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Review of the uncertainties associated to hull girder hydroelastic response and wave load predictions

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ABSTRACT

This paper reviews the importance of uncertainties in hull girder loads influenced by flexible fluid structure interactions. The focus is on developments in the field of hydroelastic modelling, simulation and model tests of practical relevance to the prediction hull girder wave load predictions and their validation. It is concluded that whereas hydroelastic methods for use in design development and assessment become increasingly useful, challenges in realizing and modelling uncertainties can be attributed to: (1) the limitations of numerical methods to suitably model nonlinearities; (2) the ambiguity of model tests; and (3) the systematic use of data emerging from computational, model- or full-scale methods. An approach is recommended to assess the uncertainty in the hydroelastic responses to wave loading and an example is provided to demonstrate the application of the procedure.

1. Introduction

Over the last 30 years improved living standards, globalization and the economies of scale contributed to shipping and trading expansion. This trend influenced the design development of mega ships and floating offshore installations that can provide for the demands of the economies of scale over long distances and at a variable speed range. A prime example are ultra large container ships. These ships have relatively small block coefficients and large bow flare that may be sensitive to wave induced loads. Their slenderness, deck openings and low bending stiffness may result in noticeable elastic hull distortions that resonate in- and out of-plane with encounter wave frequencies leading to a phenomenon known as "*springing*" [1]. "*Whipping*" induced loads may also be excited by nonlinear impulsive wave actions associated with bow flare-, bottom- or stern-slamming (e.g. Refs. [2,3]). The phenomenon is evident on passenger ships, container ships, LNG carriers and war ships. Highly impulsive "*sloshing loads*" on the cargo containment systems of gas ships may also induce local structural damage to tank walls (e.g. Refs. [4,5]). Sloshing loads may also affect overall seaworthiness in those cases that the coupling of ship motions caused by the complex interaction between external forces exerted by sea waves on the

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ship's hull.

Understanding springing, whipping and sloshing induced wave load effects is the main focus of "Hydroelasticity of Ships". The subject, also termed as "Flexible Fluid Structure Interactions - FFSI", was originally introduced by Bishop and Price in 1979 [6]. As a counterpart of aeroelasticity the method is concerned with the interaction between hydrodynamic and inertia forces and their effects on flexible hull dynamics. In recent years, advances in engineering practice, mathematics, computational methods and model experiments made possible the development and implementation hydroelastic methods. For example, the importance of hydroelastic contributions to the hull girder stresses of slender ships in moderate to severe sea states became possible through the development of frequency or time domain numerical methods (e.g., see section 2.2). Selective validation of these methods against data streams from full-scale measurements and model tests is evident in literature (see section 2.1). Sloshing model experiments have been also used for the validation of engineering multiphysics (see section 3.3). Following these developments, Classification Societies introduced design assessment procedures, computational tools and Rules to assure the implementation of new technology in ship design ([7-13]).

To date reviews by the International Ships and Offshore Structure Congress (ISSC) and the International Towing Tank Conference (ITTC) technical committees ([1,14–16]) indicate that (i) high-frequency components of the vertical bending moment due to whipping can be as large as the wave-frequency component; (ii) the total bending moment can exceed traditional rule design values of slender vessels; (iii) both springing and whipping loads are relevant for the assessment of fatigue limit states; (iv) ultimate limit states are primarily influenced by whipping responses; (v) sloshing loads may lead to local damages of cargo containment systems. Notwithstanding this, uncertainties associated with the uniform application and validation of engineering assumptions for unified use in design development and assessment remain broadly incomplete and ambiguous (e.g. Refs. [17,18]).

This paper reviews uncertainties associated to hull girder hydroelastic response and wave load predictions. Following this brief introduction, section 2 outlines concurrent hydroelastic methods for use in ship design. Section 3 pays special emphasis to the assessment of springing, whipping and sloshing wave induced loads. Challenges pretraining to uncertainties in model experiments are summarized in section 4 followed by the presentation of approaches to quantify the model uncertainty and an example of application in section 5, with conclusions in section 6.

2. Uncertainties of relevance to ship design

Learning from ships in service is fundamental in terms of understanding the influence of extreme events on ship design (e.g. Ref. [19]). Accident records and full-scale measurement (FSM) campaigns present the pathways for research and design updating especially for the case of novel ship designs for which empirical methods may not be valid and computational tools may have limitations. Since accident records of practical relevance to flexible ship dynamics may be rare, FSMs represent the most authentic means to understand the influence of wave loads and hydroelastic responses in realistic seaways. Despite genuine attempts from both ISSC and ITTC communities (e.g. Refs. [1,14-16]) the experience of the maritime community based on FSM is not systematically developed and remains relatively scarce especially for the case of ships operating in extreme conditions. This is because FSM data processing requires large amount of human resources and know-how for the rapid and meaningful processing of big data streams. The confidential nature of the vast majority of available data records also prohibits the transparent use and implementation of lessons learnt. With latter in mind this paper refers only as much as practically possible to available accident records and FSMs for those cases that published results are available.

2.1. Learning from accidents

In recent years, increased stresses have been recognized as relevant for the loss of ultra large container ships MSC Napoli and MOL Comfort (see Fig. 1). The former was a United Kingdom-flagged ship that developed a hull breach due to rough seas and slamming in the English Channel on January 18, 2007. An investigation conducted by the UK Marine Accident Investigation Branch (MAIB), Det



(a)





Fig. 1. Recent Container ship accidents (a) MSc Napoli - © gcaptain.com (b) MOL Comfort - © shipengineer worldpress.com.

Norske Veritas (DNV) and Bureau Veritas (BV), demonstrated that the ship broke due to inadequate frame strengthening in the engine room area. The investigation recognized that during the accident wave loads were amplified by 30% possibly due to whipping [20].

The MOL Comfort experienced a fracture of midship part while in the Indian Ocean on June 17, 2013. The ship was split into two halves and sank. The accident report suggested that ship structural strength could be exceeded because of wave loads [21,22]. The analysis accounted for uncertainties associated to lateral load effects on hull girder strength, the deviations of yield stress, the effects of welding induced residual stress, weather, and operational conditions, whipping effects and cargo loads, etc. It was recommended that future rule requirements should account for the effects of lateral and whipping loads for the evaluation of ship structural strength. Postmortem accident surveys confirmed hull bottom plastic deformations in way of amidships for 5 out of her 6 sister ships and 1 out of 4 similar ships in service [23]. However, the root cause of the casualty cannot be specified with any certainty [24].

An interesting argument that followed up from MOL Comfort studies, is whether a ship can survive loads exceeding her ultimate strength over short time. With the latter in mind Iijima et al. [25] and Xu et al. [26] discussed the dynamic behaviour of a hull girder undergoing large vertical bending moment that exceeds its strength capacity. The authors suggested, that in such cases (i) the excess of the vertical bending moment that is larger than the ultimate strength capacity can be redistributed to the inertia and hydrostatic restoring moments associated with plastic hull girder deformations; (ii) the plastic deformation develops to a much smaller degree due to whipping moment than due to normal wave-induced loads possibly because of the limited time during which the plastic deformations grow following whipping. In a follow up study it is shown that the plastic deformation can accumulate gradually under a series of extreme whipping moments that exceed the ultimate strength, and the rate of accumulation can grow after large accumulation of the plastic deformation [27].

