curve obtained from octave tuning has characteristic stretching behavior in both the low and high registers.

The shape of the combined tuning curve of the subjective octave experiment conformed well to the Railsback curve, as seen in Fig. 7. There were no noticeable differences between the tuners and the musicians.

Figures 8 and 9 represent comparisons between our previous subjective octave study with orchestra instruments (Jaatinen *et al.*, 2019) and the tuning experiment with inharmonic piano tones (Fig. 8), or the grand average curve of the subjective octave experiments of this study (Fig. 9). Although there are differences in certain parts of the curves, the overall tendency is reminiscent. Differences between curves in the low register can be explained by the higher perceived pitch of inharmonic tones (see Sec. IV C) and the influence of the harmonic spectrum of a clarinet-type instrument on perceived pitch (see Jaatinen *et al.*, 2021). In the case of orchestra instruments, all sampled tones were strictly harmonic. There was also a difference between the experimental designs: In the orchestra instrument experiment, the upper tone was always adjustable, while in this study, depending on the case, either the lower or upper tone was controllable and the reference tone A4 was fixed. In subjective octave datasets, all one- and two-octave pairs in mezzoforte dynamics have been included in the GAM analysis.

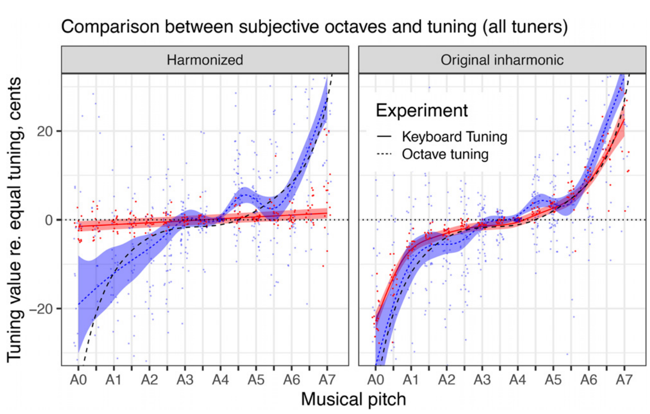

FIG. 6. (Color online) Comparison between results of subjective octave listening experiments and tuning experiments by all tuners. Solid red line/opaque dots, piano tuning; dotted blue line, subjective octave listening experiment; dashed separate line, the Railsback curve. Dots indicate individual measured values.

C. Results of the pairwise comparison experiment

According to our data (visualized in Fig. 10), in the low and middle registers (up to $C\sharp 7$), most of the inharmonic versions of the tones were perceived higher. In the highest register (above $C\sharp 7$), the uncertainty increased and no clear trend was observed.

Statistical tests indicated that A6 was the lowest tone, where the unadjusted p-value of the inharmonic piano tone judged higher exceeded 0.05. Above C $\sharp$ 7, neither of the harmonized conditions showed a nonsignificant proportion of the higher perceived pitch.

According to our hypothesis, the pitch of an inharmonic tone, especially in the low register, could be perceived as higher due to the elevated pitches of its individual harmonics (Järveläinen *et al.*, 2002). Jackson and Moore (2013) and Moore *et al.* (1985) suggest that the six lowest harmonics are the most important cues for the perception of the pitch of low-register tones. The lower the fundamental frequency, the weaker the influence of the lowest harmonics on the perceived pitch due to the insensitivity of the human ear (Suzuki and Takeshima, 2004). This may emphasize the influence of the upward-stretched higher harmonics and, at the same time, raise the perceived pitch of the lowest tones.

In the highest register, most of the harmonics were above the temporal pitch detection limit of the human auditory system, about 5 kHz (Bachem, 1948; Semal and Demany, 1994). Although alternative values have been proposed for the pitch detection limit of a human listener (Verschooten *et al.*, 2019), the influence of inharmonicity could no longer be a significant issue for accurate pitch detection.

D. Results of the damped string experiment

We evaluated the contribution of covibrating strings to the measured inharmonicity of a string under different damping conditions. However, there were no significant differences in inharmonicity coefficients (B) between conditions. A one-way ANOVA of independent variables (piano tones A0-A6) and dependent variables (damping conditions) yielded statistics $F(2, 18) < 0.01, p > 0.99$. Therefore, the variation in inharmonicity between the analyzed grand piano tones was substantially larger than the variation between damping conditions.

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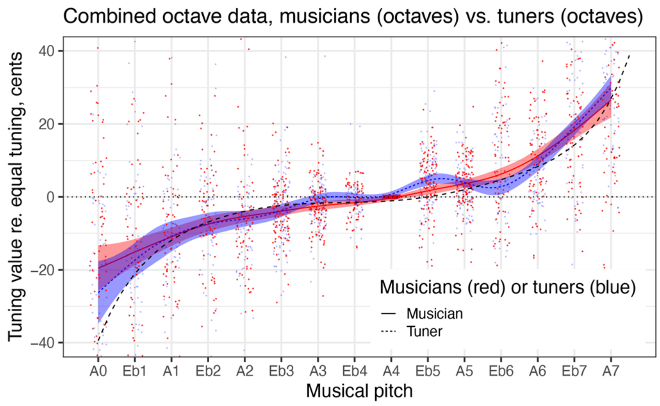

FIG. 7. (Color online) Results of subjective octave experiment. Solid red line/opaque dots, musicians; dotted blue line, piano tuners; dashed black line, the Railsback curve. Combined data of harmonic and inharmonic octave pairs. Dots indicate individual measured values.

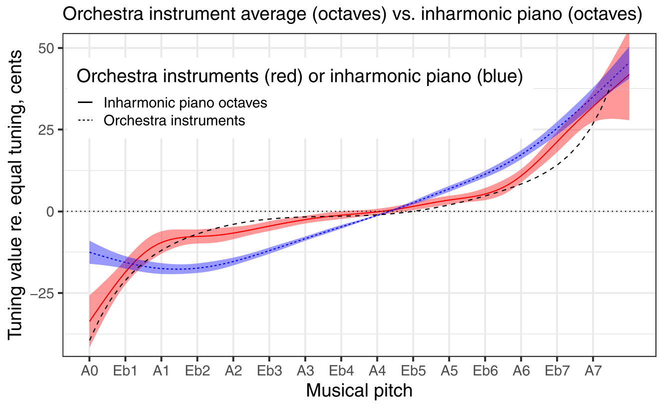

FIG. 9. (Color online) Comparison between subjective octave listening experiments. Solid red line, grand average of all participants (1- and 2-octaves) on the listening experiment by inharmonic piano tones; dotted blue line, grand average of orchestra instruments (1- and 2-octaves) (Jaatinen *et al.*, 2019); dashed separate line, the Railsback curve.

In summary, this result suggests that the estimated inharmonicity of the recorded piano string spectrum is not influenced by partially vibrating strings of adjacent piano keys or instrument structure.

V. DISCUSSION

Individual tuners may have differences in the tuning method due to background, school, or tuning philosophy. However, the resulting tuning curve is quite consistent, especially in the case of inharmonic tones.

The results of a range of experiments, in general, showed that the beating-based piano tuning method is highly dependable on the degree of inharmonicity. The tones themselves (harmonized or inharmonic) are not easily distinguishable by timbre, but in a pairwise comparison, the pitch of an inharmonic tone is mainly perceived as higher. In subjective octave experiments, the results of harmonized and inharmonic tones did not differ significantly from each other.