Despite practical uncertainties and the inconclusiveness of the impact of flexible ship dynamics during accidents, these investigations lead to the revision of IACS longitudinal strength requirements and the selective introduction of Classification Society notations. The new longitudinal strength standard for container ships URS 11 A [28] introduced a partial safety factor that accounts for the influence of increased stresses in way of container ships double bottom. This new standard may be considered important especially in the hogging condition when a double bottom topology may be exposed to high compressive stresses that can exceed the buckling capacity of shell structures or double-bottom plating and consequently trigger the progressive collapse of the hull. The same standard introduced an update on the vertical wave-induced bending moment via the introduction of a nonlinear correction factor.

Forensic analysis demonstrated that the influence of whipping induced loads on of hull girder strength are still a matter of research and Classification Society procedures still differ considerably. Although whipping is included in the functional requirements for large container vessels, because of the uncertainties influencing design factors it is left up to the individual assurance body to consider its relevance. For example, DNV classification guidelines introduced partial safety factor of 0.9 reducing the effectiveness of whipping during collapse [29]. BV are not using that reduction claiming that there is no evidence for factor of 0.9 [30]. Yet, both Lloyd's Register and BV introduced notations and design assessment procedures for the assessment of springing and whipping loads [9,10]. Along these lines it may be suggested that despite considerable progress in the development of Classification Rules and guidance notes, a rational and unified method that allows for the influence of springing and whipping effects into structural design development and assurance is still not available. This could be attributed to uncertainties associated with (i) existing computational tools and hydroelastic modelling approaches (see section 2.2); (ii) the definition of the influence of dynamic failure modes which may different than those understood within the context of rigid body ship dynamics [31]; (iii) the lack of consensus on the definition of representative design or extreme operational conditions [32]; (iv) limited understanding of methods that can be used to model the influence of extreme events on fatigue accumulation based on data from in service experience [19].

2.2. Hydroelasticity in ship design

Hydroelastic idealisations used for the prediction of springing and whipping loads can be carried out in the frequency or time domains. Studies on modelling and influence of springing and whipping phenomena on ship design have been presented in literature (e.g. see Hirdaris et al. [33] and Tuitman and Malenica [185]). Irrespective to mathematical assumptions the analysis is practically divided in two parts namely "*dry*" and "*wet*" (see Table 1).

In the dry analysis stage, ship structural dynamics are modelled on the basis of Timoshenko beam theory implemented in a Finite Difference (FD) scheme or a Finite Element Analysis (FEA) model. The wet analysis stage can consider the influence of linear, weakly nonlinear or fully nonlinear hydrodynamic actions depending on the level of fidelity of hydrodynamic assumptions as defined by ISSC [1,14,15]. Ship hydrodynamics can be idealized in two-dimensional domain (2D) by a linear or weakly nonlinear strip theory (ST) (e.g. Refs. [6,34,35]). Linear three–dimensional (3D) domain idealisations are possible via the use of a Green function (GF) method with pulsating sources as introduced by Bishop et al. [183] and demonstrated in Refs. [36,38,184]. A GF with translating/pulsating sources [39] or a Rankine panel method [37] may also be used by weakly nonlinear 3D hydroelasticity methods. The use of 3D transient free

Table 1

Hydroelasticity of Ships - idealisations for use in ship design (NB: The taxonomy of hydrodynamic nonlinearity is outlined in Refs. [2,14,15,43]	n Refs. [2,14,15,43]).
--	------------------------

Domain	Dry Analysis	wet Analysis		
		Linear	Weakly nonlinear	Fully nonlinear
2D	Beam FEA/FD scheme	ST	ST	-
3D	3D FEA	GF method (pulsating source)	GF method (pulsating/translating source) or RP method	RANS CFD
Hybrid	2D FEA/FD			

surface Green function allows the coupled BEM-FEM hydroelastic response to be studied [40].

The influence of nonlinear effects is possible via the implementation of (Reynold Averaged Navier Stokes Computational Fluid Dynamics) RANS CFD (e.g. Refs. [41,42]). Accordingly, uncertainties in wave load estimations may relate with fundamental beam theory assumptions (e.g. suitable modelling of in and out of plane distortions, rotary inertia and shear deformation effects) to hydrodynamic modelling assumptions (e.g. radiation, diffraction, large amplitude, forward speed, viscous effects), or even with the numerics of CFD discretization.

Irrespective to the type of hydroelastic idealization it is notable that uncertainties in modelling ship structural dynamics may lead to onerous results. For example, as demonstrated in Refs. [44,45] and confirmed by the experiments under [46], differences in the predictions between springing loads predicted by 2D and 3D FFSI models may be significant for the case of prismatic hulls with large openings where beam theory fails to suitably model out of plane ship structural dynamics. In these cases, the influence of shear on bending and torsion and the contribution of transverse bulkheads to hull stiffness that are replete with uncertainties may be significant. Therefore, structural discontinuities may lead to significant overestimation of antisymmetric dynamic responses and advanced beam theory models should be used with care [47,48]. The latter is significant especially considering that although present experience demonstrates that 3D FEM simulations can simulate well ship structural dynamics for strength analysis as a prerogative for fatigue estimation, beam models are computationally efficient and thus may be preferable for time domain FFSI.

To date, linear "*frequency domain*" hydroelasticity methods have been applied for the prediction of symmetric (i.e. vertical bending induced) and antisymmetric (i.e. horizontal bending/torsion coupled) springing (i.e. steady state) responses of ships in regular waves [33]. A convolution integral may be used to predict symmetric transient responses due to slamming in irregular seaways, using different time integration schemes (e.g. Refs. [49,50]). On the other hand, "*time domain*" potential flow hydroelastic methods are derived on the basis of the expanded Cummins' equation or by direct coupling assumptions. When Cummins's equation is employed, hydrodynamic modelling is achieved by a boundary element method (BEM) (e.g. Refs. [51,52]).

Direct coupling hydroelastic methods can be more beneficial for strongly nonlinear problems as they allow for the implicit implementation of wet modes and the easier inclusion of hydrodynamic nonlinearities in radiation and diffraction potentials. These



(a) Comparison with literature data

(b) Comparison with URS11A requirement



(c) Amplitude of the first harmonic VBM amidships at height / ship length ratio 20 [42]

Fig. 2. Uncertainties in the nondimensional VBM for S175 container ship. Results display differences between the novel CFD based FFSI simulations introduced in Refs. [41,42] against partly validated potential flow analysis results [74,75], the ISSC benchmark [76] and IACS URS 11 A [28].

approaches are still computationally expensive and vague in terms of the hydrodynamic assumptions employed. The implementation of *"Impulse Response Functions - IRF"* may be considered simpler in terms of FFSI modelling although conclusive results for use in ship design remain unknown (e.g. Refs. [53,54]). However, these methods allow modelling complex situations such as cross seas [55].

Developments in time domain methods inevitably imply the need to question the status and uncertainties associated with nonlinear hydroelasticity methods [56,57]. The debate is supported by recent results from FSM [58,59], model experiments [60–62] and nonlinear rigid body seakeeping predictions (e.g. Refs. [43,63–66]). The first nonlinear hydroelasticity theory that accounted for the contribution of first-order velocity potential and response (including rigid body motion and structural elastic deformation) to second-order hydrodynamic forces in way of the average wet surface is presented in Ref. [39]. Since then, the same method has been extended to account for second order hydroelastic responses including the influence of variable draft and air cushion of VLFS (Very Large Floating Structure) [67] and catamarans progressing in irregular seas [68].