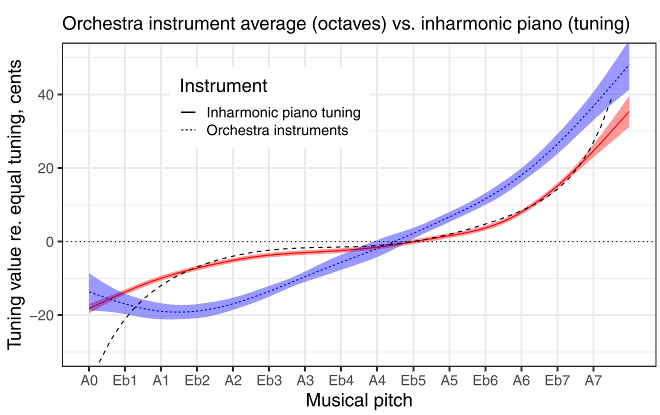

FIG. 8. (Color online) Comparison between tuning experiment by inharmonic piano tones and subjective octave listening experiment by orchestra instruments (Jaatinen *et al.*, 2019). Solid red line, grand average of all tuners; dotted blue line, grand average of orchestra instruments (1-octave; Jaatinen *et al.*, 2019); dashed separate line, the Railsback curve.

Since the aural tuning method is based on listening to beats of the common harmonics, it is evident that strings' inharmonicity strongly influences the shape of a tuning curve. However, in a high register, the deviation of the tuning result from the theoretically calculated 2:1 or 4:2 unison seems to be larger. In the highest tones, the frequencies of the compared harmonics are above the temporal pitch perception limit of a human auditory system. Therefore, a decision on the correct size of an octave interval must be made on the basis of the fundamental frequencies of the octave pairs or by using other judging features, like the brightness of a tone or the sharpness of an attack. That is, in those cases, the subjective octave listening method potentially overrides the beat listening method.

In the keyboard tuning experiment, most tuners reported that they were unable to distinguish the overall difference between inharmonic and harmonized tones by ear. This means that they did not know what type of tones they were tuning. That is, there were no perceptible differences in timbre or perceived pitch. This result was interesting, since the tuning curves for harmonized and inharmonic tones differed significantly from each other, although the degree of

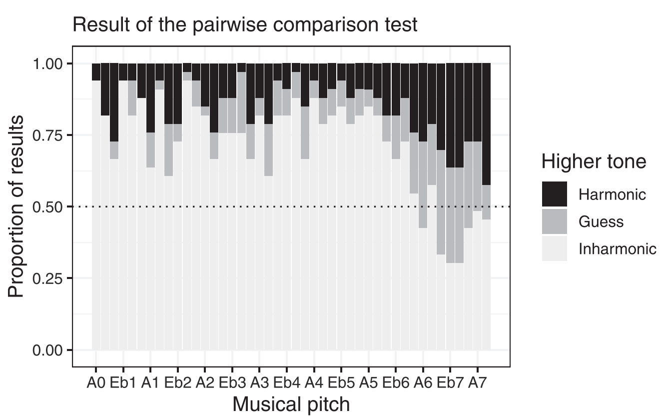

FIG. 10. Result of the pairwise comparison test with inharmonic and harmonized piano tones by relative proportion of answers.

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inharmonicity strongly influences the beating frequency of the compared common harmonic unisons (see Sec. II B).

In the subjective octave experiment, beating could not occur due to successive presentation of tones. Nevertheless, the inharmonic tones yielded a stretched shape, which closely follows that of the keyboard tuning experiment. Harmonized tones increased uncertainty, especially in the low register (see Fig. 6). Although some of the tuners did not have any performing musical background, the results of the subjective octave experiment were consistent with those of professional musicians (see Fig. 7).

The surprising contrast between tuning curve results from keyboard and octave tuning experiments with harmonized tones (see Fig. 6) raises the question of a learned effect among piano tuners. When evaluated simultaneously, harmonized tones produced a linear non-stretched tuning curve. However, when evaluated successively in an alternating sequence, the identical harmonized tones yielded a stretching behavior that closely represents the typical tuning curve of inharmonic tones. Without the simultaneous tuning reference from other tones and their harmonics, it could be theorized that the tuners assume that low- and high-register tones should be tuned further away from the middle register than the mathematical intervals.

The keyboard tuning experiment can be seen to represent a traditional setting for tuning evaluation. The overall similarity of the results from the octave experiment provides further evidence to support the assumption of the additive nature of individually adjusted octaves and the formation of a tuning curve over a wider range of pitches. Although the shapes of the curves between piano tones and orchestra instruments deviate significantly, the tendencies are parallel. Differences in the middle register can be mainly explained by methodological divergence. In the case of piano tones in the lowest register, the influence of inharmonicity can be regarded as indubitable.