Recently, research focused on the utilization of advanced potential flow methods [69,70] and strongly coupled RANS CFD – FEA schemes available in commercial packages (e.g. STAR CCM + for RANS CFD and ABAQUS FEA) [42,71,182]. For example [70], shows that even when Froude-Krylov forces are accounted for in hydroelastic idealizations extreme pitch peak values and hogging/sagging vertical bending moments in way of the ship's bow may be underestimated when compared to experimental values. This could possibly be attributed to body surface nonlinearities that are not considered in the calculation of radiation and diffraction forces. In Ref. [69] comparisons against experimental results for a 13,000 TEU container ship demonstrated that in beam and quartering seas, the nonlinearity of restoring force is dominant and the total sagging Vertical Bending Moment (VBM) at the ship's bow is also underestimated. This could be attributed to the fact that (i) high-frequency load component is more sensitive to wave direction, and (ii) the theoretical model does not take under consideration the influence of second order springing that may be significant especially in bow quartering waves where slamming forces may become prevalent.

To date computational modelling methods have been developed within the context of ITTC guidelines [72]. However, uncertainties in modelling, simulation and interpretation of results for use in design prevail. Comparisons for the nondimensional longitudinal vertical bending moment (VBM = of the S175 container ship depicted in Fig. 2(a) and (b) [41] demonstrate that different methods generally agree well irrespective to incident wave heights and forward speeds. However, the hogging and sagging curves show significant asymmetry and differences against the IACS URS 11 in sagging. The trough–peak of the total sagging VBM curve appears in front of the crest peak of total hogging VBM curve because bow slamming-induced whipping loads can contribute more to total sagging than hogging. In addition, the IACS requirements are based on the extreme design wave with a height of 7.07 m which is much higher than the wave height of 4.8 m used in simulations. The comparisons of the first VBM harmonics depicted in Fig. 2(c) [42] demonstrate generally better agreement of two-way coupled FFSI in wave matching region. However, differences between predictions and measurements in short and long waves vary and in short waves ($\lambda/L < 0.9$), all the numerical predictions are higher than the measurements. The majority of the methods over-predict the amidships bending moment in the region of λ/L 0.7–1.2 and the uncertainties in the predictions for the range of wave frequencies investigated are quite high. Since uncertainties in the measurements are not addressed in literature, it is not possible to quantify uncertainties.

In summary, research in nonlinear hydroelasticity may be considered valuable within the scope of both ISSC and ITTC proceedings [1,14,73]. However, high CPU costs and computational modelling uncertainties prevail and therefore the practical use of nonlinear hydroelastic methods for design development should be faced as a medium to long term objective.

2.3. The structural reliability perspective

Structural reliability methods present appropriate means to quantify the safety margin between load and resistance in novel ship designs or when new methods for load or strength evaluation are introduced into design practice [111] or under extreme operational conditions [77]. This is because reliability analysis takes rationally into account various uncertainties in loading, response and structural strength by random variables, each with an associated distribution type and parameters. By using suitable procedures, the probability of structural failure (or its counterpart namely "the reliability index") may be calculated [78]. On this basis Classification Societies introduced the Load and Resistance Factor Design code (LRFD), in particular in their Common Structural Rules [79], which implemented an ultimate limit state condition as formulated in Ref. [189] These rules had implications on the reliability of tankers as discussed in Refs. [190,191]. The partial safety structural reliability approach has been applied to calibrate some of the safety factors for double hull oil tankers [79,192], including plates in the double hull [192]. Notwithstanding this progress, the influence on ultimate collapse of double hull oil tankers remains uncertain and may be different in comparison to container ships that may be prone to whipping loads.

The structural safety of container ships assuming rigid hull responses in waves is assessed in Refs. [80,193]. A method accounting for whipping loads is presented in Refs. [81,194]. A study on the structural reliability of container ships is presented in Ref. [78]. A practical method for the short-term structural reliability assessment of container ships subject to whipping in relatively well-defined environmental conditions is presented in Ref. [82]. In this work the authors extend the commonly used limit state function for structural reliability analysis of oil tankers and bulk carriers to consider the whipping response due to the bow flare slamming. The ultimate vertical bending moment capacity of the hull girder is considered as a fundamental safety variable in the limit state function, while the stochastic model of the loads consists of static still water bending moments, low frequency rigid-body vertical wave bending moments and high frequency whipping bending moments [168]. Probabilistic load combinations of these three global load components are also considered as described in Ref. [82]. An important conclusion from that study is that allowable safety indices which are recommended for double hull oil tankers may be too conservative for container ships because the whipping load is an impulsive event of rather short duration.

Reliability methods may be useful for encompassing uncertainties associated with nonlinear wave loads [196] and extreme events. Examples on the latter are given in Refs. [80,84] that present comparisons of the maxima of measured and simulated vertical bending moments in severe short-term sea states using 10^{-3} probability of exceedance. Since such approach may be computational expensive an alternative approach is to use a deterministic irregular wave episode and a First Order Reliability Method (FORM) [85]. Latest research confirms the value of this method for the case of freak wave events [86] and accordingly it may be concluded that irregular wave approaches may be useful in terms of detecting extreme loads. The dynamic response under extreme loads may exceed the linear elastic region. With the later in mind [87,88] present and validate a hydro-elasto-plastic model. Their method demonstrated that for a given magnitude of the vertical bending moment, the collapse behaviour under extreme loads with short- and long-time duration may be different.

In harsh sea states, a ship master may attempt to secure ship cargo and safety by voluntarily reducing the ship speed or by changing the ship heading angles [19]. Whereas simulation tools can be used to reduce uncertainty in operations by communicating ship load and response curves the development and validation of intelligent monitoring systems utilising digital twins is at embryonic stage [89]. Reliability theories may be utilised to improve hull girder safety while maintaining ship performance. Fig. 3 depicts a schematic diagram which explains the information to be derived from such a digital twin system. The hull girder response is estimated in real-time from sensors of limited numbers attached to the ship (model) in the tank test. The measurement and simulation system are integrated into a system called 'i-SAS' and the system is applicable to a full scale ship to monitor the ship onboard and check the response/load against the criteria [90]. In Fig. 3b the solid lines show the hull girder ultimate strength capacity estimated by simulations. The broken lines show the total vertical bending moment consisting of wave bending moment and still-water bending moment. The vertical bending moments may be predicted by the simulation, taking account of the hydroelastic vibrations, and then corrected by the full-scale measurement. In the diagram, the upper two standard deviation and lower two envelope curves respectively demonstrate loads and capacity. The concept demonstrates that reliability theory backed up by suitable big data analytics could be used for pre-mortem estimation of the probability of failure.

3. Uncertainties in wave load predictions

3.1. Springing induced loads and responses

In literature the significance of springing induced loads for ship design and operations are demonstrated on the basis of results from Full Scale Measurements (FSM) under different operational conditions (e.g. environmental conditions, loading, routes, length of service, etc.) [91–94,98,197–199].