Although our synthesized piano tones were generally experienced to sound quite natural, some participants expressed uncertainty in the pitch perception of the highest inharmonic and harmonized tones. The synthesized tones did not contain frequencies below the fundamental frequency or outside synthesized partials. In contrast, in a real piano tone, many audible noise components exist below and above the fundamental frequency (i.e., vibration of the soundboard and other strings, or mechanical noise from keys).

VI. CONCLUSIONS

The effect of piano tone inharmonicity on piano tuning result and pitch perception was studied with a series of experiments. In all tuning and listening experiments, the presented stimuli were based on synthesized piano tones.

Beats play a crucial part in tuning simultaneously sounding tone pairs due to the aural piano tuning approach. In other words, the stretch tuning must be significantly influenced by the inharmonicity of the strings. The tuning curves with different degrees of inharmonicity diverged greatly from each other. Although beats caused harmonized tones to be adjusted entirely differently, the tuning curves for the harmonized tones considerably departed from the equal-tempered scale. Although it is undeniably seen that the inharmonicity of strings is the main reason for the stretched tuning of piano-type instruments, the general octave stretching phenomenon also plays an important role in the tuning of the upper register. That is, the final tuning is a compromise between the beat-based tuning and the stretched subjective octaves. In the high register, the beat-based method is superseded by the subjective octave-based method.

The tuning curve produced with a subjective octave experiment and inharmonic tones matched closely the Railsback curve. With harmonized tones, the curve obtained through the same experiment setting displays typical stretching behavior in both the low- and high-registers, in contrast to the essentially linear result produced by keyboard tuning.

Although there are variances in some areas of the curves, the overall curve shapes are typical for stretched tuning and correspond to comparisons between our earlier subjective octave study using orchestra instruments and the tuning experiment using inharmonic piano tones. Furthermore, variations in experimental approaches could explain the observed differences, especially in the middle register.

Several participants reported that it was difficult to say whether a single tone was harmonized or inharmonic. There were no noticeable differences between the timbres, but the pitches of the inharmonic tones were perceived to be higher in the low and middle registers in the pairwise comparison experiment. In the highest register, this trend in pitch perception became considerably weaker.

We can theorize that the increased pitches of an inharmonic tone's individual harmonics may cause the pitch of the inharmonic tone to be heard as higher, particularly in the lowest register (see Sec. IV C). Due to the sound level insensitivity of the human ear in lower frequencies, the lower the fundamental frequency, the lowest harmonics have less impact on the perceived pitch. That is, the contribution of the upward stretched higher harmonics may emphasize the raising of the perceived pitch of the lowest tones. Since most harmonics in the highest register tones were above the temporal pitch detection limit of the human auditory system, inharmonicity could no longer be a substantial factor in correcting pitch identification.

The difference in inharmonicity between the investigated grand piano tones was significantly greater than the difference in damping settings. This finding indicates that the instrument's structure or partially co-vibrating strings from adjacent piano keys do not affect the estimated inharmonicity of the recorded piano string spectrum, and it is not necessary to completely dampen the other strings during recording a harmonic structure of a single piano string. Hence, such effects can be disregarded in the perceptual properties in piano tone inharmonicity and tuning results.

The current results and experiments offer several avenues for future research. For example, it could be interesting to evaluate the amount of inharmonicity which is still acceptable and enables the possibility of using the beating-based tuning method. Another topic for deeper scrutiny could be to measure all the used harmonic unisons and analyze the preferable alternatives in the extremities (see Sec. II B). This could be achieved on the basis of the data from the current experiments.

From a methodological aspect, the keyboard tuning experiment setup could be employed relatively directly for educational purposes and training of new piano tuners.

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[1]The first author requested the original studies of Railsback from Eastern Illinois University and they were reported as disappeared.

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