For example, spectral analysis records from FSMs on the "*Bohai Changqing*" 221 m tanker demonstrated that deck stresses may amplify between 13% and 17% due to springing under 7–9 wind force conditions [91]. A similar type of study on a large iron ore carrier travelling from St. Lawrence Bay to Rotterdam confirmed that springing induced vibration stresses link with vibration damping coefficients of the order of 0.5% [92]. For this ship, under ballast conditions the magnitude of springing and wave-induced hull stresses



Fig. 3. Schematic of an onboard display for use on intelligent operational guidance systems [90].

(a)

were similar. However, in full-load conditions the stresses were lower. This could be attributed to the fact that wave pressures at the ship's bottom decrease exponentially with increasing draught in following seas. FSMs on a 281 m long ore carrier operating over 4 months in the Pacific route (USA to China and Japan) demonstrated that for significant wave height up to 3 m or more, the vibration damage of the lowest bending mode accounts for nearly 50% of the total fatigue damage [93].



Fig. 4. Display of hydroelastic effects on predictions of total force and slamming pressures for flat plates. Comparisons from (a) ALE [121] and (b) CFD RANS two way coupled method [116] against experiments [134].

Whereas the importance of the hydroelastic effects of springing in linear or random waves under different loading conditions are understood, the influence of nonlinear springing dynamic response remains largely inconclusive [94]. This is critical considering that linear springing analysis assumes that the ship response has the same frequency as the encounter frequency of the incident wave and linear hydroelastic models assume that linear springing occurs in way of the wave matching region (wavelength/ship length = 1.0). Nonlinear springing has been observed even in regular wave model experiments for hulls that have low bending stiffness and progress in medium to high speed (e.g. Refs. [95–97]). In these cases, the encounter frequency is equal to 1/nth ($n\epsilon\mathbb{Z}$) for each of the structural natural frequencies. Thus, linear and nonlinear springing loads on ships may reduce by up to 50% the fatigue life of bulk carriers. Similar observations for the case of large container ships relate to the fact that wave induced resonances are caused by antisymmetric loads [98].

3.2. Slamming induced loads and responses

To date engineering experience suggests that springing and whipping events manifest simultaneously and since associated wave induced resonances are superimposed on wave frequency loading they may contribute to fatigue damage accumulation. Along these lines distinguishing whipping and springing phenomena is naturally a practical cause for uncertainty. Recent research demonstrates that the main source of structural vibration of slender ships is slamming and, in this sense, better understanding on the influence of springing and whipping effects may prove useful [99]. While slamming tends to occur with waves in the bow sectors, in which case the shape of the bow is of paramount importance [100–102], there are ship forms that are equally prone to bow and stern slamming [103, 104].

Slamming is a primary local loading event and possibly the major cause of local structural damage for high-speed crafts. High speed ferries, typically have operational limitations based on maximum motions, acceleration and slamming events. Monohulls tend to suffer of high slamming pressures near the lowermost chine from the bow up to 1/3 of the hull length towards midship [105]. Slamming load analyses considering fast ferries as built, found that fluid-structure interactions were not affecting local pressure loads for hulls built in aluminium or steel. However, fluid-structure interactions may indeed play an important role for high speed hulls built in reinforced plastic [106–108]. Cross deck bottom slamming is an important design constraint for high speed catamarans which can be equipped with wave piercing bows [109] to mitigate slamming loads to avoid large clearance from the waterline [110].

The non-stationarity of waves and the nonlinearities of fluid actions associated with wave induced loads add on an additional uncertainty of relevance to the estimation of slamming induced fatigue loads. Available methods estimate fatigue cycles in the time domain by rain-flow counting methods that are computationally uneconomic. Based on Rayleigh's superposition method the total hydro-elastic response can be the aggregate of narrow-banded processes [36] and accordingly the use of frequency domain solutions could present a useful alternative in terms of decomposing the problem and calibrating uncertainties [32].

In hydroelasticity methods the entry velocity and location of local slamming events are traditionally estimated by analytical momentum theory models [111], embedded in Classification Rules and Guidance notes (e.g. Refs. [10,11]). In recent years local slamming problems have been investigated by a number of numerical methods namely Smoothed Particle Hydrodynamics - SPH (e.g. Refs. [112,113]), Arbitrary Lagrangian Eulerian algorithm – ALE [114,115] and RANS CFD [103,116–118]. Although uncertainties associated with different theories and computational models have not been systematically investigated [119] engineering experience suggests that numerical computations may convey good comparisons against experiments [120]. Numerical uncertainties such as round-off, iterative convergence and discretization errors influence the solution of such FFSI idealisations [121,122]. The two of the most popular methods that may be used to analyse the grid and time step uncertainties are known as "correction factor" [123] and "factor of safety" [124] methods and are based on "Richardson's extrapolation" [125]. Using these methods, uncertainties due to discretization errors on the predictions of the peak values of slamming induced loads and responses of a flat plate are quantified in Ref. [121] (see Fig. 4). Both systematic and random uncertainties may be important [126]. Recent research discussed the practical influence of slamming induced physics (e.g. trapped air, compressibility effects, hydro-elastic interaction, viscosity of fluid, three-dimensional effects) [127,128]. The influences of these factors on slamming loads were extensively investigated, regarding the air compressibility [130,131]. Analysis of experimental uncertainties related to the measured impact loads and deformation of wedge-shaped structures demonstrated the importance of repeatability in measurements, sampling rate effects and hydroelastic effects on slamming pressures [132].

A number of potential flow numerical whipping models are summarized by ISSC [15]. For simplified (two-dimensional) whipping analysis beam models may be used to idealise the influence of the first few modes of vibration on symmetric responses (see section 2). Confidence in the use of three-dimensional seakeeping models for use in whipping analysis is limited primarily to head sea conditions. This is because of uncertainties associated with nonlinear hydrodynamic methods and issues associated with mathematical idealization of the incident waves in oblique seas [1,133].

A guidance for uncertainties in measurements along the lines of ISO GUM method is presented in Refs. [186,187]. ITTC published a comprehensive procedure with recommendations for uncertainty analysis for ship model testing [73]. The primary sources of differences resulting from numerical methods relate with different mathematical modelling of the boundary value problem, non-converged or inaccurate hull geometry modelling, insufficient or incorrect knowledge on mass distribution and human errors during experiments. Possibly, the emergence of strongly coupled RANS CFD tools and FEM solvers could resolve modelling and simulation issues associated with incident waves and dynamic response in stochastic seaways (e.g. Refs. [42,71,85]). This is because two-way implicit coupling algorithms account for the influence of hydrodynamic actions at different headings, wave profiles and the impact of stochasticity at each step of the dynamic response. Yet, unified engineering principles for the application of these methods are not evident and demands for computational economy makes almost impossible their coupling with advanced 3D FEA models. On

this basis it could be suggested that uncertainty in seakeeping may associate to uncertainties in whipping [129].

Recent research confirmed the hypothesis that the buckling modes may be amplified by cyclic compressive loads caused by waveinduced vertical bending moments due to whipping [135,136]. Despite this, it remains uncertain how much of the limited energy associated with the high-frequency whipping impulses contributes to global hull girder collapse [137]. The experimental work presented in Refs. [87,88] demonstrates that the hull girder collapse mechanism may not have time to develop significantly before whipping loads start to decrease. Therefore, the extent of the influence of whipping on ultimate limit states could be smaller than originally assumed. Whereas this finding converges to indications from previous theoretical work [138] it is difficult to prove the case because of lack of economy of implicit non-linear FEA methods. Possibly, simplified models such as the one presented in Ref. [30] could shed some light on the cyclic elasto-plastic behaviour of the hull girder in slamming conditions.

3.3. Sloshing loads

Sloshing is a highly nonlinear impulse load with local effects that couple with global wave-induced motions of Liquefied Natural Gas (LNG) carriers and vessels with moon pools. Because of liquid sloshing in the inner walls of the ship tanks and the pump towers installed inside those may be subject to enormous hydraulic forces. The phenomenon has been reviewed by the ITTC Ocean Engineering Committee [16]. A critical review of available research demonstrates that the typical temporal duration of sloshing loads is generally comparable with the lowest natural period of tank structures. In this sense, hydroelasticity effects may also be critical [2,14, 15,112,188]. Recent research demonstrates that tank fluid motions and sloshing induced loads are sensitive to water filling levels, the size and position of the containment system and the energy spectrum of global platform motions. Additional uncertainties may be

(a) Ship sloshing experimental set up



(b) Prismatic tank experiments



(c) Bilope tank experiments



Fig. 5. Sloshing experimental set up on hexapod displaying wiring, data acquisition system and sensors [146-148] (© https://mhl.snu.ac.kr/).

attributed to the idealization of fluid flow physics (e.g. liquid surface breaking, air bubble dynamics, gas compressibility, etc.) and limited operational experience.

In general, experimental results are sensitive to scaling laws as well as the suitable utilization of state-of-the-art measuring equipment (e.g. sensor type and topology, data acquisition and processing, temperature variations, etc. – see Fig. 5) [4,139,140]. A critical review of published work demonstrates that (i) the topology of sensor positioning is critical in terms of estimating sloshing impact pressures irrespectively to the size of the containment systems, (ii) integrated circuit piezoelectric sensors generally give good results irrespective to temperature changes, (iii) the influence of experimental similarity laws is critical especially considering that gas pocket pressures are greatly affected by compressibility. Results from numerical studies with focus on the validation of fluid flow physics are broadly available in literature (e.g. Refs. [112,113,141]). For example [141], assessed sloshing loads on the LNG Tank of a floating offshore bunkering terminal using an unsteady CFD RANS solver. Comparisons against experiments for various filling conditions under irregular excitations demonstrated good agreement and suggested that impact loads may not be significant even for partial filling cargo levels. This could possibly be attributed to the positive influence of lower chamfer height of the containment system.

As demonstrated in the open-FOAM RANS CFD studies presented in Ref. [142] the use of optimized porous baffles could present an alternative positive measure for the reduction of sway induced sloshing loads. Similar reduction has been found by the introduction of invar edges [143], by means of unsteady finite volume multiphase viscous flow solvers and meshless smoothed particle methods validated by experimental tests [106]. The influence of coupled sloshing and FLNG hull motions using potential flow methods are presented in Ref. [144]. In this study a 3D finite difference method used to idealise sloshing physics is coupled with a time domain seakeeping solver. Comparisons against experimental results demonstrated that in principle the influence of internal liquid free surface motions may be significant when coupled with sway and roll induced excitations. The use of advanced CFD (e.g. Smooth Particle Hydrodynamics - SPH and Moving Particle Semi implicit – MPS) methods for the simulation of sloshing flows is emerging. Recent studies demonstrate that SPH predictions are computationally uneconomic and highly dependent on modelling assumptions associated with the choice of particle size and speed of sound [112,113]. MPS when coupled with FEM could be useful for the idealization of hydroelasticity induced sloshing loads [145]. However, modelling fluid physics remains ambiguous and therefore the practical application of the method for use in design is under development.

From an overall perspective, best practices for the simulation of sloshing phenomena and quantification of uncertainties due to modelling parameters and solvers are still under development. This is because, of uncertainties associated with the maturity seakeeping approaches used for motion predictions, the idealization of fluid flow in partially filled tanks and gaps in the repeatability of model scale experiments.

4. Uncertainties in model scale experiments

It is well known that model testing can be carried out in tank facilities or open waters. Validation is based on the notion that the simulation model should reproduce the motions and responses of the prototype. Scaled models are usually towed on a carriage in the tank facility to maintain forward speed and keep course. In the case of ocean basin or in open waters the scaled model may be self-propelled. In laboratories wave makers are equipped on one end of the tank (towing tank), or along the periphery of the tank (ocean basin). The waves can be either regular, irregular, short/long - crested or oblique. On this basis uncertainties may be attributed to the quality of waves in the tank, insufficient number of wave and response cycles, calibration of the instrumentation, the properties inherently implemented to the scaled model design, sufficient capturing of realistic environmental conditions, the repeatability of experimental values, etc.

Over the years ITTC developed experimental procedures for use in the seakeeping of ships and floating offshore installations (e.g. see ITTC seakeeping committee procedures 7.5-02-07-02.1 and 7.5-02-07-03.1 in Ref. [16]). According to ISO-GUM methodology Type An uncertainties can be evaluated from repeated measurements. Type B uncertainties are assessed using available scientific judgements such as previous measurements, experiences and knowledge of instruments, calibration data, and precisions from manuals of instruments, etc. Whereas the focus of this paper is on uncertainties associated with flexible ship dynamics good practise requires sound understanding and calibration of uncertainties associated with both traditional seakeeping model testing (particularly the validation of hydrodynamic nonlinearities captured from rigid body model experiments) and hydroelastic model tests as explained in the following sections.

4.1. Rigid body model testing

A practical review of the implication of uncertainties associated with the experimental measurements of nonlinear motions and loads (e.g. Refs. [73,149–152]) leads to the observations that (i) waves and responses in model testing facilities are not stationary even under regular wave conditions because of fluctuations of the tank carriage; (ii) large amplitude effects in irregular wave conditions may be influenced from differences between the target and measured wave spectra; (iii) uncertainties due to sensors, calibrations, model characteristics (e.g. speed, heading, geometry, mass distribution) and hardware equipment (e.g. wave probes, load cells, motion capture system, power supply) are critical; (iv) floating assets involving deep water mooring lines, risers and dynamic positioning systems are more sensitive to scaling and highly nonlinear multiphase fluid flow phenomena (e.g. water breaking, air bubbles, sloshing, slamming and green water on deck).

From an overall perspective it may be concluded that generally Type B uncertainties are more significant than Type A and the significance of well structured data management (e.g. acquisition and processing) should not be underestimated. However, expanded

Type A and Type B uncertainties in the measurements of model motions, drift forces and wave elevations are not strictly independent and therefore further studies on how they may be combined and how they influence in specific nonlinear seakeeping is essential.

4.2. Hydroelastic model testing

Scaled model design techniques for flexible (i.e. hydroelastic) ship models developed significantly over the last 20 years [153]. Traditionally, hydroelastic model scale rigs are classed into segmented and non-segmented elastic (also known as hydro-structural models without a backbone).

Segmented models may include elastic backbone beams (e.g. Refs. [97,99]) or mechanical connections with instrumentations that can re-produce bending flexibility (e.g. Refs. [154,155]). Alternatively, segmented open sea tests may be possible. An example of the latter is given in for the case of a prototype ship with a scale ratio 1/25 that comprised of seven segments made of FRPs connected to a steel backbone is given in Ref. [70]. Comparison of results form these open seas experiments against a 1/50 model scale model demonstrated that open sea tests are more realistic in terms of wave directional spreading. Another example used a large-scale segmented model with a box-type backbone beam in short-crested irregular seas provided new insights into the relationship between nonlinear VBM and HBM of the ship sailing in short-crested sea waves [156]. Other examples can be found in Ref. [59].

For ships with large deck openings (e.g., container ships and bulk carriers) the torsional and vertical deflection modes can be of major concern (see section 2 and e.g. Refs. [32]). Yet, the number ship model tests considering torsional vibrations are very limited. For example, in Ref. [60] experimental results are presented from an elastic backbone (U- and H-shape) hydroelastic model. The authors considered several cut-outs and attempted to adjust the natural frequencies to match the symmetric and antisymmetric dynamics of the flexible model as closely as possible using FEA. Although strain gages were carefully calibrated against torsional moments, the target torsional natural frequency was not attained because the shear centre locations and consequently warping torsional stiffness between the model and the prototype ship were different.

Traditionally, non-segmented flexible hull models have been mainly used for the measurement of vertical bending moments [95]. In these models the model's structure is regarded to behave as a thin-walled cross-section beam and the global structural properties of the prototype ship are implemented in the beam model. In such cases special attention should be attributed so that the scaled model local structures retain their strength properties under hydrostatic and hydrodynamic pressures. Notwithstanding this, the plate thickness of the model cannot be very thin and foam material (e.g. urethane), resin or composite material with sufficient plate thickness is used. These materials may have large variations in the mechanical properties as compared to metal materials such as steel and Aluminium. Uncertainties regarding quality control during fabrication, difficulty in strain measurements due to material softness, creep effects and large damping ratio are broadly evident and should be treated with care.

The study presented in Ref. [86] extended the hydro-structural model technique to account for the similarity of torsional and warping stiffness in addition to vertical bending. In this work, the shear centre of the model was located below the bottom of the ship model. The static and dynamic elastic properties of the model were validated by three-point bending, torsion and decay tests. A series of towing experiments in regular and freak waves demonstrated that the vertical bending and torsional moments, including elastic vibration, can be measured successfully. The damping ratio of the hydro-structural model adopted in this research was found to be almost equivalent to that of the prototype ship and larger than the one used in segmented models whose damping ratio normally ranges from 0.1% to 2%.

Advances in rapid prototyping make possible the use of 3D printed technology for the production of ship-like sections for use in hydroelastic model testing. This is demonstrated in recent studies for the case of ship-like sections progressing in regular head waves [157]. Whereas the capability of such models for the prediction of ship torsion is still under investigation preliminary tests demonstrate that over the medium to long term the evaluation of global antisymmetric vibrations may be possible.

4.3. The influence of structural properties on hydroelastic model testing

Flexible ship responses depend on both hydrodynamic and structural properties and therefore model behaviour in tank facilities should be interpreted on the basis of the law of William Froude (RINA, 2010). This means that structural and hydrodynamic properties must also be scaled on the same or similar basis. The main sources of uncertainties in hydroelastic model tests relate with similarities in the properties of symmetric and antisymmetric flexible beam dynamics, the connectivity of rigid segments with the backbone beam, load cells and strain gauges and their installations, locations and numbers, and scaling. For example, the bending rigidity of the model should be scaled to α^5 for the geometrical scale ratio α between the prototype and the model. When the ship is modelled as a beam, the

Table 2

Structural properties to be considered in the scale model design (E: Young's Modulus; I: Inertia, A: Sectional Area, J: Torsional moment of inertia; Γ: warping moment).

Property	Symbol	Scale	Remarks
Bending stiffness Shear Axial stiffness St. Venant's torsional stiffness Warping stiffness	EI GA EA GJ ΕΓ	α^5 α^3 α^3 α^5 α^7	Two axes; vertical and horizontal bending Two axes; vertical and horizontal bending. Shear deformation is not usually designed in the scaled model. The axial elongation mode is not usually considered.

structural properties to be considered for scaling are summarized as in Table 2. It is to be noted that the scale factors can be different among the structural properties considered and in between different models corresponding to different designs (see Fig. 6 and [158]).

Current practice suggests that calibration of symmetric responses associated with vertical bending is critical in terms of calibrating symmetric whipping and springing responses. Yet, better understanding and calibration of antisymmetric dynamics should be considered critical especially for the case of prismatic hulls with large deck openings. Other uncertainty sources inherent to the scaled model design may involve.

- Differences in deflections or modal shapes that can lead to differences in stresses.
- The physical cuts between the segments can be the source of the hydrodynamic uncertainty. Although such cuts may be sealed in some cases by a rubber sheet it should be noted that in segmented models, the mass distributions are also segmented.
- The damping properties may differ depending on the scaled model techniques and should be treated with care especially considering that springing and whipping response may be sensitive to damping effects. For example, in general, backbone models encompass lower damping.
- Additional stiffness due to the fixation to the carriage may affect the structural deflections. In such cases axial forces may contribute to the measured vertical bending moment.
- In segmented models with mechanical connectors force transducers are used for estimating the hull girder loads. Similarly, in backbone models the strains are measured and integrated to give the hull girder loads, on the basis of beam theory, or by using the calibrations to fixed loads. Whereas irrespective to the type of experimental model calibration may be considered straight forward in hydro-structural models the measured strains are sensitive to stress gradients associated with local structural arrangements, stress concentration effects, deflection to local loads, etc. [159].

5. Quantification of model uncertainty in wave load predictions

A categorization of global wave load uncertainties of ships is presented in Ref. [160]. Uncertainties of relevance to ship rigid body responses are included in total uncertainty estimation when accounting for hydroelastic effects. With the latter in mind this section summarises main conclusions regarding uncertainties in ship rigid body wave-induced responses and hydroelastic contributions.

Rigid body responses are sometimes called low-frequency responses but can be misleading. This is because the lowest natural frequency of hull girder vibration of ultra-large container ships (ULCS) may be so low to interfere with the wave frequency. In such cases, the rigid-body and elastic responses are within the same frequency range and cannot be separated. Thus it is preferred to use the term "rigid-body" rather than "low-frequency" response.

5.1. Uncertainty in rigid body responses

Wave load uncertainties associated to rigid body ship responses can be categorized into two large groups associated to linear and non-linear ship responses.

Linear response models encompass the uncertainty in transfer functions that are normally calculated by strip theory or 3D methods. In those cases that time-instead of frequency – domain seakeeping methods are employed, transfer functions account for ship responses to regular waves of low steepness. In CFD idealisations the efficiency in the computations of transfer functions can be improved if wave trains are used instead of regular waves [63].

Ships operate in the random wave environment. Therefore, rigid body wave loads are commonly determined by spectral analysis using linear transfer functions. Additional environmental uncertainties are considered within the group of linear wave load uncertainties. The choice of the so called theoretical wave spectrum, or more generally, the spectral shape uncertainty [161] belongs to



Fig. 6. Difference in percentage of the natural frequencies predicted by various models compared to the frequencies predicted by the ship shell element model [158].

this category [162]. Other uncertainties related to the short-term responses are those of the wave energy spreading [163] and those related to the influence of two-peak wave spectra [164]. The influence of the wave energy spreading on the hydroelastic response is discussed in Ref. [165], where short-crested waves resulted in about 10% lower long-term response compared to the long-crested waves. The major contribution to the two-peaked wave energy spectra is the swell, where sea states combined of with wind-generated waves and swell represent about 25% of all sea states in the North Atlantic [166,167].

Uncertainties in the long-term description of the wave environment [168] is commonly assessed using linear spectral analysis. Modelling is influenced by the choice of suitable wave scatter diagram [169] and operational choices of relevance to heavy weather avoidance [170,171]. Ship operational uncertainties are important for ships prone to hydroelastic responses. This is because the amplification of slamming induced loads is influenced by speed profiles [165].

The most important source of non-linear behaviour in wave loads is the difference between sagging and hogging vertical wave bending moments. Full-scale measurements and the results of numerical calculations have shown that the long-term distribution of sagging and hogging bending moments is non - symmetric with respect to the distribution of linear moments. Since different non-linear methods are presently in use, uncertainty quantification of non-linear methods is quite challenging. Several approaches to quantify uncertainties in non-linear ship responses are reviewed in Ref. [104].

5.2. Uncertainty in ship hydroelastic responses

Hull girder transient flexural vibration (whipping) occurs due to bottom, bow flare or stern slamming. Bottom slamming is relevant for full-form ships (e.g., tankers or bulk carriers) sailing in rough seas at relatively shallow draught [172]. This type of slamming can be avoided if heavy ballast loading that also enables better ship manoeuvring and propeller emergence Bow flare slamming is relevant for ships with fine forms (e.g., container ships, war ships and passenger ships) and is relevant in all loading conditions. Thus the ability of the method employed to predict slamming load is obviously an important source of uncertainty in predicting whipping responses [131, 173]. A recent uncertainty study indicates that whipping responses may be influenced by hydroelastic modelling choices (e.g. the idealization of stiffness and mass), wave steepness and the overall strategy used to model of impact loads [57].

A benchmark of uncertainties that relate with structural dynamic idealisations (e.g., 3D shell versus 1D Timoshenko beam models) are presented in Ref. [174]. This study shows that the uncertainties in wet frequencies may be lower in comparison to those of relevance to dry modes. The damping coefficient is another important parameter that is often expressed as a percentage of the critical damping for each natural mode of vibration, and it influences both whipping and springing responses. Storhaug et al. [175] concluded, based on the full-scale measurement data, that container ships have higher damping than other blunt ships like oil tankers, or carriers and gas carriers. They proposed damping of 1.7% for container ships and 0.7% for blunt ships.

In hydroelastic investigations the assessment of the accuracy of a code is frequently made by direct comparison with experiments



Fig. 7. The experimental and numerical RAOs of the heave, pitch and vertical bending moment of an ultra-large containership at amidships in 3 m regular wave (left plot) and 8 m regular wave (right plot) (figure extracted from Ref. [61]).

that may also be prone to uncertainties. In addition to the uncertainties in rigid model tests traditionally addressed by ITTC, the main sources of uncertainties in hydroelastic model tests involving springing and whipping include the properties of the hull, load cells and strain gauges and their installation, location and the number of gauges used, scaling, data acquisition such as sampling rate, and associated data analysis. In recent years, various hydroelastic model tests have been carried out [10]. However, detailed uncertainty analyses in those hydroelastic tests are very rare [176]. There is a need to quantify the uncertainty sources in hydroelastic model tests as part of the framework of the joint ISSC-ITTC committee, which involves tests of different scale hydroelastic models, in different facilities. Main sources of uncertainties to be quantified should include initial test conditions, hydroelastic model specifications/instrumentation (e.g. scaling, sensors, load cells and strain gauges etc.), wave generation mechanisms, data analysis and as/if applicable mooring line set-up.

To date comparisons of results from numerical simulations and experiments shows a large scatter [174]. That could be the reason why a common approach for the prediction of slamming induced loads and associated whipping responses is not readily available. The challenging problem of quantifying modelling uncertainties of relevance to the prediction of long-term whipping bending moment remains prevalent [82]. Since the extreme values of those bending moments do not occur at the same time, they should be combined probabilistically [177].

5.3. Approaches for quantifying model uncertainty

Several methods for modelling uncertainties in transfer functions are reviewed in Refs. [104,152]. The transfer functions calculated with different models or codes can be compared with each other and with experiments. The aim of this process is to model uncertainties and model bias with respect to measurements. Hydroelastic responses can be represented by a transfer function that includes additional peaks of response amplitude at higher frequencies corresponding to the elastic responses (see Fig. 7).

A useful model uncertainty quantification method is the Frequency Independent Model Error (FIME) introduced by Guedes Soares [178]. The approach represents the average bias of the measured transfer functions with respect to the computed ones. It is a convenient uncertainty measure, as it gives immediate information if the theory overestimates or underestimates measurements. In addition to bias, random dispersion of experimental values around the regression line represented by FIME is required for a complete uncertainty description. This dispersion is expressed by the so-called coefficient of determination (CoD), which reflects the variance in the errors. In practice a variance that is larger than 0.9 can be considered as an excellent fit to the data points and FIME can be considered as a sufficient uncertainty measure. Another useful approach is the total difference (TD) measure that aims to assess the deviation of an individual numerical model against the mean of all models [179]. Notwithstanding this TD accounts for both bias and dispersion of individual transfer functions used in different idealisations (computer codes). In this way it is not possible to make distinctions amongst different methods or identify the source of mean differences against experimental measurements.

In the benchmark presented in Ref. [152], FIME is split into two parts namely (i) FIME of an individual code with respect to the average of all codes using the same method, and (ii) FIME of the method average with respect to experiments. Following this approach, Abdelwahab et al. [180] proposed a modified total difference to quantify the uncertainty of individual seakeeping codes against available experimental results. In this method the total difference between transfer functions is determined by assuming that the measured experimental transfer function represents the best response estimate at each frequency. The advantage of the latter is that discrepancies between the numerical methods and measurements can be utilised. However, when the dispersion of numerical results is large, it is hard to measure with accuracy the individual numerical model.

As an alternative, a method that combines FIME and a CoD could be used to evaluate the accuracy of the individual numerical model relative to measurements. FIME can be used to evaluate the systematic bias between experiments and predictions of wave-induced responses as introduced by Guedes Soares [178]:

$$\widehat{a}_{i} = \frac{\sum_{j} |H_{j}| |H_{ij}|}{\sum_{j} |H_{ij}|^{2}}$$
(1)

where i is the applied numerical model. A FIME greater than 1 indicates that the numerical model underestimates measurements. A FIME less than 1 suggests that the numerical model overestimates experimental results. The CoD, measured by the variance of the errors (R2), can be used to measure how well the regression prediction approximates the discrete data points, is given by

$$R_{i}^{2} = 1 - \frac{\sum_{j} (\hat{a}_{i}H_{ij} - \hat{H}_{j})^{2}}{\sum_{j} (\hat{a}_{i}H_{ij} - \overline{H})^{2}}$$
(2)

where *H* is the average measured transfer function across all frequencies in the measurements.

When R2 = 1, the regression predictions fully fit in the data and all measurement points are on the regression line. In such a situation, FIME may be considered sufficient to represent uncertainties associate with the transfer function. In practice, an R2 greater than 0.9 is considered an excellent fit to the data points, and FIME is an appropriate uncertainty measure.

In this study, only three numerical models based on the same theory are considered. Accordingly, the modified total difference with respect to available experimental results is calculated. The modified total difference (TD_i) for transfer functions, proposed by

Abdelwahab et al. [180] is determined according to the equation:

$$TD_i = \frac{\sum_{j=1}^{j} |H_{ij} - \hat{H}_j|}{\sum_{j=1}^{j} \hat{H}_j} \bullet 100$$

where H_j represents the measured transfer function at a frequency *j*.

5.4. Case study

Rajendran et al. [34,61] investigated the flexible vertical response of an ultra-large containership in high seas by using a body nonlinear time time-domain method. The ship moves with a speed of 15 knots which corresponds to a Froude number of 0.135. The numerical Response Amplitude Operators (RAOs) of the vertical ship responses in regular head waves (3 m and 8 m) were compared with the experimental results. In addition to the hydroelastic model (TD-flex), a rigid body model (TD-rigid) and a linear frequency domain model (FD) were adopted. Generally, the heave and pitch RAOs from the model agreed well with the measurements, except for a range of frequencies corresponding to wave/ship length ratios close to 1. Nonlinearity for the heave motion becomes an important factor only in this region. The rigid body vertical bending moment (VBM) RAO value decreases as the sea severity increases. However, the RAO values from the experimental and the numerical flexible model behave in an opposite manner.

In the above mentioned study, the systematic bias between experiments and predictions is not evident. In this paper available data from Ref. [61] are used to demonstrate the influence of hydroelasticity on modelling uncertainties. Fig. 8 depicts the numerical and the experimental vertical responses as a function of encounter frequencies in 3 m and 8 m regular waves. For 3 m regular waves, 10 frequencies are used, while for the other case, only 7 are considered. In addition to the uncertainties observed in rigid body model tests, main sources of uncertainties in hydroelastic model tests may involve uncertainties associated with springing and whipping effects (e.



Fig. 8. Measures of uncertainty in transfer functions of an ultra large containership for in heave, pitch and VBM at amidships obtained by numerical methods in 3 m regular wave (left plot) and 8 m regular wave (right plot).

(3)

g., appropriate modelling of the structural dynamic properties of the hull, use of load cells and strain gauges, scaling, data acquisition strategies such as sampling rate, data analysis, etc.).

FIME, CoD and the modified total difference for transfer functions were calculated using Eqs. (1)–(3) respectively. As shown in Fig. 9, the frequency domain (FD) method underestimates and the time domain (TD-rigid) method overestimates all the vertical motions and loads for both cases. Most CoD are larger than 0.9. Exceptions to this is the value (0.8986) which corresponds to the VBM obtained when using the model TD-rigid in 3 m regular waves and the values (0.8753 and 0.7792) corresponding to the heave RAOs obtained from TD-rigid and TD-flex models.

TD-flex model shows higher uncertainties, namely 21.3%, 19.2% and 8.6%, for the transfer functions of heave, pitch and VBM in 8 m regular waves. It seems that with the increase of the severity of the sea states, the uncertainties are not increasing, in particular for the VBM transfer functions. Compared with the values, 23.9%, 24.1%, and 9.0% obtained from the 3 m regular wave case, the estimates in the other case are 5.8%, 4.3%, and 8.6%, respectively. This is probably due to the high discrepancy in VBM between TD-rigid model and the measurement for the last frequencies as shown in Fig. 9. For 8 m regular waves, the range of frequency is small. Therefore this discrepancy is not observed. A similar comparison for the FD model relative to the experiments. As for the predictions of the VBM from the TD-flex model, the total difference for both cases is low, because of the good agreement, regardless of the range of frequencies.

To better understand the influence of frequencies on modelling uncertainties, the first 7 encounter frequencies (0.59, 0.66, 0.73, 0.84, 0.94, 0.98, 1.01 rad/s) were selected and the estimates of the total differences are plotted in Fig. 10 in 3 m regular waves. The total differences of the TD-flex model relative to measurements are 24.3%, 13.2%, and 8.9%, respectively for heave, pitch and VBM. The estimations of FD and TD-rigid models for heave and pitch motions are similar. However, the values for the VBM are reduced from 23.9%, and 24.1%, to 15.2% and 16.4% respectively. It can be concluded that the TD-flex model is capable of predicting the additional peaks of response amplitude at higher frequencies corresponding to the elastic responses, so the total difference relative to measurements is not affected by the range of frequencies.



















Fig. 9. Heave, pitch and VBM transfer functions amidships an ultra-large containership obtained by different models in 3 m (see sub plots a,c,e) and 8 m (see sub plots b,d,f) in regular waves.



Fig. 10. The total difference in transfer functions of an ultra-large containership for in heave, pitch and VBM at amidships obtained by numerical methods in 3 m regular waves with only 7 frequencies.

6. Conclusions

The influence of wave loads on ship springing, whipping and sloshing induced responses has been studied for more than 40 years. Over this period the theory of hydroelasticity of ships, computational tools, hardware for use in FSM and experimental methods developed significantly. Notwithstanding this, conclusive understanding of the influence of hydroelastic uncertainties on wave induced loads remains scarce.

In general the lack of conclusiveness could be attributed to theoretical assumptions associated with structural and fluid dynamics and their coupling, difficulties in mathematical programming and processing of big data, computing speed, wrongly applied or immature experimental methods and limited collaboration across industry and academia. In general, 3D structural models provide very detailed information about stress distribution but beam models have proven adequate to represent the global hull response to whipping and slamming. Linear frequency domain methods reached a stage of maturity and time domain potential methods are emerging. CFD methods are still under development. Notwithstanding this so far they helped qualify the importance of strong coupling in modelling hull girder loads and responses. Methods to quantify model uncertainty of rigid models can be used to quantify the uncertainty in hydroelastic response, as described by RAOs.

Whereas hydroelastic methods for use in design development and assessment become increasingly useful, challenges in realizing and modelling uncertainties can be attributed to: (1) the limitations of numerical methods to suitably model nonlinearities; (2) the ambiguity of model tests; and (3) the systematic use of data emerging from computational, model- or full-scale methods. The review presented in this paper shows that engineering confidence is limited to the linear and symmetric hydroelastic response problem. This is more of an empirical observation rather than conclusion on the basis that the limitations of linear structural mechanics and potential flow hydrodynamic methods could be considered somehow mature within the context of classic marine hydromechanics.

Comparisons between FSM, model experiments and computational approaches show that impulse response problems, antisymmetric ship dynamics and extreme events is ongoing. This is the reason why new methods and benchmarks should be considered an essential step forward in terms of quantifying uncertainties. It is essential to improve our understanding associated with ship science concepts of relevance to the hydroelastic behaviour of ships prone to features that amplify nonlinear responses in stochastic seaways (e.g. slenderness, large deck openings, large bow flare, wide stern, medium to high speed, etc.). The accuracy and efficiency of computational and experimental methods employed should be considered critical especially considering that the curiosity of researchers is continuously challenged by uncertainties associated with the rapid development of modern technology (e.g. instrumentation, multiphysics methods, the emergence of artificial intelligence methods).

While technology develops and matures, a comprehensive uncertainty analysis could remain useful within the context of de-risking experimental methods. In addition to this, the development and implementation of advanced reliability methods for the evaluation of load or strength under extreme events is essential. This is because reliability analysis takes rationally into account various uncertainties in loading, response and structural strength by random variables, each with an associated distribution type and parameter.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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